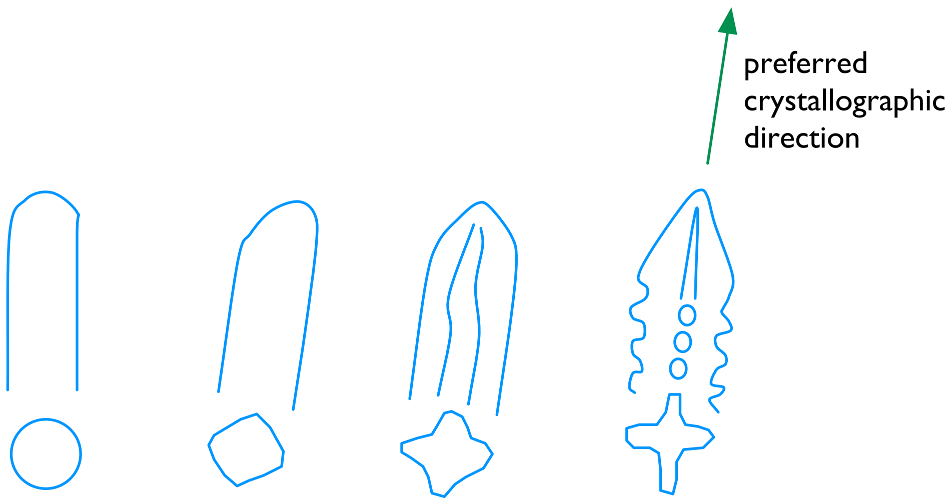


Cellular and Dendritic Solidification

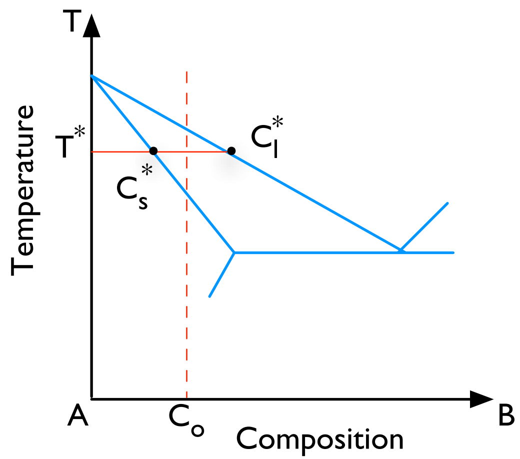
Topic 7

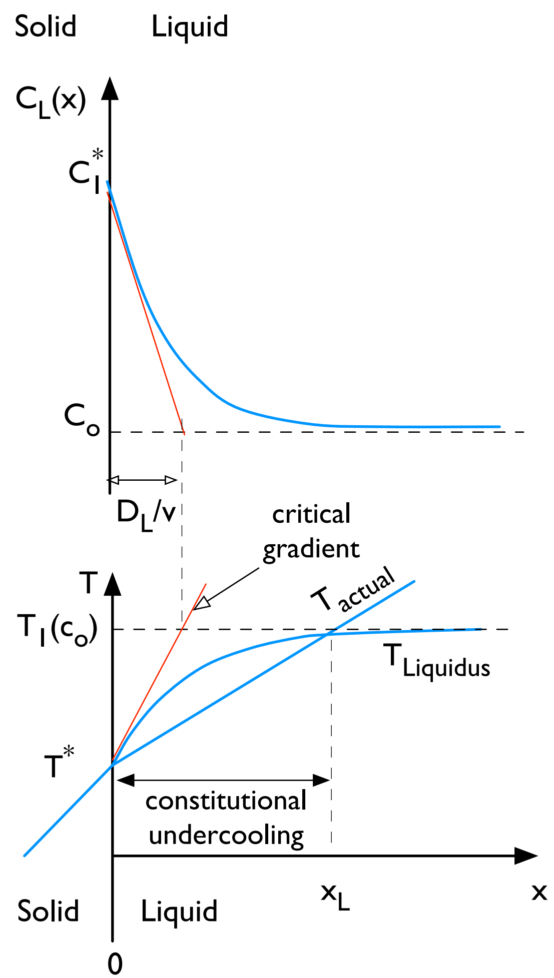
M.S Darwish

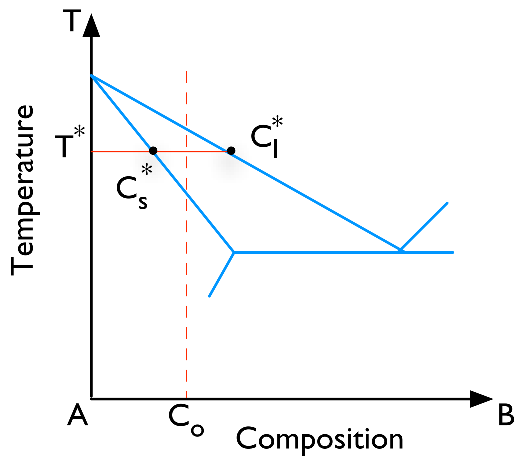
MECH 636: Solidification Modelling

Cellular and Dendritic Solidification

Constitutional Undercooling





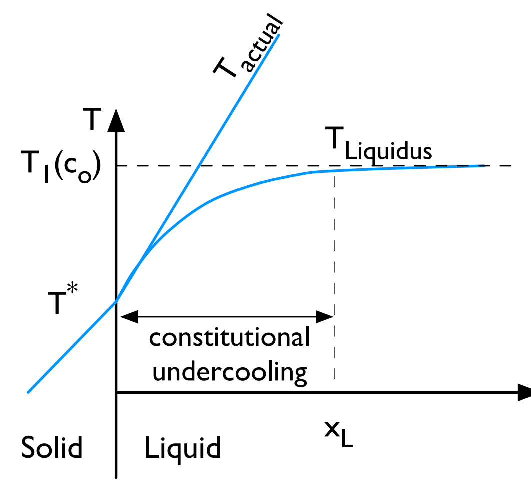


Gradient of Solute in the liquid at the interface

$$\left(\frac{dC_l}{dx'}\right)_{x'=0} = -\frac{v}{D_l} C_l^*(1-k)$$

Gradient of Solute in the liquid at the interface

$$\left(\frac{dT_{liquidus}}{dx'}\right)_{x'=0} = k_{liquidus}\left(\frac{dC_l}{dx'}\right)_{x'=0}$$



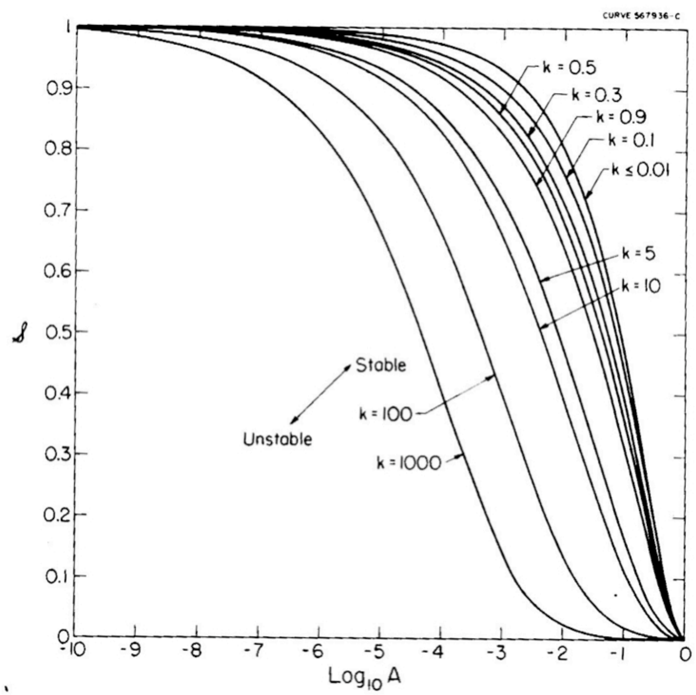
if the temperature gradient in the liquid at the interface is equal or greater that

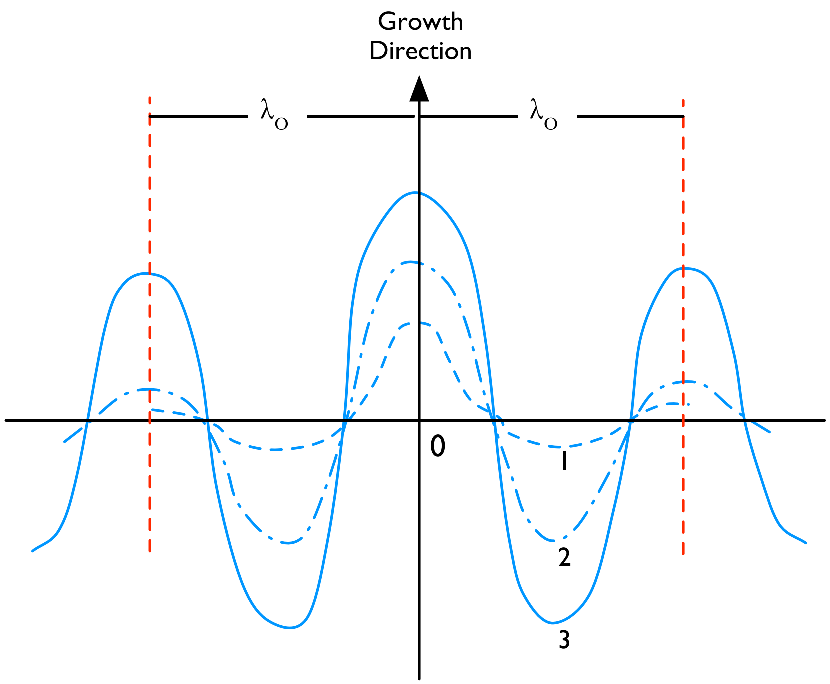
$$\left(\frac{dT_l}{dx'}\right)_{x'=0} \geq \left(\frac{dT_{liquidus}}{dx'}\right)_{x'=0} \quad no\ supercooling$$

$$\frac{\left(\frac{dT_l}{dx'}\right)_{x'=0}}{v} \geq -\frac{k_{liquidus} C_s^*}{kD_l}(1-k) \quad no\ supercooling \qquad \left(\frac{dT_l}{dx'}\right)_{x'=0} \geq \frac{T_1 - T_3}{D/v}$$

Instability Theory

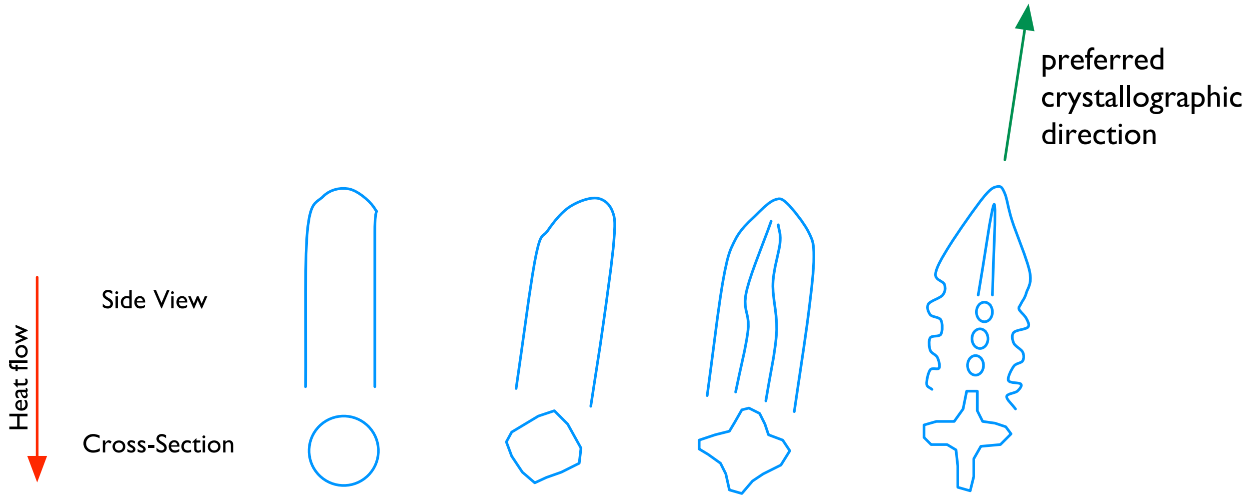
$$\frac{\left(\frac{dT_l}{dx'}\right)_{x'=0}}{v} + \frac{\rho_l \Delta H_f}{2K_l} \geq -\frac{k_{liquidus} C_s^*(1-k)}{kD_l}\,\frac{K_l + K_s}{2K_l}\,\mathcal{S}$$

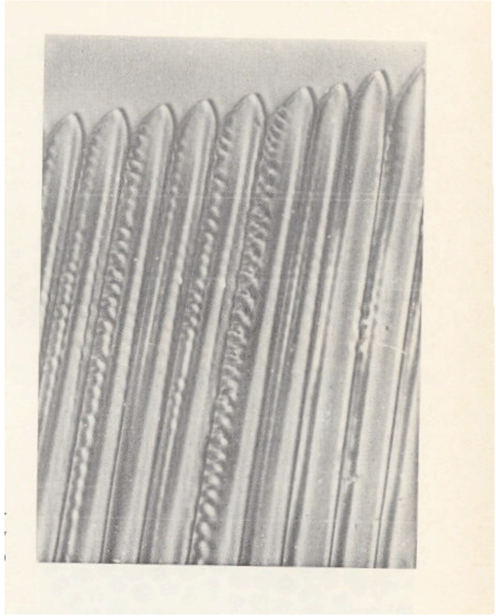




Formation of Dendrites

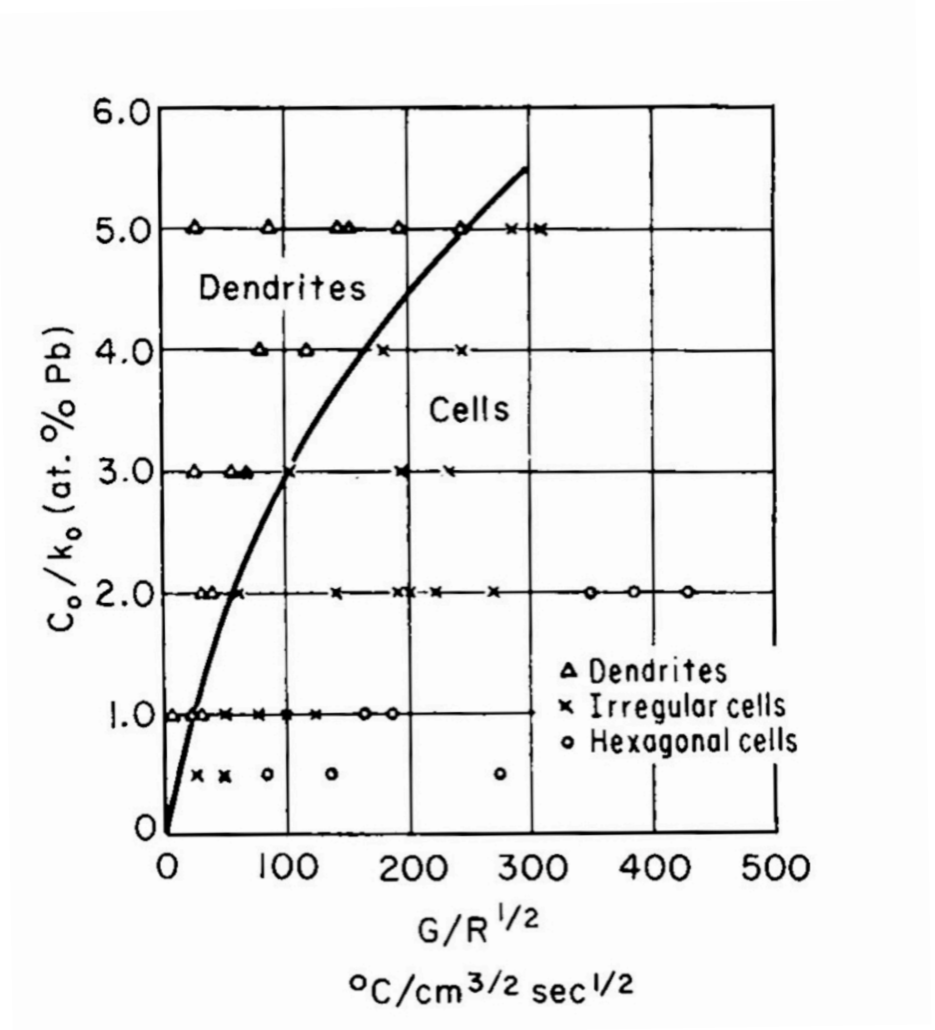
When regular cells form and grow at relatively low rates, they grow perpendicular to the liquid-solid interface regardless of crystal orientation.





When the growth rate is increased crystallography effects begin to exert an influence and the cell growth direction deviates toward the preferred crystallographic growth direction. Simultaneously the cross section of the cell generally beings to deviate from its previously circular geometry owing to the effects of crystallography

Cell to Dendrite Transition



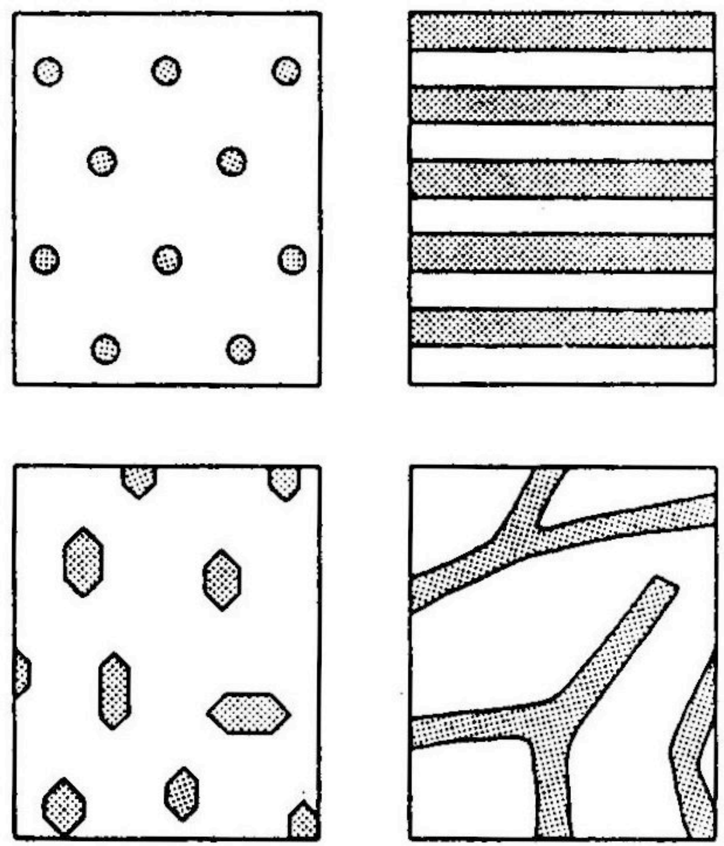
Eutectic Growth

Regular and Irregular Eutectics

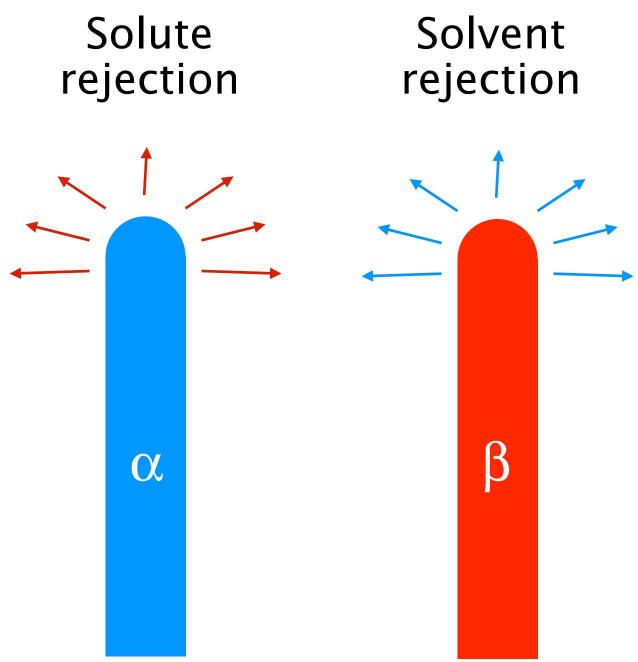
small VF large VF

low entropy

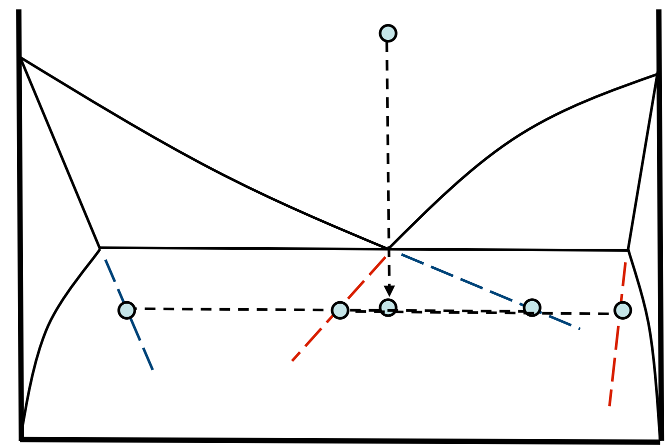
High entropy

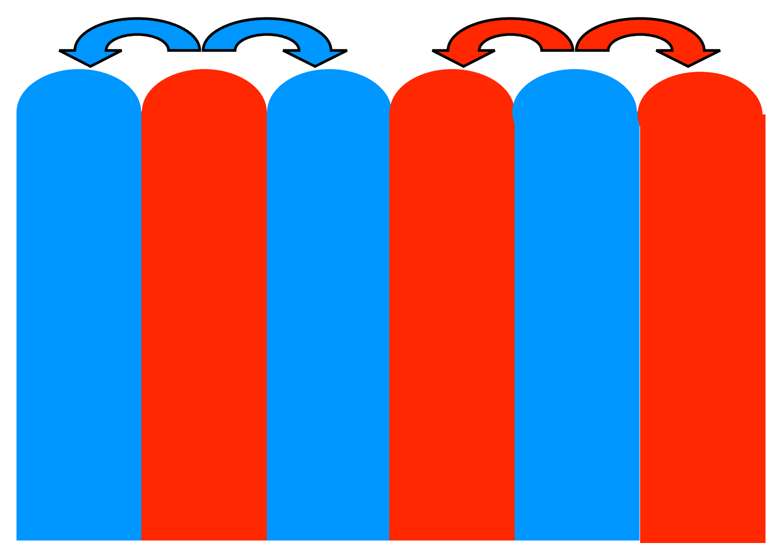


Coupled Growth

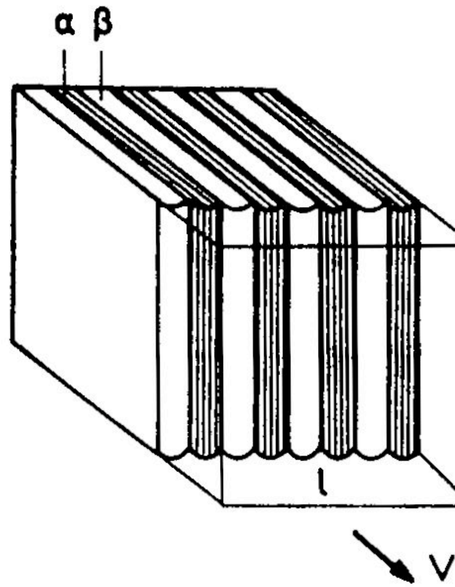


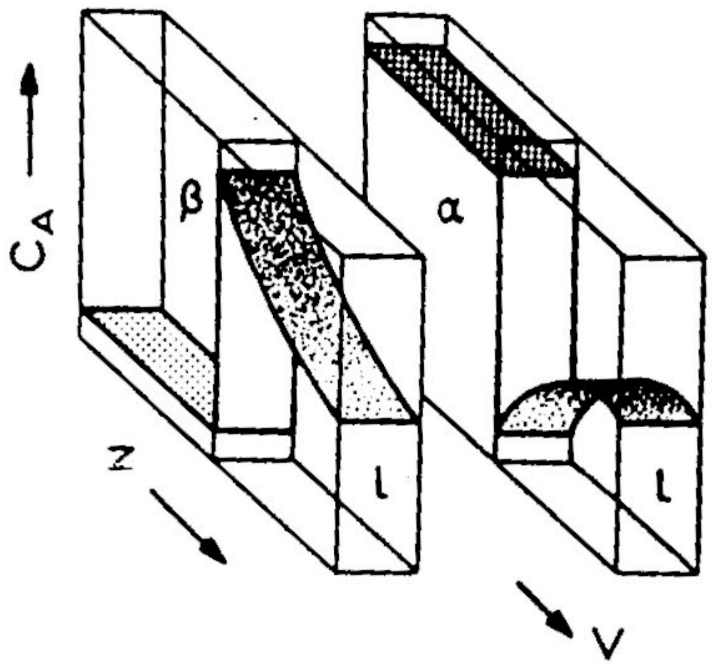
Growth of plates depends on lateral diffusion



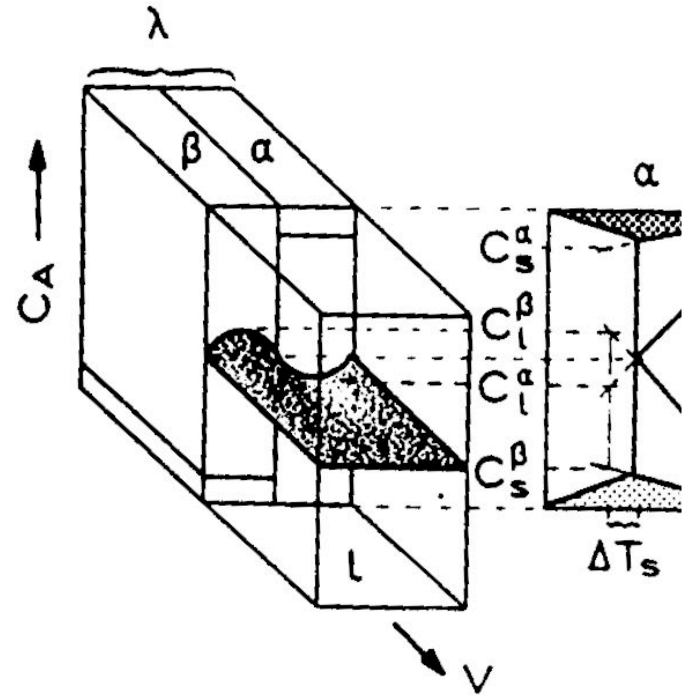


Eutectic Diffusion Field





Boundary Layer



Lateral Mixing

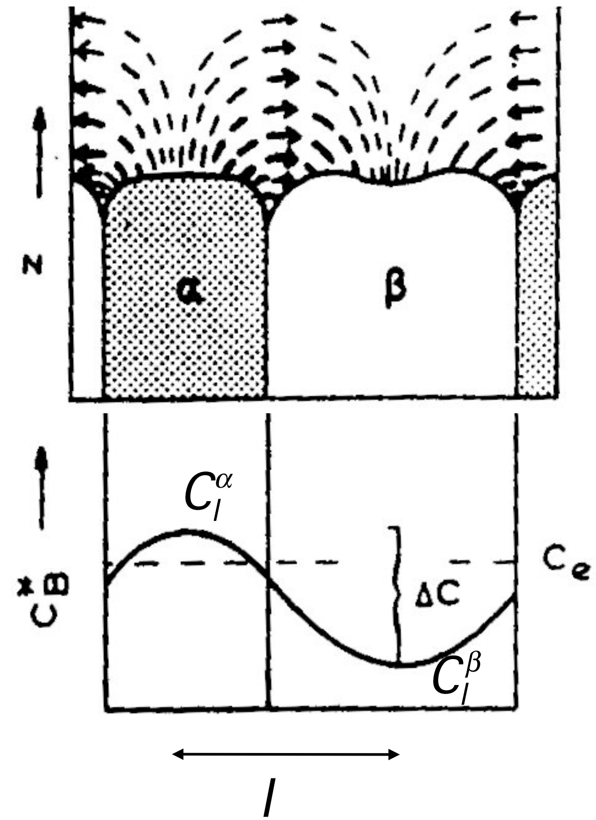
Inter-lamellar Spacing

$$\Delta G(\lambda) = -\Delta G_{\lambda\to\infty} + \frac{2\gamma_{\alpha\beta}V_m}{\lambda}$$

$$\Delta G_{\lambda\to\infty} = \frac{L_V \Delta T_0}{T_{eut}}$$

setting ΔG to 0 $$\lambda^* = \frac{2\gamma_{\alpha\beta}V_m T_{eut}}{L_V \Delta T_0}$$

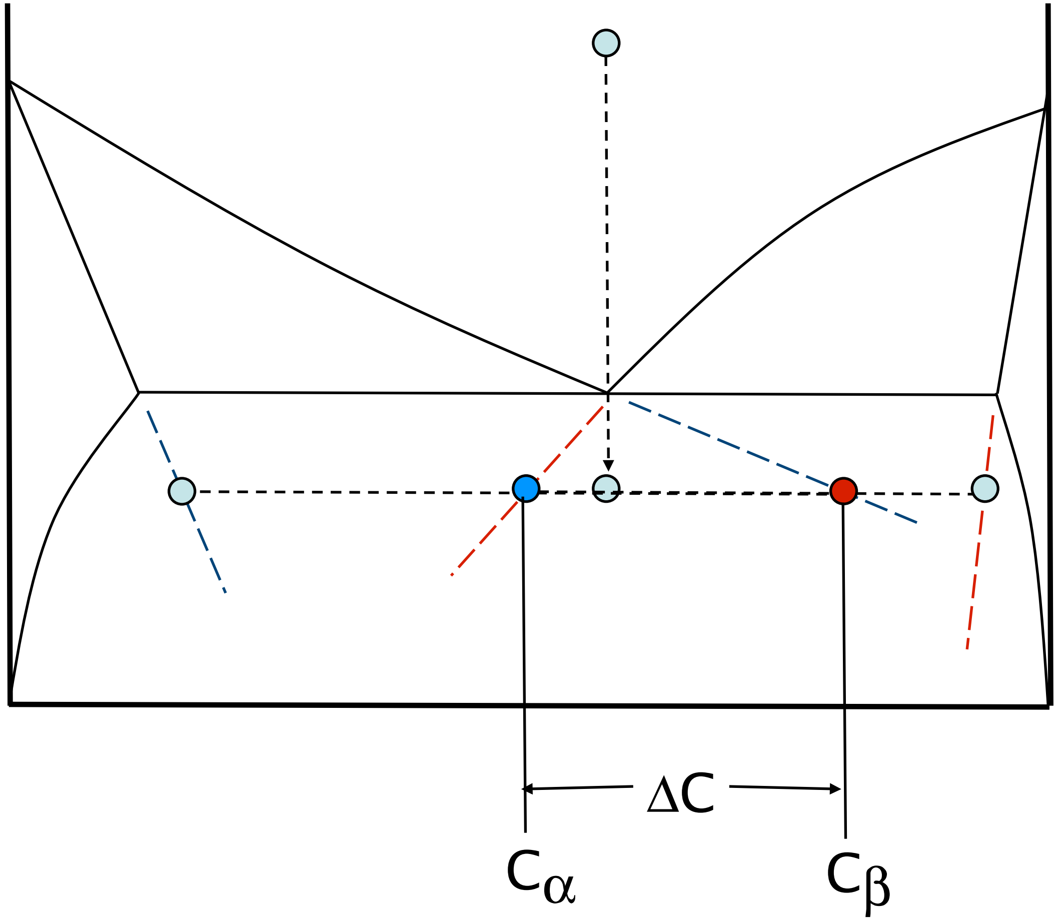
$$\Delta G(\lambda) = \frac{2\gamma_{\alpha\beta}V_m}{\lambda_{\min}}\left(1-\frac{\lambda_{\min}}{\lambda}\right) = -\frac{L_v \Delta T_0}{T_e}\left(1-\frac{\lambda_{\min}}{\lambda}\right)$$
$$= -\Delta G_{\lambda\to\infty}\left(1-\frac{\lambda_{\min}}{\lambda}\right)$$

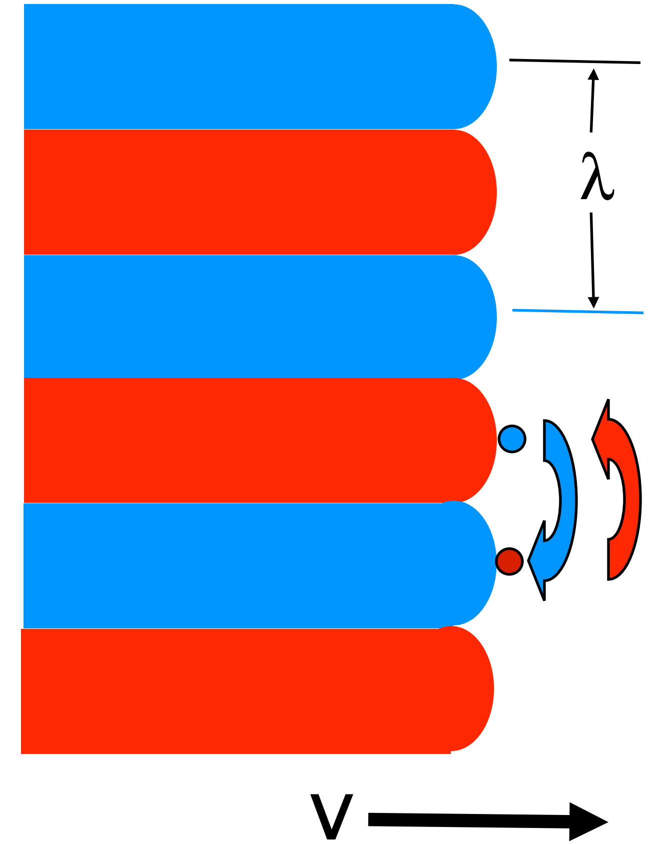


$$When\ \lambda = \lambda_\infty \Rightarrow \Delta G_\lambda\ is\ \max imum \Rightarrow (C_\alpha - C_\beta) = \Delta C_l\ is\ \max imum$$

$$When\ \lambda = \lambda_\infty \Rightarrow \Delta G_\lambda\ is\ \max imum \Rightarrow (C_\alpha - C_\beta) = \Delta C\ is\ \max imum$$

$$Assume\ \Delta C_l \propto \Delta G_\lambda \quad \Delta C_l = \Delta C_{l\,\max}\left(1-\frac{\lambda_{\min}}{\lambda}\right)$$



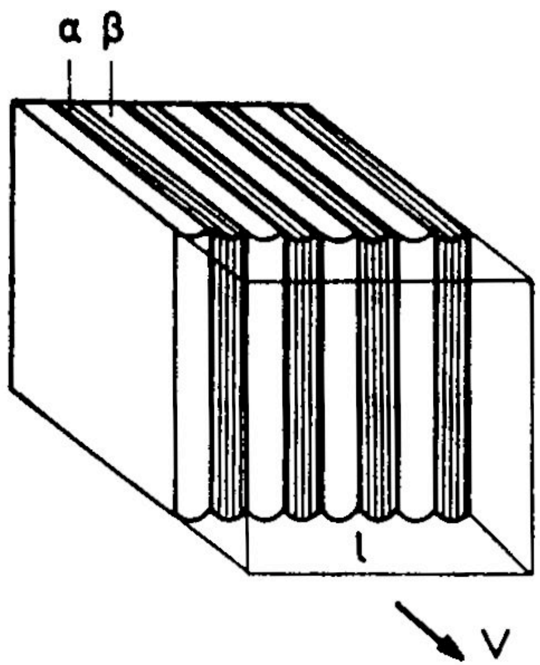


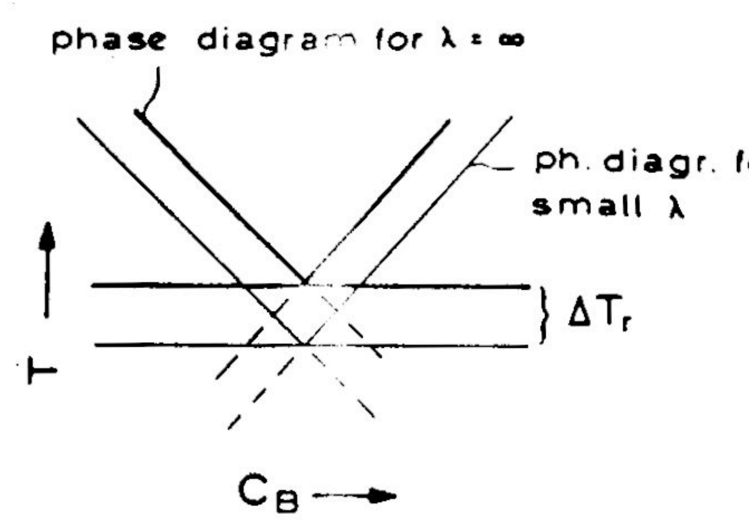
$$v \propto D\left(\frac{dC}{dx}\right) = \frac{2D}{\lambda}\left(C_\beta - C_\alpha\right) = \frac{2D}{\lambda}\Delta T_C\left(\frac{1}{m_\alpha} - \frac{1}{m_\beta}\right)$$

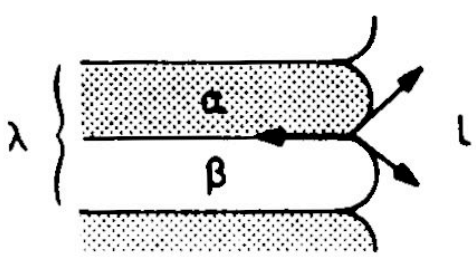
$$\Delta T_C = \frac{\lambda v}{2D}\left(\frac{m_\alpha m_\beta}{m_\beta - m_\alpha}\right) = K_c \lambda v$$

$$v \propto D\left(\frac{dC}{dx}\right) = \frac{2D}{\lambda}\left(C_\beta - C_\alpha\right) = \frac{2D}{\lambda}\Delta T_C\left(\frac{1}{m_\alpha} - \frac{1}{m_\beta}\right)$$

Curvature Effects







$$\Delta T_r = \Gamma \kappa = \frac{K_r}{\lambda}$$

Interface Temperature

$$\Delta T_r = \Gamma\kappa = \frac{K_r}{\lambda}$$

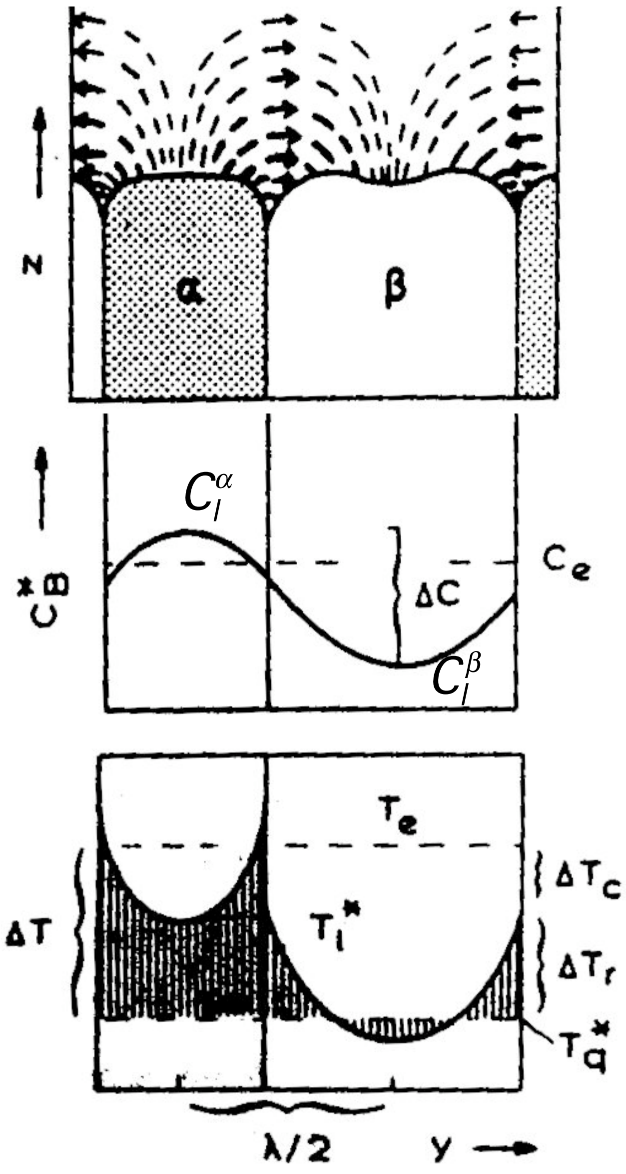
$$\Delta T_C = K_c \lambda v$$

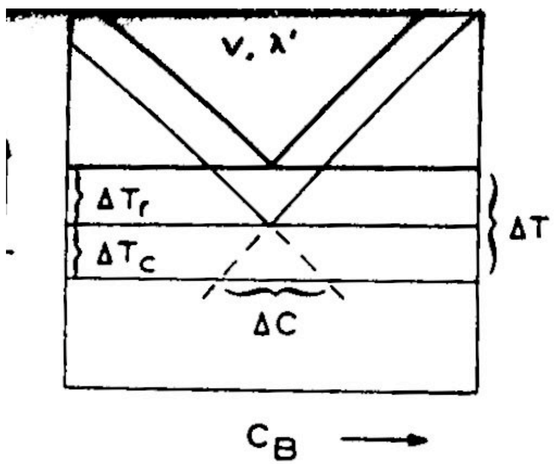
$$\Delta T = K_c \lambda v + \frac{K_r}{\lambda}$$

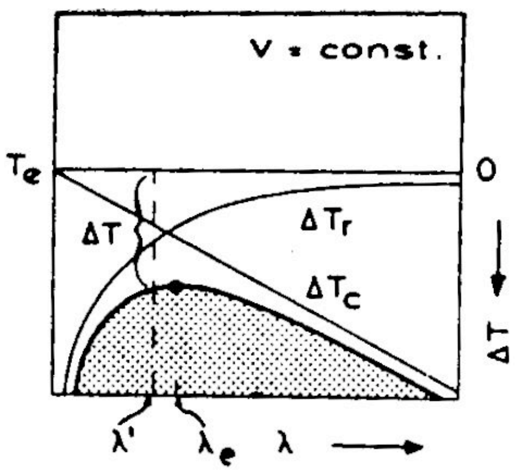
$$\frac{d(\Delta T)}{d\lambda} = 0 \qquad \frac{d(\Delta G)}{d\lambda} = 0$$

$$\lambda^2 v = \frac{K_r}{K_C} \qquad \frac{\Delta T}{\sqrt{v}} = 2\sqrt{K_r K_C}$$

$$\Delta T \lambda = 2K_r$$







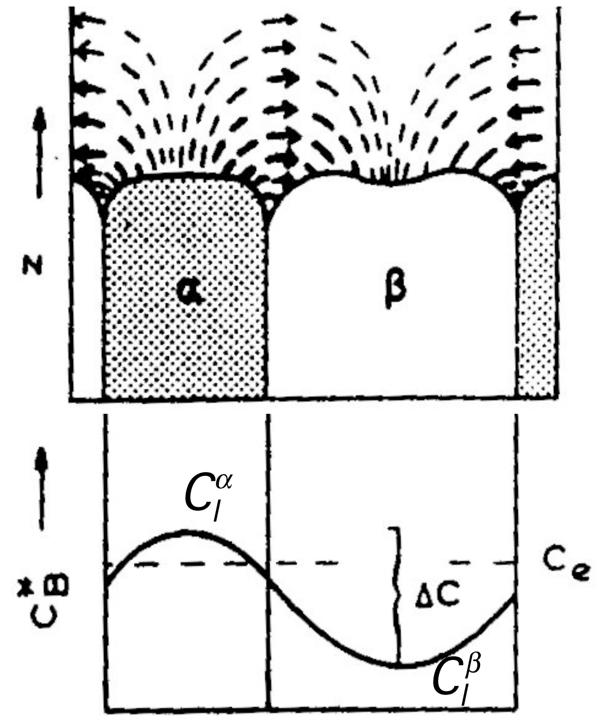
Interface Velocity

$$J_t = \frac{2D\Delta C}{\lambda}$$

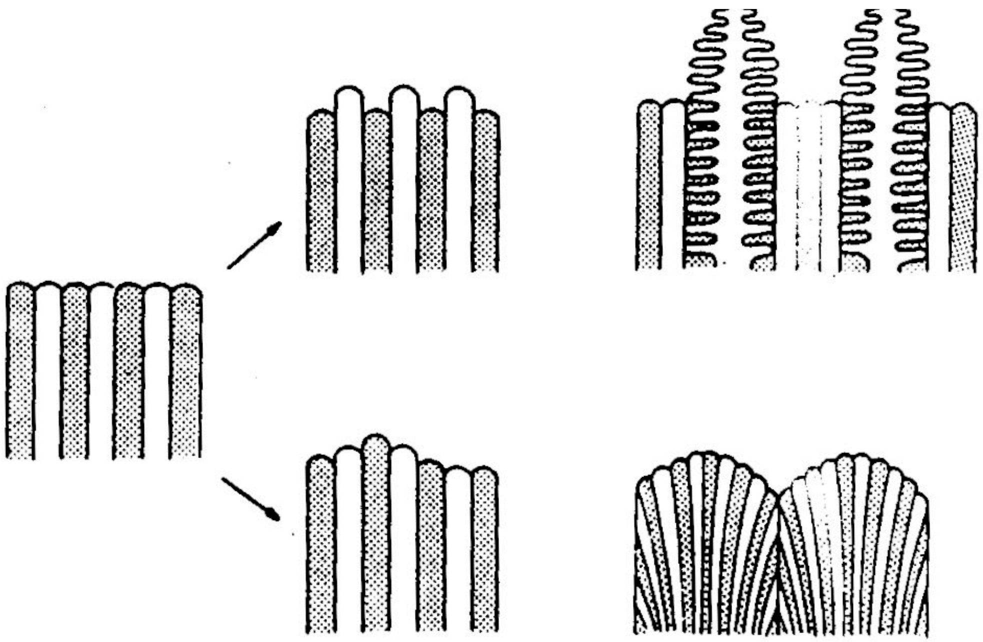
under steady state $J_t = J_r$

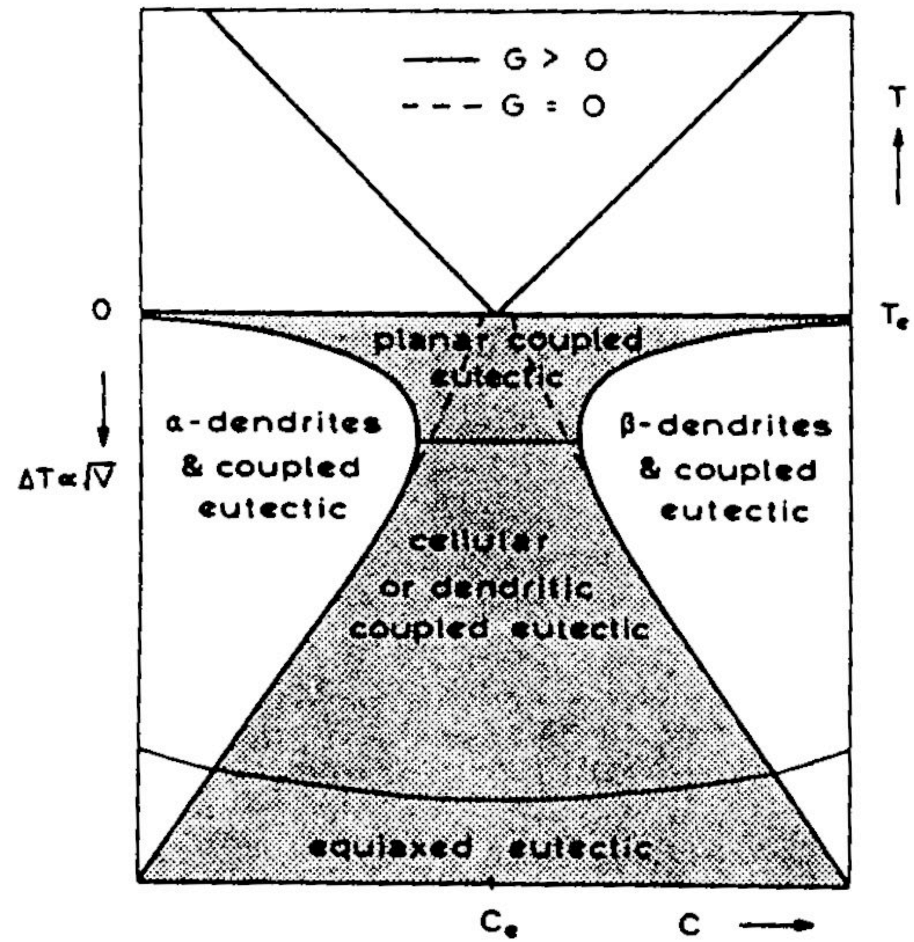
$$J_r = vC_{eut}(1-k)$$

$$\frac{\lambda v}{2D} = \frac{\Delta C}{C_{eut}(1-k)}$$



Competitive Growth of Dendritic and Eutectic Phases





G > O
G = O
T
O
T_e
planar coupled eutectic
α-dendrites & coupled eutectic
β-dendrites & coupled eutectic
$\Delta T \propto \sqrt{V}$
cellular or dendritic coupled eutectic
equiaxed eutectic
C_e
C