Pergamon

Renewable Energy, Vol. 4, No. 1, pp. 53–58, 1994
Elsevier Science Ltd
Printed in Great Britain.
0960–1481/94 \$6.00+0.00

THE EFFECT OF PLANET THERMAL CONDUCTANCE ON CONVERSION OF SOLAR ENERGY INTO WIND ENERGY

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(*Received* 26 *January* 1993; *accepted* 10 *May* 1993)

Abstract—Endoreversible thermodynamics are used for studying the influence of a planet's thermal conductance on conversion of solar energy into wind energy. Results indicate a strong dependence of the rate of wind energy generated on the amount of heat leaking by conduction from the illuminated side to the dark side of the planet. The upper boundary for the conversion efficiency of solar energy into wind energy derived in DeVos and Flater [*Am. J. Phys.* **59**(8), 251–254 (1991)] is found to be well above the actual value calculated here. Furthermore, an upper limit for the planet's dimensionless thermal conductance, a function only of the thermal conversion efficiency, is also determined.

1. INTRODUCTION

The maximum conversion efficiency of solar energy into wind energy on earth and other planets lately has been the subject of several investigations. DeVos and Flater [1] applied a model first put forward by Gordon and Zarmi [2] in a simple manner that leads to universal results. The basic idea of the model, described mathematically in detail in the next section, consists of replacing any planet by a heat engine the illuminated and dark sides of which represent its hot and cold reservoirs, respectively. Energy conversion within the engine is assumed to take place reversibly, but the heat transported to and out of the engine, which occurs by a thermal radiation mechanism, is a source of irreversibility (endoreversible heat engine [3–5]). The work output of the engine is the amount of wind energy generated.

By applying the first and second laws of thermodynamics along with the Carnot efficiency to the heat engine, DeVos and Flater [1] derived an upper bound for the conversion efficiency of solar energy into wind energy. The computed value was obtained by neglecting the heat transported by conduction from the hot side to the cold side of the planet. This assumption does not affect the derived upper bound for energy conversion, since the effect of conduction heat transfer is to reduce the thermal efficiency of the system and consequently the amount of solar energy converted into wind energy. However, this decrease may be large in situations when the thermal resistance of the surface of the planet and its envelope is relatively small, leading to actual conversion rates well below the given upper bound. Heat is conducted through the Earth, for example, at an extremely slow rate. At a few meters below the ground, daily temperature variations are hardly felt [6]. The same should hold for other planets and as such, heat conduction is only a surface effect. Furthermore, heat is also conducted from the lit side to the dark side of the planet through its surrounding atmosphere. Therefore, it is the combined influence of conduction through the surface of the planet and through the atmosphere surrounding the planet which affects wind energy generation. It should be pointed out here that this conduction phenomenon is not steady state conduction through the planet but transient heat flow in and out of the surface of the planet as it rotates. In effect this is heat convection from the hot to the cold side of the planet.

To this end, the aim of this paper is to investigate, by incorporating a heat-leak term into the model described in [1], the effect of conduction heat transfer on the efficiency of wind energy generation on Earth and other planets.

2. THE BASIC MODEL

In the model described by DeVos and Flater [1] and schematically shown in Fig. 1, the illuminated side of a planet is considered to be a heat reservoir at temperature T_1 and the dark side a reservoir at temperature T_2. The bright side receives energy from the sun which is at temperature T_s by thermal radiation through a view factor f. For our planetary system f is in the range of 10^4–10^8 [1]. The rest of the hemi-

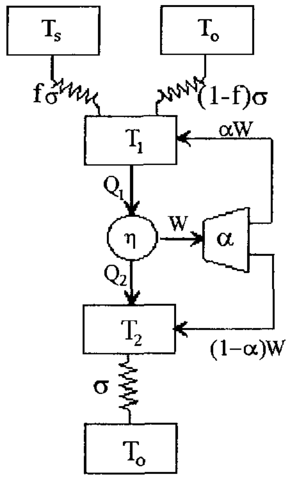

Fig. 1. Basic model for the conversion of solar energy into wind energy.

sphere, as viewed by the lit side, is a free space at temperature T_0 and its geometric view factor is $1-f$, so that the lightened side of the planet accepts a power $f\sigma T_s^4$ from the sun and emits a power $f\sigma T_1^4$ towards the sun. The planet reflects some of the radiation in the visible spectrum (albedo factor) and re-emits as well in the infra-red range (greenhouse factor). If these two factors are ρ and γ, respectively, then the net power incident on the illuminated side is $f\sigma[(1-\rho)T_s^4-(1-\gamma)T_1^4]$. Additionally, the planet exchanges $(1-f)\sigma[(1-\mu)T_0^4-(1-\gamma)T_1^4]$ with the rest of the sky; μ being the microwave reflectance of the planet. The total input power is therefore:

$$Q_1 = \sigma\{f[(1-\rho)T_s^4-(1-\gamma)T_1^4]+(1-f)[(1-\mu)T_0^4 -(1-\gamma)T_1^4]\}. \quad (1)$$

In a similar fashion the dark side of the planet loses a power:

$$Q_2 = \sigma[(1-\gamma)T_2^4-(1-\mu)T_0^4]. \quad (2)$$

Following DeVos and Flater [1], the dark side of the atmosphere is assumed to be at $T_0 \equiv 0$ K. This simplification is justified by noting that $T_1^4 \gg T_0^4$ and $T_2^4 \gg T_0^4$ which are accordingly true if T_1 and T_2 are greater than 10 K. With this assumption, the previous two equations simplify to:

$$Q_1 = \sigma[(1-\rho)fT_s^4-(1-\gamma)T_1^4] \quad (3)$$

and

$$Q_2 = \sigma(1-\gamma)T_2^4. \quad (4)$$

The work generated (wind) is actually dissipated over the planet via mechanisms such as internal friction and frictional interactions with the planet's surface and as such, it represents an additional source of heat that should be accounted for in the model. This is done by introducing a dissipation factor (α) that partitions the available dissipated work between the lit and dark sides (Fig. 1). With this modification, the input and output powers become:

$$Q_1 = \sigma[(1-\rho)fT_s^4-(1-\gamma)T_1^4]+\alpha W \quad (5)$$

$$Q_2 = \sigma(1-\gamma)T_2^4-(1-\alpha)W. \quad (6)$$

Maximum work will be extracted if a reversible engine connects the two heat reservoirs. Assuming this, the Kelvin relationship (reversibility condition), the first law of thermodynamics, and the Carnot efficiency of the heat engine can be written as,

$$Q_1/T_1 = Q_2/T_2. \quad (7)$$

$$Q_2 = Q_1 - W. \quad (8)$$

$$\eta = W/Q_1. \quad (9)$$

By combining eq. (9) with the reversibility condition [eq. (7)] and the first law [eq. (8)], the efficiency equation transforms to:

$$\eta = 1-(T_2/T_1). \quad (10)$$

At this stage, eqs (7), (8) and (10) may be solved for T_1, T_2 and W to obtain:

$$T_1 = \frac{1}{[1+(1-\eta)^4]^{1/4}}\frac{(1-\rho)^{1/4}}{(1-\gamma)^{1/4}}f^{1/4}T_s, \quad (11)$$

$$T_2 = \frac{1-\eta}{[1+(1-\eta)^4]^{1/4}}\frac{(1-\rho)^{1/4}}{(1-\gamma)^{1/4}}f^{1/4}T_s, \quad (12)$$

and

$$W = \frac{\eta(1-\eta)^4}{(1-\alpha\eta)[1+(1-\eta)^4]}(1-\rho)\sigma f T_s^4. \quad (13)$$

From eq. (13) it is seen that for constant values of (α) the wind generated over a planet depends only on the conversion efficiency. Therefore, its maximum value is found by taking $\mathrm{d}W/\mathrm{d}\eta$ and setting it equal to zero. This results in:

$$\eta^4-4\eta^3+6\eta^2-8\eta+2+\frac{(\alpha-1)\eta}{1-\alpha\eta}[1+(1-\eta)^4] = 0 \quad (14)$$

where η is now η_{max}.

The upper limit on maximum work will be obtained when $\alpha = 1$ (all wind energy is dissipated on the lit side) and may be checked by plotting $W = f(\eta)$ for different values of (α). This leads to:

$$\eta^4-4\eta^3+6\eta^2-8\eta+2 = 0, \quad (15)$$

which has the solution $\eta = 0.307$. This gives an upper limit of 8.3% on solar energy converted into wind energy. Thus,

$$W = 0.083(1-\rho)\sigma f T_s^4. \tag{16}$$

For other values of α the generated wind energy will be less.

3. THE EXTENDED MODEL

The basic model is extended in this section by embodying into the endoreversible heat engine of DeVos and Flater [1] a heat-leak term ($K(T_1-T_2)$, K being the planet's thermal conductance per unit area) accounting for the heat transferred by conduction from the hot side to the cold side of the planet (Fig. 2). It is to be noted here that DeVos and Flater [1] recognized the importance of including such a term to the model, but in view of the algebraic complexity involved, elected to tackle a simpler problem by neglecting heat transfer effects.

With the same solar energy input and the same simplifying assumptions as before, the power input and output are given by the following two equations:

$$Q_1 = \sigma[(1-\rho)fT_s^4-(1-\gamma)T_1^4]-K(T_1-T_2)+\alpha W, \tag{17}$$

$$Q_2 = \sigma(1-\gamma)T_2^4-K(T_1-T_2)-(1-\alpha)W. \tag{18}$$

Expressions for (T_1) and (T_2) as functions of the various parameters involved are obtained by substituting (Q_1) and (Q_2) given by eqs (17) and (18) in the first law and combining the resulting equation with the carnot efficiency relation. It turns out that

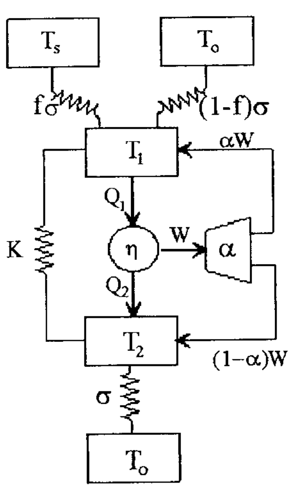

Fig. 2. Extended model for the conversion of solar energy into wind energy.

the equations for T_1 and T_2 are the ones found using the basic model. This is to be expected since the extra heat transfer terms drop out when Q_1 and Q_2 are inserted into the first law. Now, if the expressions for T_1 and T_2 and the Carnot relation are used in the reversibility condition an equation for W is found and is given by:

$$\frac{W}{(1-\rho)f\sigma T_s^4} = \frac{\eta(1-\eta)^4}{(1-\alpha\eta)[1+(1-\eta)^4]} - A\frac{\eta^2}{(1-\alpha\eta)[1+(1-\eta)^4]^{1/4}}. \tag{19}$$

Where,

$$A = \frac{K}{(1-\gamma)^{1/4}(1-\rho)^{3/4}\sigma f^{3/4}T_s^3} \tag{20}$$

is the dimensionless thermal conductance of the planet. It is obvious from eq. (19) that maximum values for W will occur when $A = 0$ which in turn means $K = 0$. If this is the case then the above equation reduces to the case of pure dissipation with no heat-leak. Thus, eq. (19) is the general form of eq. (13).

For maximum work the derivative of eq. (19) with respect to η is taken and set equal to zero; the efficiency at that condition is given by:

$$\eta^4-4\eta^3+6\eta^2-8\eta+2+\frac{(\alpha-1)\eta}{(1-\alpha\eta)}[1+(1-\eta)^4] - A\frac{\eta[1+(1-\eta)^4]^{3/4}}{(1-\alpha\eta)(1-\eta)^3}\{[1+(1-\eta)^4] + (1-\alpha\eta)[1+(1-\eta)^3]\} = 0. \tag{21}$$

By comparison, it is evident that eq. (21) is also a generalization of eq. (14). In addition, as seen from eq. (19), maximum work occurs when $\alpha = 1$ (all wind energy is dissipated to the illuminated side of the planet). For this case, we have:

$$(\eta^4-4\eta^3+6\eta^2-8\eta+2) - A\frac{\eta[1+(1-\eta)^4]^{3/4}\{2[1+(1-\eta)^4]-\eta\}}{(1-\eta)^4} = 0. \tag{22}$$

Noting that work is always a positive quantity, eq. (19) shows:

$$A \leqslant \frac{(1-\eta)^4}{\eta[1+(1-\eta)^4]^{3/4}} \tag{23}$$

which is effectively an implicit upper-bound on the dimensionless thermal conductance of a planet, function only of its thermal efficiency.

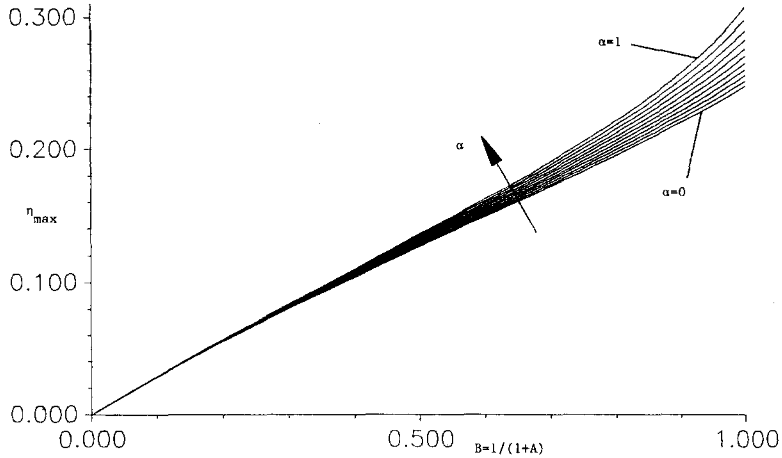

Fig. 3. Variations of maximum thermal efficiency with planet's thermal conductance and dissipation rate.

4. RESULTS AND DISCUSSION

The equations derived in the previous section are analyzed here and the effects of the planet's thermal conductance on wind energy generation are discussed.

The maximum conversion efficiency, as given by eq. (21), is function of the dissipation factor (α) ($0 \leqslant \alpha \leqslant 1$) and the dimensionless thermal conductance A ($0 \leqslant A \leqslant \infty$). In order for the variation of the parameter representing the combined conduction effect to be over a finite interval, η_{max} is plotted as a function of B ($B = 1/(1+A)$, $0 \leqslant B \leqslant 1$, B is a form of the planet thermal resistance) for different values of (α). As shown in Fig. 3, η_{max} increases with increasing α and attains its highest values at $\alpha = 1$. This is to be expected since for this case, the wind energy generated will be dissipated on the illuminated side of the planet causing an increase in T_1 and consequently in η_{max} [eq. (10)]. On the other hand, the clear effect of the planet's thermal resistance, as depicted in Fig. 3, is a decrease in the maximum conversion efficiency with the amount of this decrease being dependent upon the magnitude of A. In the limit as $A \to \infty$ (i.e. $B \to 0$), $\eta \to 0$. This behavior is anticipated since heat transferred by conduction decreases the difference in temperature between the hot and cold sides of the planet by cooling the first down and heating the second up. Furthermore, the upper limit of 0.307 derived in [1] is well above the actual upper limits displayed in Fig. 3 as a function of the planet's thermal resistance and

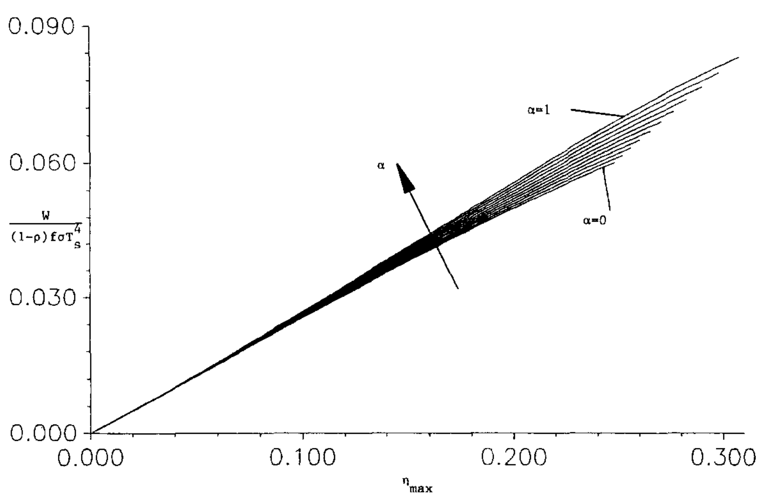

Fig. 4. Variation of maximum work with maximum thermal efficiency and dissipation rate.

dissipation rate. In fact, Fig. 3 may be seen as a generalization of the value found by DeVos and Flater [1]. In addition, the sensitivity of η_{max} to variation in α decreases as A increases. This decrease is caused by conduction heat transfer, which, as A increases, assumes a higher role in controlling the redistribution of the available thermal energy between the two sides of the planet. Thus, if a large portion of W is dissipated over the hot side then more energy will be transferred by conduction to the cold side and vice versa.

In Fig. 4, the maximum work generated is plotted as a function of η_{max} for different values of α. In calculating the maximum work, the value of A producing maximum efficiency was used. In general, the trends are similar to those found in Fig. 3 with the value of W_{max} increasing with α and with decreasing values of A and becoming less sensitive to variation in α as A increases for the same reasons explained above. Further, the values given in Fig. 4, may be seen as generalizations of the upper limit of 8.3% reported in [1].

Experimental data for the earth reported in [3], give about 3% for $W/(1-\rho)f\sigma T_s^4$. On the assumption that wind is generated with maximum efficiency, the derivations presented here produce a value of 11.5% for η. The corresponding values for T_1 (eq. 11), T_2 (eq. 12), and A (eq. 19) are 350 K, 310 K, and 1.32, respectively. The values for T_1 and T_2 are close to the actual figures of 290 K and 195 K reported in [2] and which are supposed to represent the values on the surface of the Earth (T_1) and at an altitude of about 80 km above the surface of the Earth (T_2). Knowing the values of γ, ρ, σ, f, T_s and A to be 0.35, 0.35, 5.669×10^{-8} W/m^2 K^4, 2.16×10^{-5}, 5800 K and 1.32, respectively, the average thermal conductance of the Earth and its envelope is found to be 3 W/m^2 K approximately. Since 70% of the Earth is formed of water, the value obtained should be of the same order of magnitude as that corresponding to water. Assuming the surface of the Earth to resemble a semi-infinite wall of water, the depth through which heating will be felt over a period of 12 hours is given by

$$\delta = 3.76(\alpha\tau)^{1/2}, \tag{24}$$

where α is the thermal diffusivity of water and τ is time [7]. A value of 0.3026 m is obtained for δ. Thus, the corresponding thermal conductance of water is $(k_{water}/\delta) = 2.058$ W/m^2 K. This value is very close and of the same order of magnitude as the one obtained. As a second check for the validity of the model, the value of the surface thermal conductivity of the Earth as reported in [8] is about 3.5 W/m K. Assuming temperature variations in the ground are felt through a depth of 1 m [6], the thermal conductance of the surface of the Earth participating in the energy conversion process is 3.5 W/m^2 K. The use of 1 m depth (nearly three times the depth obtained when assuming the surface of the Earth to resemble a semi-infinite wall of water) is justified by the fact that the actual thermal conductivity of the Earth is higher than that of water due to constituents of high thermal conductivity and as such temperature variation is felt over a greater depth.

A third check on the validity of the model may be obtained by determining an Earth thermal conductance which, in the formula

$$Q = (k/\delta)\Delta T, \tag{25}$$

would give the same total heat flow over a 12-hour

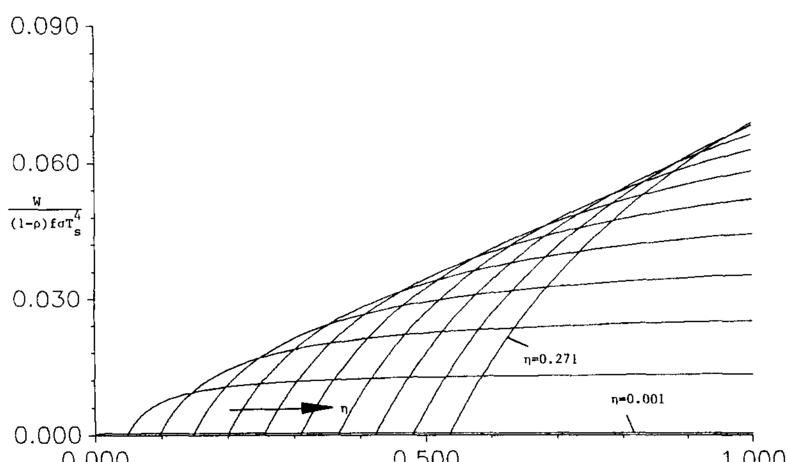

Fig. 5. Variation of work generated with the planet's thermal conductivity and thermal efficiency ($\alpha = 0.5$).

period as the transient heat flow into the Earth's surface. The average solar irradiation per unit area of the Earth's surface is about 223 W/m^2 [2]. The hot and cold sides' average temperatures are 290 K and 195 K, respectively [2]. Using these values in eq. (25), the thermal conductance of the Earth is found to be 2.35 W/m^2 K. Again this value is nearly equal to the value predicted by the model.

In general, it is not necessary for energy conversion to take place at maximum efficiency and as such, the work output may not be a maximum. In fact, for any value of the efficiency A may assume values varying between 0 and A_{max} [eq. (23)]. Therefore, the actual work generated by the heat engine should be computed from eq. (19) with η, α, and A varying between their minimum and maximum values. Such calculations are performed for a value of α of 0.5 and are presented in Fig. 5. In this figure, the variation of the wind energy as function of the planet's thermal resistance is shown along lines of constant η. As depicted, curves start from 0 at the point of minimum thermal resistance to a maximum at maximum thermal resistance.

5. CONCLUSION

The model described by DeVos and Flater [1] is extended by the inclusion of a conductance term that allows heat-leak between the hot and cold reservoirs, and the effects of conduction heat transfer in Earth and other planets on wind energy generation is studied. The analysis undertaken has resulted in an implicit upper bound for the planet's dimensionless thermal conductance and has demonstrated the profound influence of conduction heat transfer on the conversion of solar energy into wind energy. The upper bounds on energy conversion derived in [1] were found to be well above the actual values calculated here.

Acknowledgements—The authors thank Dr N. Ghaddar for her suggestions and valuable comments. This work was supported by the University Research Board of the American University of Beirut under grant 48220.

NOMENCLATURE

- A dimensionless thermal conductance
- B dimensionless thermal resistance
- f view factor—fraction of sun's emitted energy intercepted by the planet
- K thermal conductance (W/m^2 K)
- k thermal conductivity (W/m K)
- Q_1 power gained by the lit side of the planet (W/m^2)
- Q_2 power lost by the dark side of the planet (W/m^2)
- T_1 temperature of the hot reservoir (K)
- T_2 temperature of the cold reservoir (K)
- W work output of heat engine (W/m^2).

Greek symbols

- σ Stefan–Boltzman constant (5.669×10^{-8} W/m^2 K^4)
- γ greenhouse factor—fraction of the longwave (infrared) radiation from the atmosphere returning to the planet
- μ microwave reflectance—fraction of the microwave radiation from the atmosphere returning to the planet
- ρ albedo factor—fraction of incident solar radiation that is reflected back to space
- α dissipation factor
- η thermal efficiency.

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