Solving the Navier-Stokes Equations

Chapter 15

Introduction

- For the solution of the scalar equation we have assumed that the velocity field is known

$$\dot{m} = \rho \mathbf{v} \cdot \mathbf{S}$$

- How do we actually compute the velocity field is the subject of this lecture

Velocity Pressure Coupling

Navier-Stokes Equations

$$\underbrace{\frac{\partial(\rho\phi)}{\partial t}}_{transient\ \ term} + \underbrace{\nabla\cdot(\rho\mathbf{v}\phi)}_{convection\ \ term} = \underbrace{\nabla\cdot(\Gamma\nabla\phi)}_{diffusion\ \ term} + \underbrace{Q}_{source\ \ term}$$

Navier-Stokes equations are a special case of the general scalar equation with $\mathbf{\Phi}$ = 1, u or v and $\mathbf{\Gamma}$=0 or $\boldsymbol{\mu}$ and appropriate Q

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\frac{\partial(\rho\underline{\mathbf{v}})}{\partial t} + \nabla\cdot(\rho\mathbf{v}\underline{\mathbf{v}}) = \nabla\cdot\underline{\tau} - \nabla\underline{p} + \mathbf{B}$$

$$\tau = -\mu(\nabla\mathbf{v} + \nabla\mathbf{v}^T) + \frac{2}{3}\mu\mathbf{I}\nabla\cdot\mathbf{v}$$

NS equations are non-linear

- Not by itself a problem
- Can do Picard iteration

Momentum Equation**s** (x,y,z)

- not a problem
- Can solve each sequentially

Source terms?

- stress tensor sources can be computed

Main Issue is that pressure is not known

- should find a way to compute it
- what PDE to use?
- Special case of incompressible flow

Options

- Option 1
  - Vorticity-Stream function formulation
- Option 2
  - Density Based formulation
- Option 3
  - Use the Momentum equations to compute the velocity field
  - Use the Continuity equation to form a pressure equation to compute the Pressure field
  - This is known as the Primitive Variables Formulation

Option I

- Used in 70's-80's
  - Derive one PDE for stream function Ψ and one PDE for the single component vorticity in 2D
  - Eliminates pressure as variable
  - Only 2 PDE;s as opposed to 3 for primitive variable formulation (u,v,p)
- However no stream function definition in 3D flows
- Vorticity-vector potential formulation in 3D
  - 6 components (3 for vorticity, 3 vector potential)
  - Fewer components for primitive variable formulation
- Difficult to derive bc's

Option 2: Density Methods

- Popular in compressible flow community
- (u,v,w,ρ,E) used as unknowns
- Artificial compressibility/preconditioning schemes
- Effectively add some artificial compressibility to enable use of density-based schemes for incompressible flows
- Widely used in aeronautics industry

Option 3: Pressure Methods

- Will focus on pressure-based methods since they are the most widely used in the incompressible flow community.
- At convergence, the solution satisfies the discrete continuity and momentum equations
- Choice of pressure-based/density-based solution technique only governs
  - whether we get a solution
  - how fast/cheaply we can get it
  - how much memory/storage we need

SIMPLE Algorithm

- Semi-Implicit Method for Pressure-Linked Equations
- Proposed by Patankar and Spalding (1972)
- Idea is to start with discrete continuity equation
- Substitute into it the discrete u and v momentum equations
- Discrete momentum equations contain pressure differences
- Hence get an equation for the discrete pressures
- SIMPLE actually solves for a related quantity called the pressure correction

Flow in Pipes

A portion of a water-supply system is shown in the figure below.

The flow rate in a pipe is given by

$$\dot{m} = C\,\Delta P$$

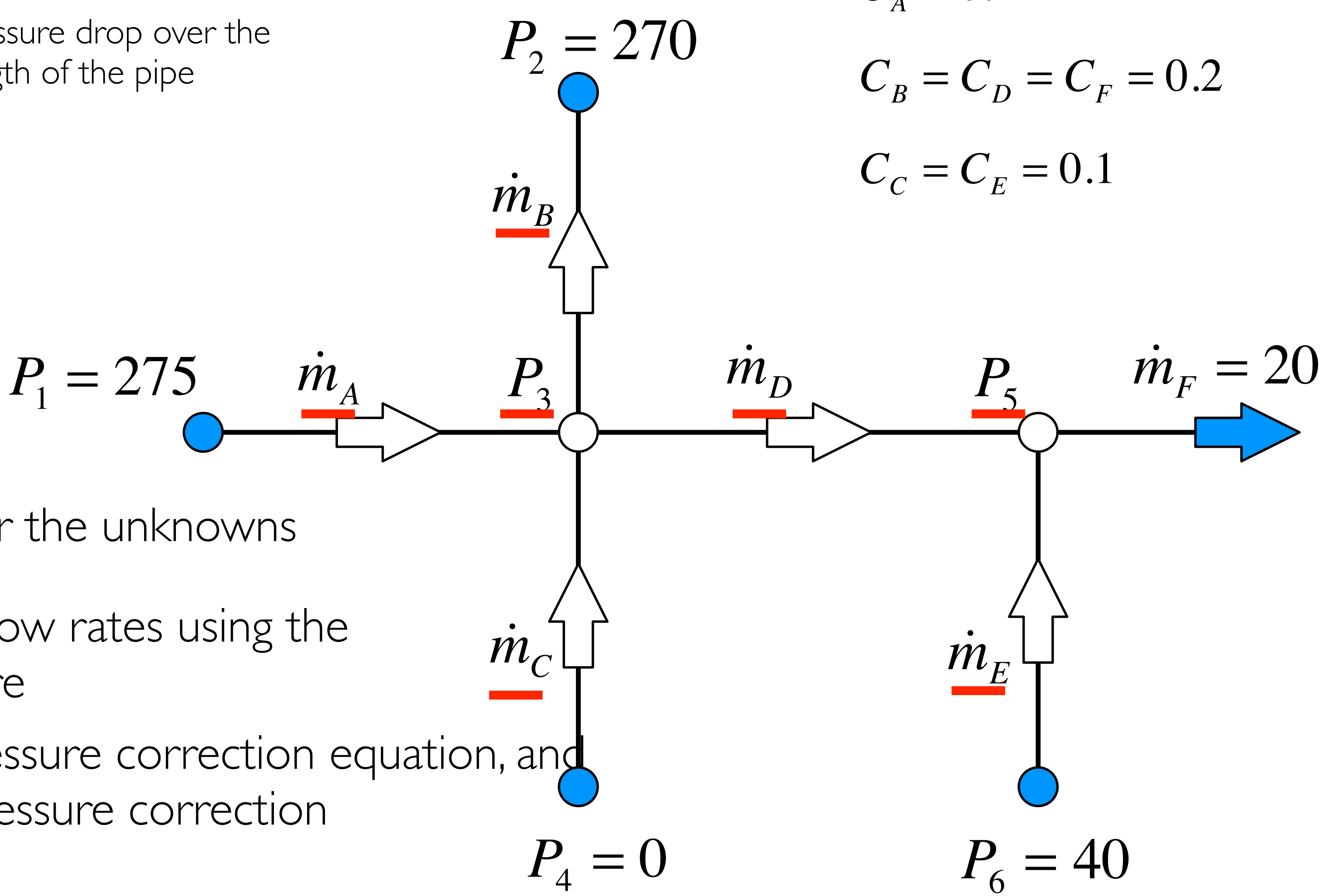
Pressure drop over the length of the pipe

Hydraulic conductance

$C_A = 0.4$

$C_B = C_D = C_F = 0.2$

$C_C = C_E = 0.1$



1. Guess values for the unknowns
2. Compute the flow rates using the guessed pressure
3. Construct a pressure correction equation, and solve for the pressure correction

Solution

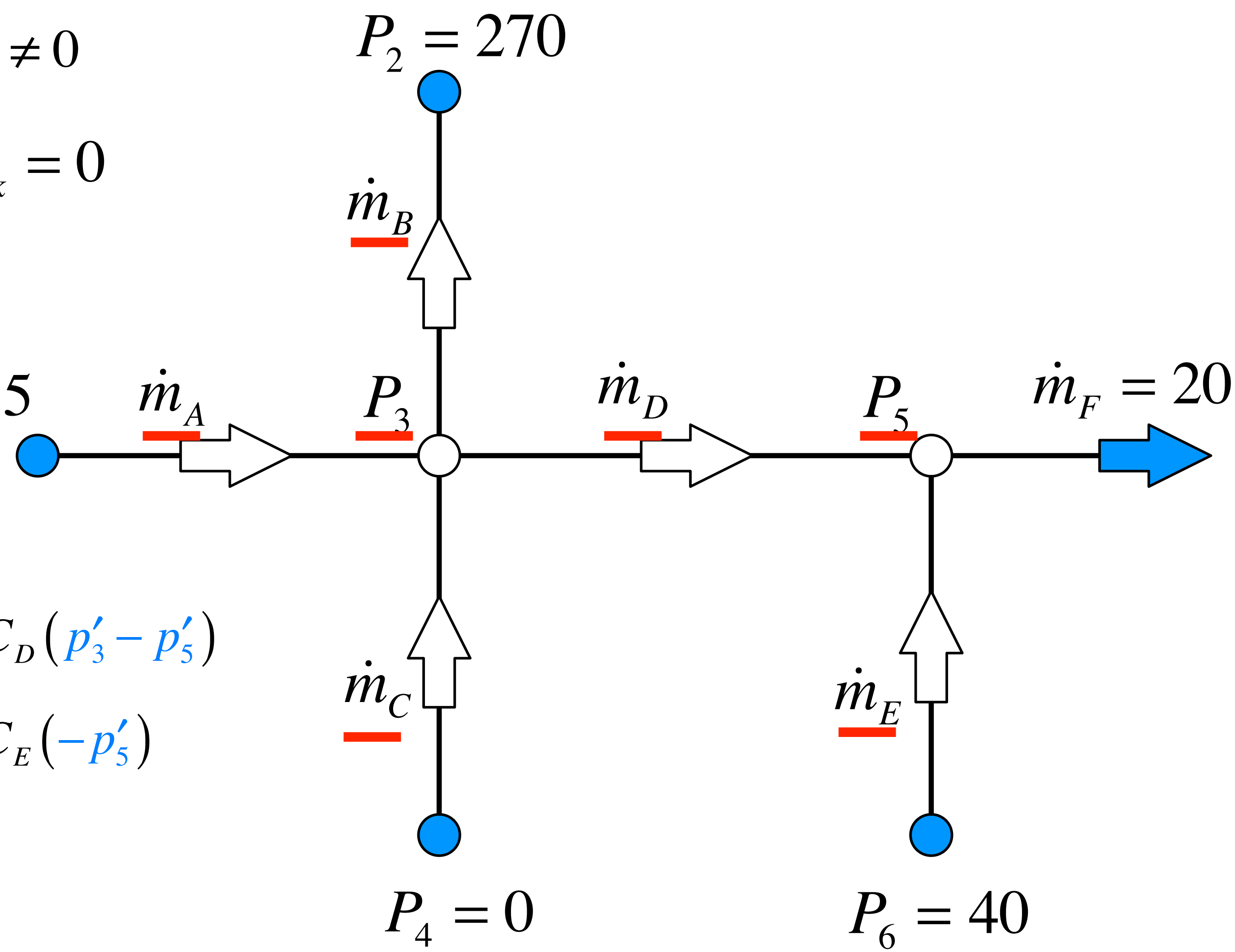
1. $P_3 = 200 \quad P_5 = 30$

2. $\dot{m}_A^* = C_A\left(P_1 - P_3^*\right) \quad \dot{m}_B^* = C_B\left(P_3^* - P_2\right) \quad \dot{m}_C^* = C_C\left(P_4 - P_3^*\right) \quad \dot{m}_D^* = C_D\left(P_3^* - P_5^*\right) \quad \dot{m}_E^* = C_E\left(P_6 - P_5^*\right)$

3. $\dot{m}^* = C\,\Delta P^* \longrightarrow \sum_{\sim k} \dot{m}_k^* \neq 0$

$$\dot{m} = C\,\Delta P \longrightarrow \sum_{\sim k} \dot{m}_k = 0$$

$$\dot{m}' = C\,\Delta\left(P'\right)$$



$$\sum_{\sim k}\left(\dot{m}_k^* + \dot{m}_k'\right) = 0$$

$$\dot{m}_A' = C_A\left(-p_3'\right)$$

$$\dot{m}_B' = C_B\left(p_3'\right) \qquad \dot{m}_D' = C_D\left(p_3' - p_5'\right)$$

$$\dot{m}_C' = C_C\left(-p_3'\right) \qquad \dot{m}_E' = C_E\left(-p_5'\right)$$

$$\sum_{\sim k} \dot{m}_k' = -\sum_{\sim k} \dot{m}_k^*$$

3 $\displaystyle\sum_{k=nb(3)} \left(\dot m_k^* + \dot m_k'\right) = 0$

- $\dot m_A' - \dot m_B' + \dot m_C' - \dot m_D' = -\left(\dot m_A^* - \dot m_B^* + \dot m_C^* - \dot m_D^*\right)$

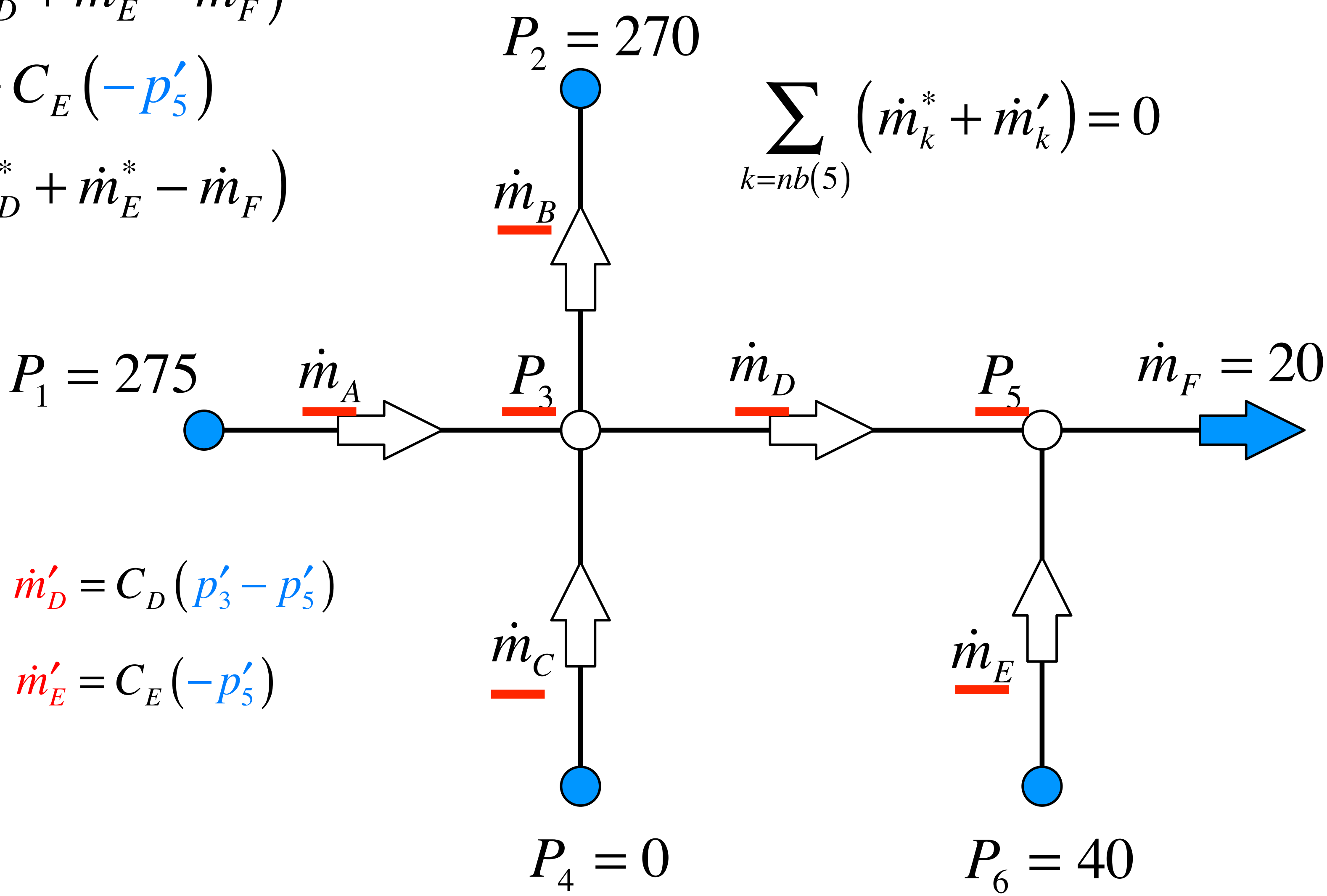
  $C_A\left(-p_3'\right) - C_B\left(p_3'\right) + C_C\left(-p_3'\right) - C_D\left(p_3' - p_5'\right) = -\left(\dot m_A^* - \dot m_B^* + \dot m_C^* - \dot m_D^*\right)$

- $\dot m_D' + \dot m_E' = -\left(\dot m_D^* + \dot m_E^* - \dot m_F\right)$

  $C_D\left(p_3' - p_5'\right) + C_E\left(-p_5'\right)$

  $= -\left(\dot m_D^* + \dot m_E^* - \dot m_F\right)$

$\displaystyle\sum_{k=nb(5)} \left(\dot m_k^* + \dot m_k'\right) = 0$



$\dot m_A' = C_A\left(-p_3'\right)$

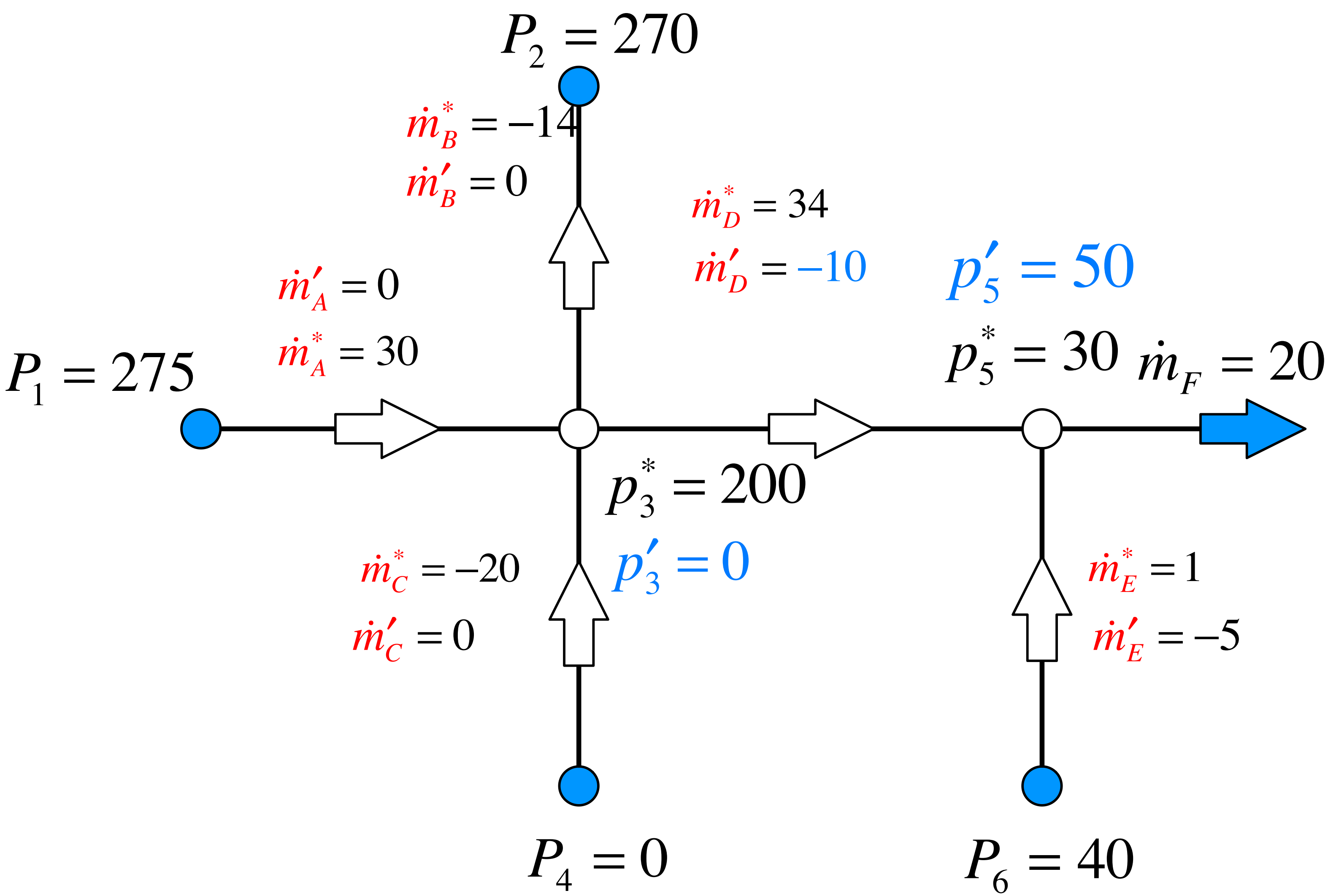
$\dot m_B' = C_B\left(p_3'\right)$

$\dot m_C' = C_C\left(-p_3'\right)$

$\dot m_D' = C_D\left(p_3' - p_5'\right)$

$\dot m_E' = C_E\left(-p_5'\right)$

$$-0.7\left(p_3'\right)+0.2\left(p_5'\right)=10$$
$$0.2\left(p_3'\right)-0.3\left(p_5'\right)=-15$$
$$\Rightarrow \begin{cases} p_3' = 0 \\ p_5' = 50 \end{cases}$$



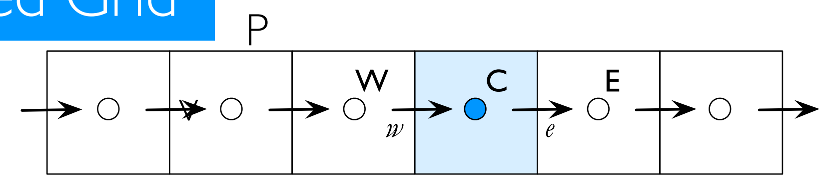
SIMPLE Based Algorithms

The SIMPLE Algorithm

$$\nabla.(\rho \mathbf{v}\mathbf{v}) = -\nabla p + \nabla\cdot\tau + \mathbf{B}$$

$$\nabla\cdot(\rho\mathbf{v}) = 0$$

Staggered Grid



Predictor $^{n)}, P^{(n)}$

$$a_C^{\mathbf{v}}\mathbf{v}_C^{\bullet} + \sum_{F=NB(C)} a_F^{\mathbf{v}}\mathbf{v}_F^{\bullet} = -V_C\left(\nabla p^{(n)}\right)_C \qquad \therefore \mathbf{v}^{\bullet}, P^{(n)}$$

$$a_e^u u_e^{\bullet} + \sum_{f=NB(e)} a_f^u u_f^{\bullet} = -V_e\left(\nabla p^{(n)}\right)_e$$

$$\sum_{f=NB(C)} \rho\mathbf{v}_f^{\bullet}\cdot S_f = \rho u_e^{\bullet}A_e - \rho u_w^{\bullet}A_w$$

$$u_e^{\bullet} + \sum_{f=NB(e)} \frac{a_f^u u_f^{\bullet}}{a_e^u} = -\frac{V_e}{a_e^u}\left(\frac{\partial p^{(n)}}{\partial x}\right)_e$$

$$u_w^{\bullet} = H_w\left(u^{\bullet}\right) - d_w\left(\frac{\partial p^{(n)}}{\partial x}\right)_w$$

$$u_e^{\bullet} = H_e\left(u^{\bullet}\right) - d_e\left(\frac{\partial p^{(n)}}{\partial x}\right)_e$$

Corrector P'

$$\sum_{f=nb(C)} \dot{m}_f^{\bullet} \neq 0 \qquad \sum_{f=nb(C)} \dot{m}_f = 0$$

$$\sum_{f=nb(C)} \left(\dot{m}_f^{\bullet} + \dot{m}_f'\right) = 0 \qquad \dot{m}_f^{\bullet} = \rho\mathbf{v}_f^{\bullet}\cdot S_f \qquad \dot{m}_f' = \rho\mathbf{v}_f'\cdot S_f$$

$$\sum_{f=nb(C)} \dot{m}_f' = -\sum_{f=nb(C)} \dot{m}_f^{\bullet}$$

$$u_e' = H_e\left(u'\right) - d_e\left(\frac{\partial p'}{\partial x}\right)_e$$

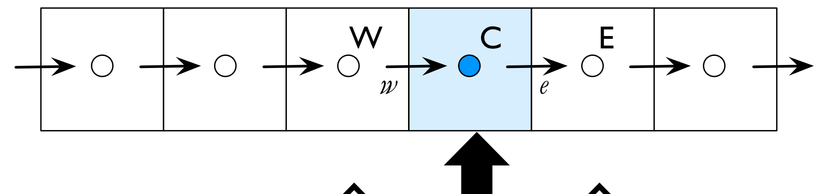
$$\mathbf{v}_f' = H_f\left(\mathbf{v}'\right) - d_f\left(\nabla p'\right)_f$$

Pressure Equation

$$\sum_{f=nb(C)} \rho \mathbf{v}'_f \cdot \mathbf{S}_f = -\sum_{f=nb(C)} \dot{m}^{\bullet}_f$$

Substituting $\mathbf{v}'_f = H_f(\mathbf{v}') - d_f(\nabla p')_f$

$$\sum_{f=nb(C)} \left[H_f(\mathbf{v}') - d_f(\nabla p')_f\right]\cdot \mathbf{S}_f = -\sum_{f=nb(C)} \dot{m}^{\bullet}_f$$



$$\boxed{\sum_{f=nb(C)} -d_f(\nabla p')_f \cdot \mathbf{S}_f} = \boxed{-\sum_{f=nb(C)} \dot{m}^{\bullet}_f} - \boxed{\sum_{f=nb(C)} H_f(\mathbf{v}')\cdot \mathbf{S}_f}$$

$$-\sum_{f=nb(C)}\left(-\sum_{ff=NB(f)} \frac{a^u_{ff}\mathbf{v}'_{ff}}{a^u_f}\right)\cdot \mathbf{S}_f$$

SIMPLE

SIMPLEC

SIMPLER

SIMPLEST

SIMPLE-M

PISO

$$\sum_{f=nb(C)} -d_f(\nabla p')_f \cdot \mathbf{S}_f = \sum_{f=nb(C)} -\frac{d_f A_f}{\|CF\|}\left(p'_F - p'_C\right)$$

$$= a^p_C p'_C + \left(a^p_E p'_E + a^p_W p'_W\right)$$

$$-\sum_{f=nb(C)} \dot{m}^{\bullet}_f = -\left(\rho u^{\bullet}_e A_e - \rho u^{\bullet}_w A_w\right)$$

$$\Rightarrow \sum_{f=nb(C)} -\frac{d_f A_f}{\|CF\|}\left(p'_F - p'_C\right) = -\left(\rho u^{\bullet}_e A_e - \rho u^{\bullet}_w A_w\right)$$

$$a^p_C p'_C + \left(a^p_E p'_E + a^p_W p'_W\right) = b^p_C \qquad \Rightarrow \mathbf{v}'_f = -d_f(\nabla p')_f \qquad \Rightarrow \mathbf{v}^{\bullet\bullet}_f = \mathbf{v}^{\bullet}_f - d_f(\nabla p')_f$$

The SIMPLE Algorithm

$$\therefore \mathbf{v}^{(n)}, P^{(n)}$$

$$a_C^{\mathbf{v}} \mathbf{v}_C^{\bullet} + \sum_{F=NB(C)} a_F^{\mathbf{v}} \mathbf{v}_F^{\bullet} = -V_C \left(\nabla p^{(n)}\right)_C$$

$$\therefore \mathbf{v}^{\bullet}, P^{(n)}$$

$$a_C^{p} p_C' + \left(a_E^{p} p_E' + a_W^{p} p_W'\right) = b_C^{p}$$

$$\therefore \mathbf{v}^{\bullet\bullet}, P'$$

$$p_C^{\bullet} = p_C^{(n)} + p_C'$$

$$\mathbf{v}_f^{\bullet\bullet} = \mathbf{v}_f^{\bullet} - d_f \left(\nabla p'\right)_f$$

SIMPLE iteration

start with nth iteration values
$\dot{m}_f^{(n)}$, **v**$^{(n)}$, $\rho^{(n)}$, $P^{(n)}$

Predictor

emble and **solve** momentum
equation for **v***

assemble $\dot{m}_f^{*}$ using
Rhie-Chow interpolation

Corrector

emble and **solve** Pressure
orrection equation for P'

correct $\dot{m}_f^{*}$, **v***, $\rho^{(n)}$ and $P^{(n)}$ to get
improved $\dot{m}_f^{**}$, **v****, ρ^{*} and P^{*}

repeat until convergence

Sequential Solution

- Pressure methods were also developed as sequential methods as opposed to coupled methods where all the Navier-Stokes equation are solved simultaneously
- In sequential methods each of the following equation is solved individually in sequence
  - Solve u-momentum equation for u velocity component, with all other variables assumed known
  - Solve v-momentum equation for v velocity component
  - Modify the continuity equation to yield a pressure equation and solve for pressure
  - Repeat process until convergence

Collocated Grids

Collocated Grids

$$\sum_{f=nb(C)} \dot{m}'_f = -\sum_{f=nb(C)} \dot{m}^{\bullet}_f$$

$$\sum_{f=nb(C)} \rho \overline{\mathbf{v}}'_f \cdot \mathbf{S}_f = -\sum_{f=nb(C)} \dot{m}^{\bullet}_f$$

$$\begin{aligned}\overline{\mathbf{v}}'_f &= \tfrac{1}{2}\left(\mathbf{v}'_C + \mathbf{v}'_F\right)\\ &= \tfrac{1}{2}\left(H_C(\mathbf{v}') - D_C\left(\frac{\partial p'}{\partial x}\right)_C + H_F(\mathbf{v}') - D_F\left(\frac{\partial p'}{\partial x}\right)_F \right)\end{aligned}$$

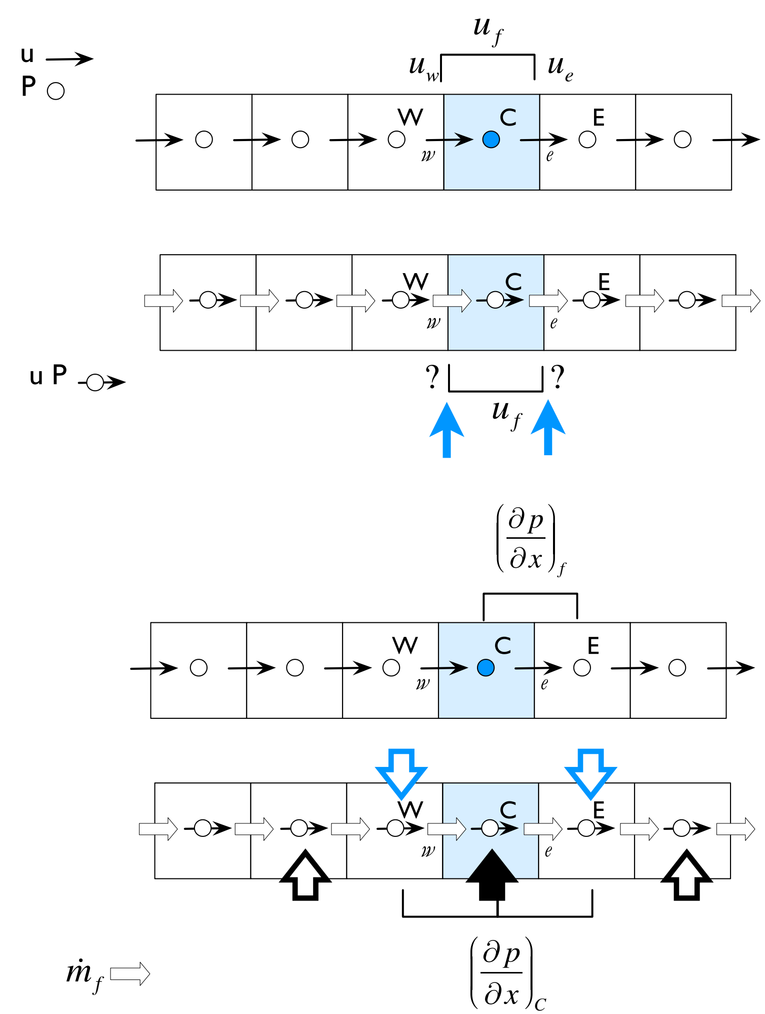
$$\overline{u}'_e = \overline{H}_e(u') + \overline{D}_e\overline{\left(\frac{\partial p'}{\partial x}\right)}_e \qquad \overline{u}'_w = \overline{H}_w(u') + \overline{D}_w\overline{\left(\frac{\partial p'}{\partial x}\right)}_w$$

$$\begin{aligned}\overline{u}'_e - \overline{u}'_w &= \overline{H}_e(u') - \overline{H}_w(u') + \overline{D}_e\overline{\left(\frac{\partial p'}{\partial x}\right)}_e - \overline{D}_w\overline{\left(\frac{\partial p'}{\partial x}\right)}_w\\ &\approx D\frac{1}{2}\left[\left(\frac{\partial p'}{\partial x}\right)_E + \left(\frac{\partial p'}{\partial x}\right)_C - \left(\frac{\partial p'}{\partial x}\right)_C - \left(\frac{\partial p'}{\partial x}\right)_W\right]\\ &\approx \frac{D}{\Delta x}\frac{1}{2}\left[p'_{EE} - 2p'_C + p'_{WW}\right]\end{aligned}$$

Similarly

$$\dot{m}^{\bullet}_e - \dot{m}^{\bullet}_w = \rho A\left(u^{\bullet}_E - u^{\bullet}_W\right)$$

$$\boxed{a^p_C p'_C + a^p_{EE} p'_{EE} + a^p_{WW} p'_{WW} = -\left(\dot{m}^{\bullet}_E - \dot{m}^{\bullet}_W\right)}$$



Rhie Chow Interpolation

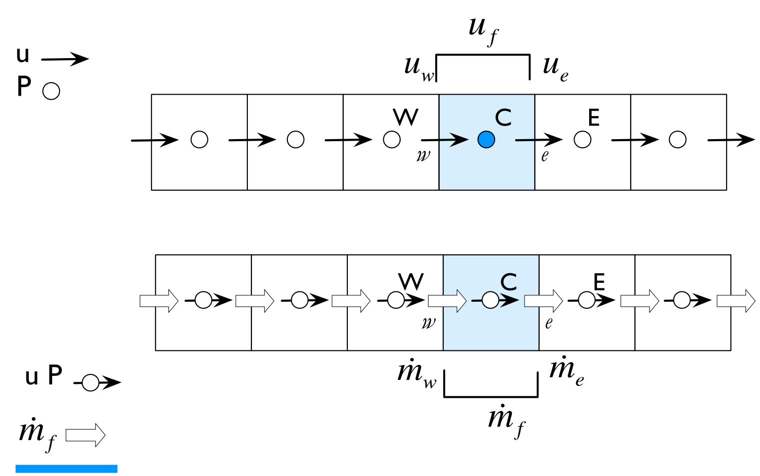
$$\mathbf{v}_P = H_C(\mathbf{v}) - D_C \nabla p_C$$

$$\mathbf{v}_F = H_F(\mathbf{v}) - D_F \nabla p_F$$

$$\mathbf{v}_f = \overline{H}_f(\mathbf{v}) - \overline{D}_f \nabla p_f$$

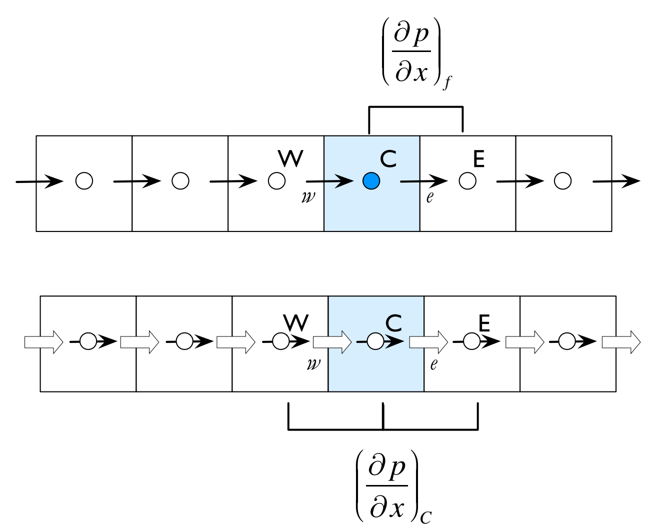
$$H_P(\mathbf{v}) = -\sum_F \frac{a_F^{\mathbf{v}} \mathbf{v}_F}{a_C^{\mathbf{v}}}$$

$$D_P = \frac{V_C}{a_C^{\mathbf{v}}}$$



1
$$\overline{H}_f(\mathbf{v}) = g_P H_P(\mathbf{v}) + g_F H_F(\mathbf{v})$$
$$= g_P\left(\mathbf{v}_P + D_P \nabla p_P\right) + g_F\left(\mathbf{v}_F + D_F \nabla p_F\right)$$
$$\approx \overline{\mathbf{v}}_f + \overline{D}_f \overline{\nabla p}_f$$

$$\overline{D}_f = \frac{\overline{V}_f}{\overline{a}_f^{\mathbf{v}}}$$



2
$$\mathbf{v}_f = \left(\overline{\mathbf{v}}_f + \overline{\nabla p}_f\right) - \overline{D}_f \nabla p_f$$
$$= \overline{\mathbf{v}}_f - \overline{D}_f\left(\nabla p_f - \overline{\nabla p}_f\right)$$

3
$$\dot{m}_f^{\bullet} = \rho \mathbf{v}_f^{\bullet} \cdot S_f \qquad \dot{m}'_f = \rho \mathbf{v}'_f \cdot S_f$$

Pressure Correction

$$\sum_{f=nb(C)} \dot{m}'_f = -\sum_{f=nb(C)} \dot{m}^{\bullet}_f$$

$$\sum_{f=nb(C)} \rho \mathbf{v}'_f \cdot \mathbf{S}_f = -\sum_{f=nb(C)} \dot{m}^{\bullet}_f$$

$$\mathbf{v}'_f == \bar{\mathbf{v}}_f - \bar{D}_f\left(\nabla p_f - \overline{\nabla p}_f\right)$$

$$u'_e = \bar{u}'_e - \bar{D}_e\left(\left(\frac{\partial p'}{\partial x}\right)_e - \overline{\left(\frac{\partial p'}{\partial x}\right)}_e\right) \qquad u'_w = \bar{u}'_w - \bar{D}_w\left(\left(\frac{\partial p'}{\partial x}\right)_w - \overline{\left(\frac{\partial p'}{\partial x}\right)}_w\right)$$

$$u'_e - u'_w = \left[\cancel{\bar{u}'_e} - \bar{D}_e\left(\left(\frac{\partial p'}{\partial x}\right)_e - \cancel{\overline{\left(\frac{\partial p'}{\partial x}\right)}_e}\right)\right] - \left[\cancel{\bar{u}'_w} - \bar{D}_w\left(\left(\frac{\partial p'}{\partial x}\right)_w - \cancel{\overline{\left(\frac{\partial p'}{\partial x}\right)}_w}\right)\right]$$

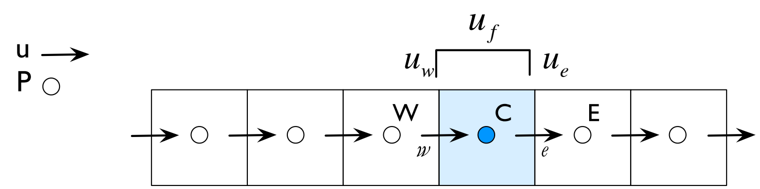
$$\approx D\frac{1}{2}\left[\left(\frac{\partial p'}{\partial x}\right)_e - \left(\frac{\partial p'}{\partial x}\right)_w\right]$$

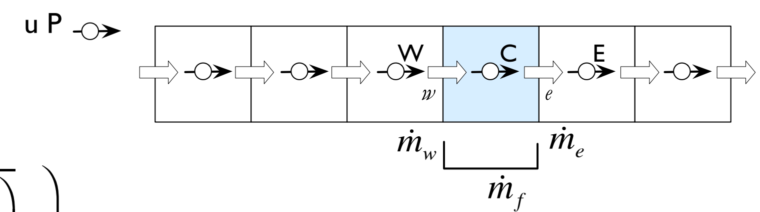
$$\approx \frac{D}{\Delta x}\frac{1}{2}\left[p'_E - 2p'_C + p'_W\right]$$

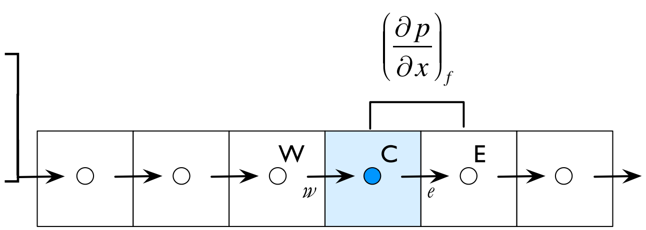
Similarly

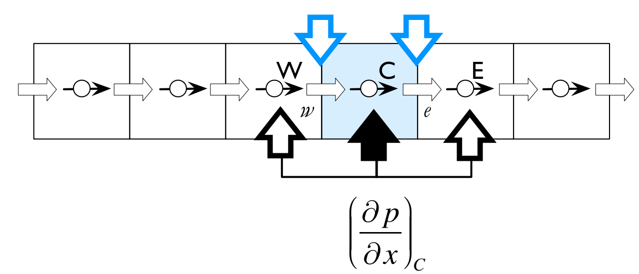
$$\dot{m}^{\bullet}_e - \dot{m}^{\bullet}_w = \rho_f u^{\bullet}_e A_e - \rho_w u^{\bullet}_w A_w$$

$$\boxed{a^p_C p'_C + a^p_E p'_E + a^p_W p'_W = -\sum_{f=nb(C)} \dot{m}^{\bullet}_f}$$









The Pressure-Correction Equation

$$\therefore \mathbf{v}^{(n)}, P^{(n)}$$

$$a_C^{\mathbf{v}}\mathbf{v}_C^{\bullet} + \sum_{F=NB(C)} a_F^{\mathbf{v}}\mathbf{v}_F^{\bullet} = -V_C\left(\nabla p^{(n)}\right)_C$$

$$\therefore \mathbf{v}^{\bullet}, P^{(n)}$$

$$\dot{m}_f^{\bullet} = \rho^{(n)}\mathbf{v}_f^{\bullet}\cdot\mathbf{S}_f = \rho^{(n)}\left[\overline{\mathbf{v}}_f^{\bullet} - \overline{\mathbf{D}}_f\left(\nabla p'_f - \overline{\nabla p'}_f\right)\right]\cdot\mathbf{S}_f$$

$$\dot{m}'_f = -\rho^{(n)}\left(\overline{\mathbf{D}}_f\nabla p'_f\right)\cdot\mathbf{S}_f$$

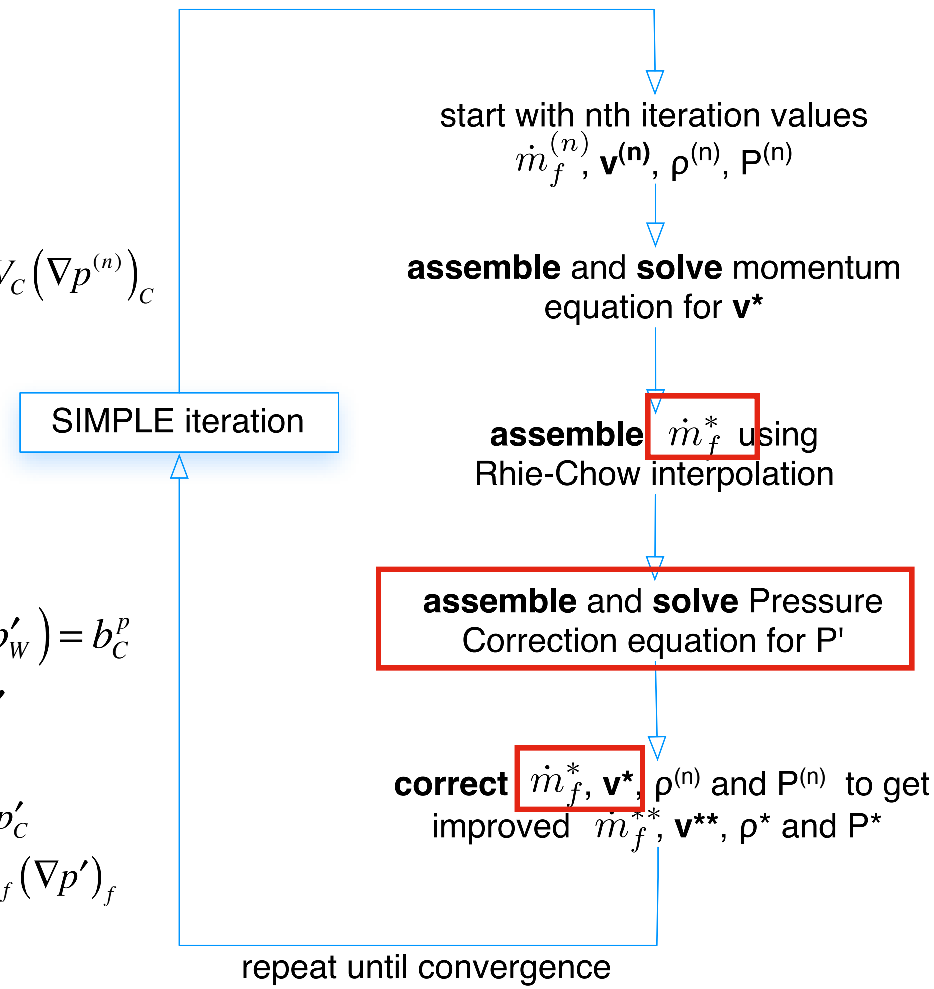
$$a_C^p p'_C + \left(a_E^p p'_E + a_W^p p'_W\right) = b_C^p$$

$$\mathbf{v}'_f = -\overline{\mathbf{D}}_f\nabla p'_f$$

$$\therefore \mathbf{v}^{\bullet\bullet}, P'$$

$$p_C^{\bullet} = p_C^{(n)} + p'_C$$

$$\mathbf{v}_f^{\bullet\bullet} = \mathbf{v}_f^{\bullet} - d_f\left(\nabla p'\right)_f$$



Pressure-Velocity Coupling

SIMPLE
SIMPLEC
SIMPLER
SIMPLEST
SIMPLE-M
PISO

collocated variable arrangement

Rhie-Chow Interpolation

$$\sum_f\left(\rho_f^*\overline{D}_f(\nabla P')_f\cdot S_f\right) = -\sum_f\left(\rho_f^* U_f^*\right) - \boxed{\sum_f\left(\rho_f' v_f'\cdot S_f\right)} - \boxed{\sum_f\left(\rho_f^* H[v']_f\cdot S_f\right)}$$

neglected

treatment leads to variety of schemes

First Predictor

$$v_P^* = H[v^*]_P - D_P\left(\nabla P^{(n)}\right)_P$$

First Corrector

$$(v',P',\rho')\left(v^{**}=v^*+v',\ P^*=P^{(n)}+P',\ \rho^*=\rho^{(n)}+\rho'\right)$$

$$v_P^{**} = H[v^{**}]_P - D_P\left(\nabla P^*\right)_P = H[v^*+v']_P - D_P\left[\nabla\left(P^{(n)}+P'\right)\right]_P$$

$$\begin{cases} v'_P = H[v']_P - D_P(\nabla P')_P \\ \rho' = C_\rho P' \end{cases}$$

Condition

$$\frac{(\rho_P^* - \rho_P^o)}{\delta t}V + \Delta\left[\rho^* v^{**} S\right]_P = 0 \Leftrightarrow$$

$$\frac{(\rho_P^{(n)} + \rho'_P - \rho_P^o)}{\delta t}V + \Delta\left[\left(\rho^{(n)}+\rho'\right)\left(v^* + H[v'] - D(\nabla P')\right)S\right]_P = 0 \Leftrightarrow$$

$$\frac{VC_\rho}{\delta t}P'_P + \Delta\left[C_\rho U^* P'\right]_P - \Delta\left[\rho^{(n)} D(\nabla P')S\right]_P = -\frac{(\rho_P^n - \rho_P^o)}{\delta t}V - \Delta\left[\rho^{(n)}U^*\right]_P - \Delta\left[\rho^{(n)}H[v']S\right]_P - \Delta\left[(\rho'v')S\right]_P$$

Approximation

Neglect $H[v']$ $\Delta\left[(\rho'v')S\right]_P$ $\Rightarrow$ $v'_P = -D_P(\nabla P')_P$

Approximation Equation

$$\frac{VC_\rho}{\delta t}P'_P + \Delta\left[C_\rho U^* P'\right]_P - \Delta\left[\rho^{(n)} D(\nabla P')S\right]_P = -\frac{(\rho_P^n - \rho_P^o)}{\delta t}V - \Delta\left[\rho^{(n)}U^*\right]_P$$

Second Predictor

$$v_P^{***} = H^{**}[v^{**}]_P - D_P^{**}\left(\nabla P^*\right)_P$$

Second Corrector

$$(v'',P'',\rho'')\left(v^{****}=v^{***}+v'',\ P^{**}=P^*+P'',\ \rho^{**}=\rho^*+\rho''\right)$$

$$v_P^{****} = H^{**}[v^{****}]_P - D_P^{**}\left[\nabla\left(P^*+P''\right)\right]_P$$

$$v_P^{***} = H^{**}[v^{**}]_P - D_P^{**}\left(\nabla P^*\right)_P$$

$$\begin{cases} v''_P = H^{**}[v^{****} - v^{**}]_P - D_P^{**}(\nabla P'')_P = H^{**}[v^{***} - v^{**} + v'']_P - D_P^{**}(\nabla P'')_P \\ \rho'' = C_\rho P'' \end{cases}$$

Condition

$$\frac{(\rho_P^{**} - \rho_P^o)}{\delta t}V + \Delta\left[\rho^{**} v^{****} S\right]_P = 0 \Leftrightarrow$$

$$\frac{(\rho_P^* + \rho''_P - \rho_P^o)}{\delta t}V + \Delta\left[\left(\rho^*+\rho''\right)\left(v^{***} + H^{**}[v^{***} - v^{**} + v''] - D_P^{**}(\nabla P'')\right)S\right]_P = 0 \Leftrightarrow$$

$$\frac{VC_\rho}{\delta t}P''_P + \Delta\left[C_\rho U^{***} P''\right]_P - \Delta\left[\rho^* D_P^{**}(\nabla P'')S\right]_P = -\frac{(\rho_P^* - \rho_P^o)}{\delta t}V - \Delta\left[\rho^*U^{***}\right]_P - \Delta\left[\rho^*H^{**}[v^{***} - v^{**} + v'']S\right]_P - \Delta\left[(\rho''v'')S\right]_P$$

Approximation

Neglect $H^{**}[v^{***} - v^{**} + v'']_P$ $\Delta\left[(\rho''v'')S\right]_P$ $\Rightarrow$ $v''_P = -D_P^{**}(\nabla P'')_P$

Approximation Equation

$$\frac{VC_\rho}{\delta t}P''_P + \Delta\left[C_\rho U^{***} P''\right]_P - \Delta\left[\rho^* D_P^{**}(\nabla P'')S\right]_P = -\frac{(\rho_P^* - \rho_P^o)}{\delta t}V - \Delta\left[\rho^*U^{***}\right]_P$$

Problem 1- Staggered Grid

- Use the SIMPLE procedure to compute p2, uB, and uC from the following data:

$$\Delta x = 2,\ \mathrm{c_B} = 0.25, \mathrm{c_C} = 0.2$$

$$A_B = 5, A_C = 4, p_1 = 200, p_3 = 38$$

- As an initial guess, set

$$u_B = u_C = 15 \text{ and } p_2 = 120$$