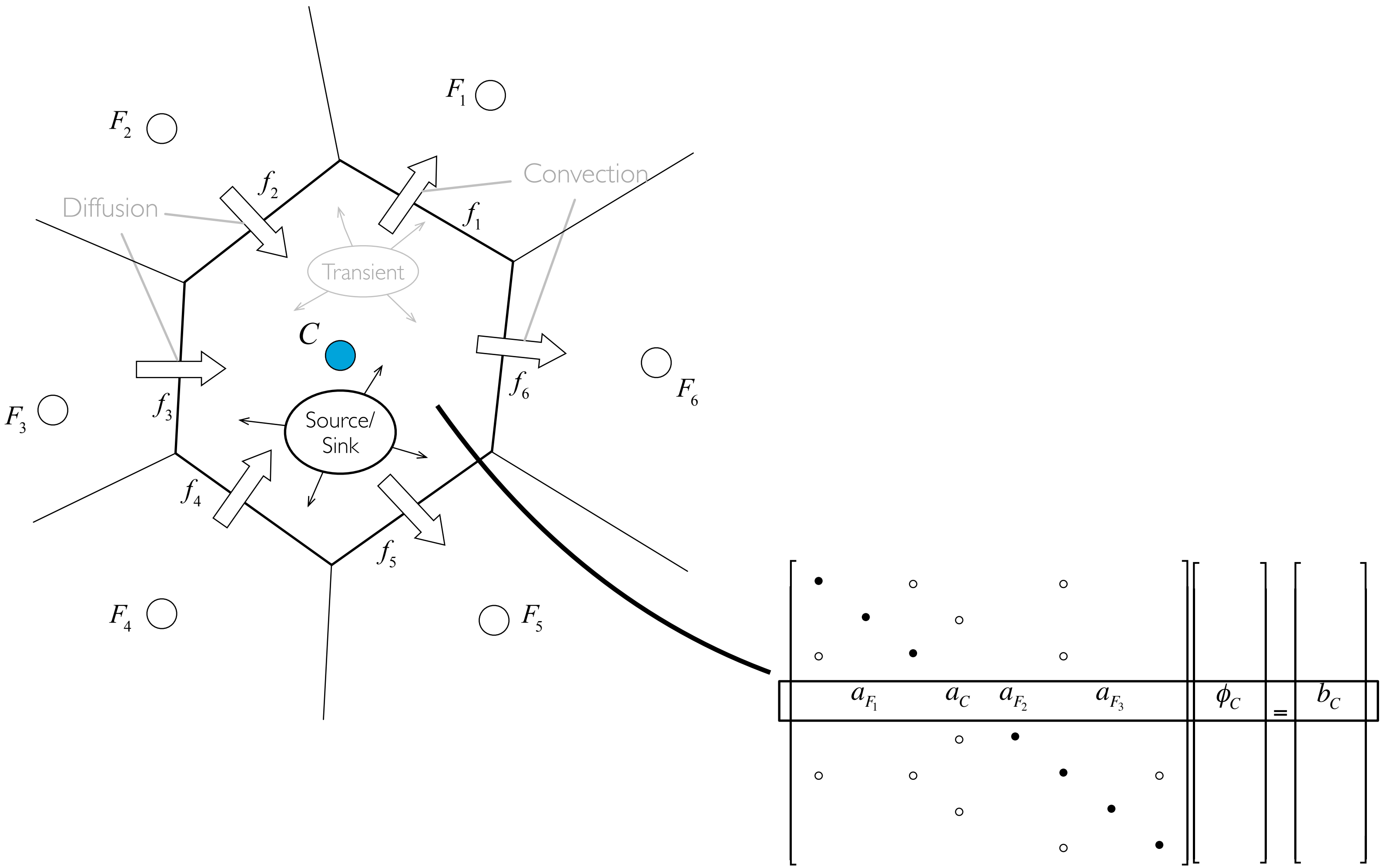
Transient, Source Terms and Relaxation

Chapter 14 to review

Source Term Discretization

Source Term



F1
F2
Convection
Diffusion
f2
f1
Transient
C
F3
f3
f6
F6
Source/
Sink
f4
f5
F4
F5
aF1
aC
aF2
aF3
φC
=
bC

Source Term Linearization

$$a_P\phi_P^{(n+1)} + \sum_{NB(P)} a_{NB}\phi_{NB}^{(n+1)} = QV_P$$

$$Q = Q(\phi)$$

$$Q^{(n+1)} = Q^{(n)} + \frac{\partial Q(\phi)}{\partial \phi}\left(Q^{(n+1)} - Q^{(n)}\right)$$

$$= \underbrace{Q^{(n)} - \frac{\partial Q(\phi)}{\partial \phi}Q^{(n)}}_{\text{Previous iteration value}} + \underbrace{\frac{\partial Q(\phi)}{\partial \phi}Q^{(n+1)}}_{\text{Current iteration value}}$$

-ve

$$\left(a_P - \frac{\partial Q(\phi)}{\partial \phi}V_P\right)\phi_P^{(n+1)} + \sum_{NB(P)} a_{NB}\phi_{NB}^{(n+1)} = \left(\phi^{(n)} - \frac{\partial Q\left(\phi^{(n)}\right)}{\partial \phi}\phi^{(n)}\right)V_P$$

Example

Solution

Consequence

Relaxation

Under-Relaxation

- For non-linear problems, a common solution procedure is Picard Iteration
  - Guess **Φ**
  - Find coefficients ap, anb and bp
  - Use Iterative solver to find new **Φ**
  - Repeat until convergence
- No Guarantee to converge for strong non-linearities
- Control the amount of **Φ** from iteration to iteration using under-relaxation

Relaxation

- At each iteration, at each cell, a new value for variableφ in cell P can then be calculated from that equation.

- It is common to apply relaxation as follows:

$$\phi_P^{new,used} = \phi_P^{old} + \lambda(\phi_P^{new,predicted} - \phi_P^{old})$$

- Here α is the relaxation factor:

- $\alpha < 1$ is underrelaxation. This may slow down speed of convergence but increases the stability of the calculation, i.e. it decreases the possibility of divergence or oscillations in the solutions.

- $\alpha = 1$ corresponds to no relaxation. One uses the predicted value of the variable.

Recommendations

- Underrelaxation factors are there to suppress oscillations in the flow solution that result from numerical errors.
- Underrelaxation factors that are too small will significantly slow down convergence, sometimes to the extent that the user thinks the solution is converged when it really is not.
- The recommendation is to always use underrelaxation factors that are as high as possible, without resulting in oscillations or divergence.
- Typically one should stay with the default factors in the solver.
- When the solution is converged but the pressure residual is still relatively high, the factors for pressure and momentum

Under-Relaxation

Consider **Φ**' the correction to field **Φ**, from one iteration to another

$$\phi = \phi^{\bullet} + \phi'$$

We want to control the amount of change introduced

$$\phi = \phi^{\bullet} + \lambda\phi'$$

$$= \phi^{\bullet} + \lambda\left(\phi^{new\ iteration} - \phi^{\bullet}\right)$$

Where **λ** is some factor with 0<**λ**<1

For the discrete equation

$$a_C\phi_C + \sum_{NB(C)} a_{NB}\phi_{NB} = b_P$$

$$a_C \leftarrow \frac{a_C}{\lambda}$$

$$b_C \leftarrow b_C + \frac{(1-\lambda)}{\lambda}a_C\phi_C^*$$

The updated **Φ** value could be written as

$$\phi_C = \frac{b_C - \sum_{NB(P)} a_{NB}\phi_{NB}}{a_C}$$

$$\phi_C = \phi_C^* + \lambda\left(\frac{b_C - \sum_{NB(C)} a_{NB}\phi_{NB}}{a_C} - \phi_C^*\right)$$

$$\boxed{\frac{a_C}{\lambda}\phi_C + \sum_{NB(C)} a_{NB}\phi_{NB} = b_C + \frac{(1-\lambda)}{\lambda}a_C\phi_C^*}$$

False Time-Step

$$\frac{\partial(\rho\phi)}{\partial t} + \nabla\cdot(\rho\mathbf{v}\phi) = \nabla\cdot(\Gamma\nabla\phi) + S(\phi,\ldots)$$

$$\frac{\rho V}{\Delta t}\left(\phi_P - \phi_P^{old}\right) \rightarrow \frac{\rho V}{\Delta t}\left(\phi_P - \phi_P^{\bullet}\right)$$

$$\left(a_P + a_P^o\right)\phi_P + \sum_{NB(P)} a_{NB}\phi_{NB} = b_P + a_P^o\phi_P^{\bullet}$$

$$a_P^o = \frac{\rho V}{\Delta t}$$

Relation between Transient and URF Factor

$$a_P\phi_P = \lambda\left(b_P - \sum_{NB(P)} a_{NB}\phi_{NB}\right) + (1-\lambda)a_P\phi_P^{\bullet}$$

$$\lambda = \frac{\mathrm{E}}{\mathrm{E}+1}$$

$$a_P\phi_P = \frac{E}{1+E}\left(b_P - \sum_{NB(P)} a_{NB}\phi_{NB}\right) + \left(1-\frac{E}{1+E}\right)a_P\phi_P^{\bullet}$$

$$a_P\left(1+\frac{1}{E}\right)\phi_P + \sum_{NB(P)} a_{NB}\phi_{NB} = b_P + \frac{1}{E}a_P\phi_P^{\bullet}$$

$$\Delta t = E\,\Delta t^{\bullet}$$

$$\Delta t^{\bullet} = \frac{\rho\,\Delta V}{a_P}$$

Implicit Relaxation

$$a_P\phi_P + \sum_{NB(P)} a_{NB}\phi_{NB} = \alpha b_P + (1-\alpha)b_P^\bullet$$

$$a_P\phi_P + \sum_{NB(P)} a_{NB}\phi_{NB} = \alpha b_P + (1-\alpha)\left(a_P\phi_P^\bullet + \sum_{NB(P)} a_{NB}\phi_{NB}^\bullet\right)$$

$$a_P\left(\phi_P^\bullet + \phi_P'\right) + \sum_{NB(P)} a_{NB}\left(\phi_{NB}^\bullet + \phi_{NB}'\right) = b_P$$

$$a_P\phi_P' + \sum_{NB(P)} a_{NB}\phi_{NB}' = b_P - \left(a_P\phi_P^\bullet + \sum_{NB(P)} a_{NB}\phi_{NB}^\bullet\right)$$

$$a_P\phi_P' + \sum_{NB(P)} a_{NB}\phi_{NB}' = RES$$

$$a_P\phi_P' + \sum_{NB(P)} a_{NB}\phi_{NB}' = \alpha b_P + (1-\alpha)\left(a_P\phi_P^\bullet + \sum_{NB(P)} a_{NB}\phi_{NB}^\bullet\right) - \left(a_P\phi_P^\bullet + \sum_{NB(P)} a_{NB}\phi_{NB}^\bullet\right)$$

$$a_P\phi_P' + \sum_{NB(P)} a_{NB}\phi_{NB}' = \alpha\left(b_P - \left(a_P\phi_P^\bullet + \sum_{NB(P)} a_{NB}\phi_{NB}^\bullet\right)\right)$$

$$a_P\phi_P' + \sum_{NB(P)} a_{NB}\phi_{NB}' = \alpha RES$$

$$a_P\left(\phi_P^\bullet + \phi_P'\right) + = \alpha\left(b_P - \sum_{NB(P)} a_{NB}\left(\phi_{NB}^\bullet + \phi_{NB}'\right)\right) + (1-\alpha)a_P\phi_P^\bullet$$

$$a_P\phi_P' + \alpha\sum_{NB(P)} a_{NB}\phi_{NB}' = \alpha\left(b_P - \sum_{NB(P)} a_{NB}\left(\phi_{NB}^\bullet + \phi_{NB}'\right)\right) + (1-\alpha)a_P\phi_P^\bullet - a_P\phi_P^\bullet$$

$$a_P\phi_P' + \alpha\sum_{NB(P)} a_{NB}\phi_{NB}' = \alpha\left(b_P - \left(a_P\phi_P^\bullet + \sum_{NB(P)} a_{NB}\left(\phi_{NB}^\bullet + \phi_{NB}'\right)\right)\right)$$

$$a_P\phi_P' + \alpha\sum_{NB(P)} a_{NB}\phi_{NB}' = \alpha RES$$

Convergence Criteria

Convergence

- The iterative process is repeated until the change in the variable from one iteration to the next becomes so small that the solution can be considered converged.
- At convergence:
  - All discrete conservation equations (momentum, energy, etc.) are obeyed in all cells to a specified tolerance.
  - The solution no longer changes with additional iterations.
  - Mass, momentum, energy and scalar balances are obtained.
- Residuals measure imbalance (or error) in conservation equations.
- The absolute residual at point P is defined as:

$$R_P = \left| a_P \phi_P - \sum_{nb} a_{nb} \phi_{nb} - b \right|$$

- Residuals are usually scaled relative to the local value of the property φ in order to obtain a relative error:

$$R_{C,scaled} = \frac{\left|a_C\phi_C + \sum a_N\phi_N - b_C\right|}{\left|a_C\phi^{\otimes}\right|}$$

- They can also be normalized, by dividing them by the maximum residual that was found at any time during the iterative process.

- An overall measure of the residual in the domain is:

$$R = \sum_{all\ cells} \frac{\left|a_C\phi_C + \sum a_N\phi_N - b_C\right|}{\left|a_C\phi^{\otimes}\right|}$$

Recommendations

- Always ensure proper convergence before using a solution: unconverged solutions can be misleading!!
- Solutions are converged when the flow field and scalar fields are no longer changing.
- Determining when this is the case can be difficult.
- It is most common to monitor the residuals.

Conclusion