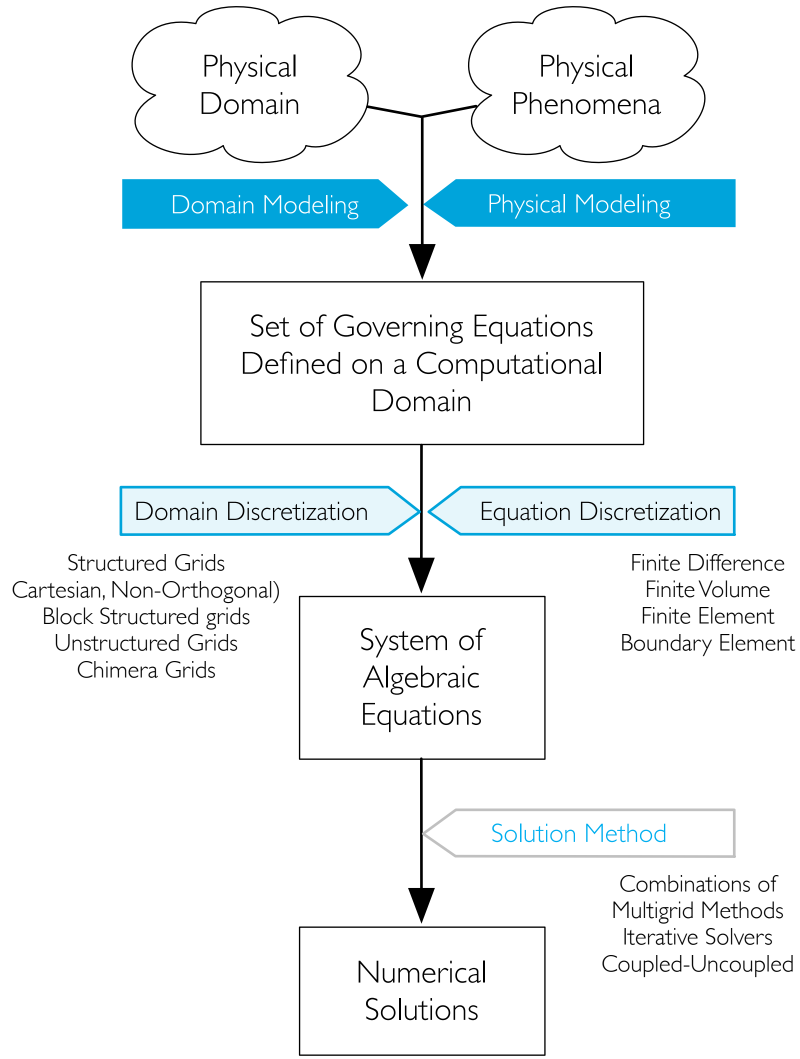
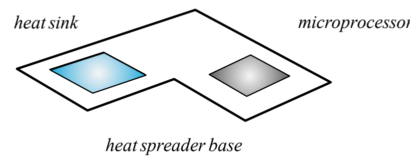
The Finite Volume Mesh

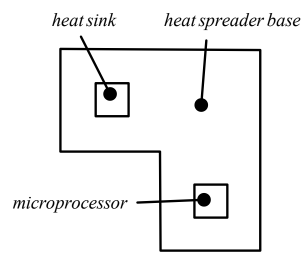
Chapter 06

The Process

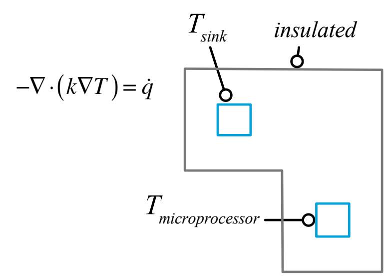

Physical
Domain
Physical
Phenomena
Domain Modeling
Physical Modeling
Set of Governing Equations
Defined on a Computational
Domain
Domain Discretization
Equation Discretization
Structured Grids
Cartesian, Non-Orthogonal)
Block Structured grids
Unstructured Grids
Chimera Grids
System of
Algebraic
Equations
Finite Difference
Finite Volume
Finite Element
Boundary Element
Solution Method
Combinations of
Multigrid Methods
Iterative Solvers
Coupled-Uncoupled
Numerical
Solutions



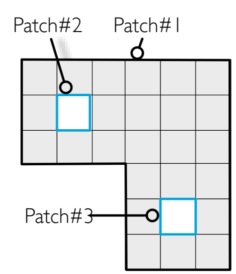
Domain Modeling



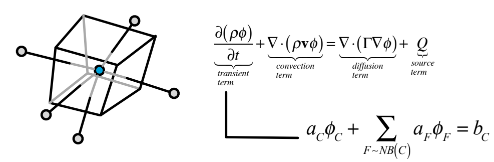
Physical Modeling



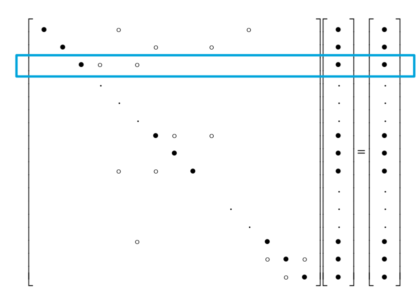
Domain Discretization

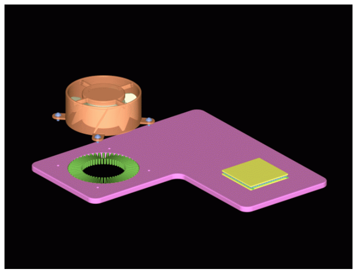


Equations Discretization

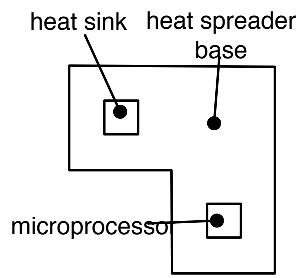


Solution Method

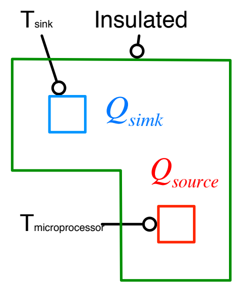




Domain Modeling

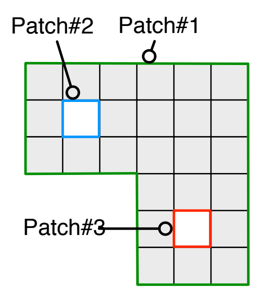


Physical Modeling

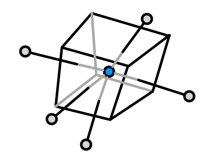


$$-\nabla\cdot\left(k\nabla T\right)=Q$$

Domain Discretization



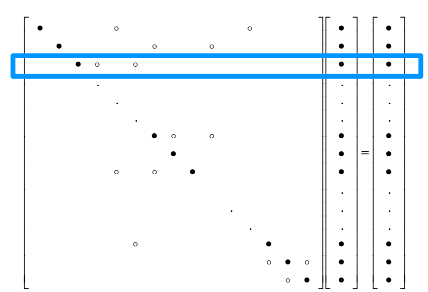
Equations Discretization



$$\underbrace{\frac{\partial(\rho\phi)}{\partial t}}_{\substack{\text{transient}\\ \text{term}}}+\underbrace{\nabla\cdot(\rho\mathbf{v}\phi)}_{\substack{\text{convection}\\ \text{term}}}=\underbrace{\nabla\cdot(\Gamma\nabla\phi)}_{\substack{\text{diffusion}\\ \text{term}}}+\underbrace{Q}_{\substack{\text{source}\\ \text{term}}}$$

$$a_C\phi_C+\sum_{NB(C)}a_{NB}\phi_{NB}=b_P$$

Solution Method



Domain Discretization

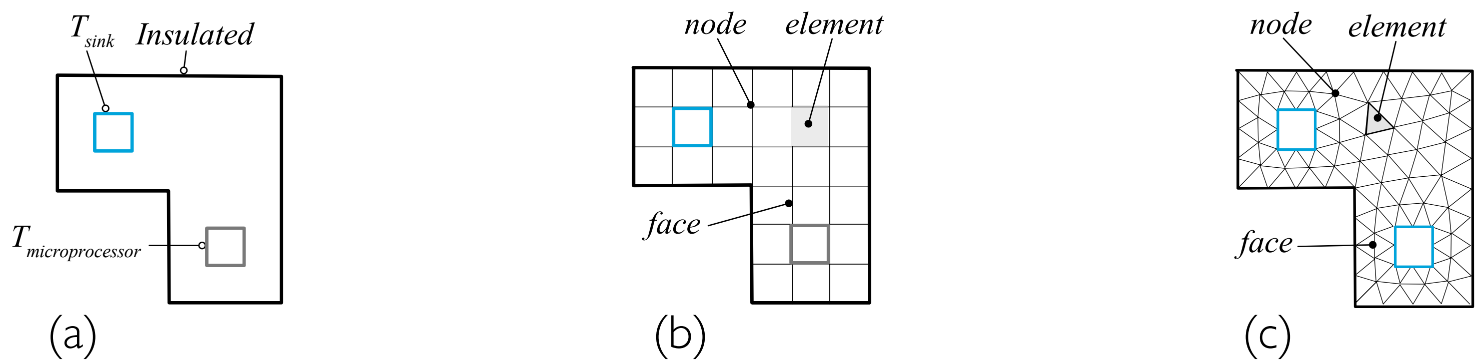


Figure 6.1 (a) Domain of interest, (b) domain discretized using a uniform grid system, and (c) domain discretized using an unstructured grid system with triangular elements.

Gradient Computation

Given a field defined over a mesh, we want to compute the field gradient

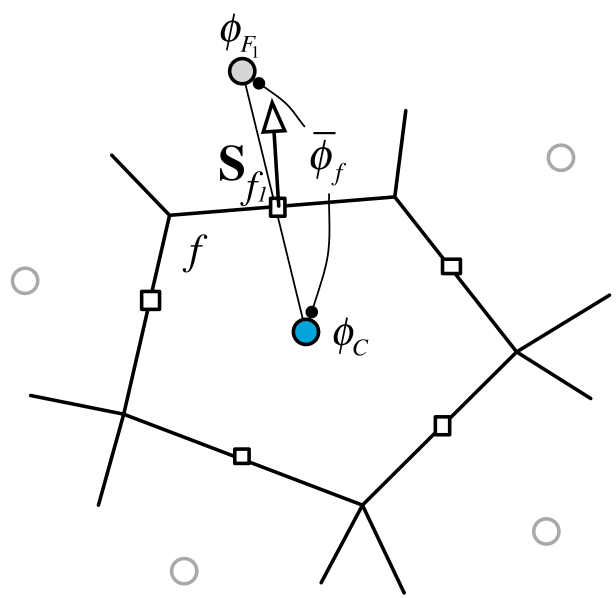
given ϕ_C compute $\nabla\phi_C$

$$\overline{\nabla\phi}_C V_C = \int_{V_C} \nabla\phi \, dV$$

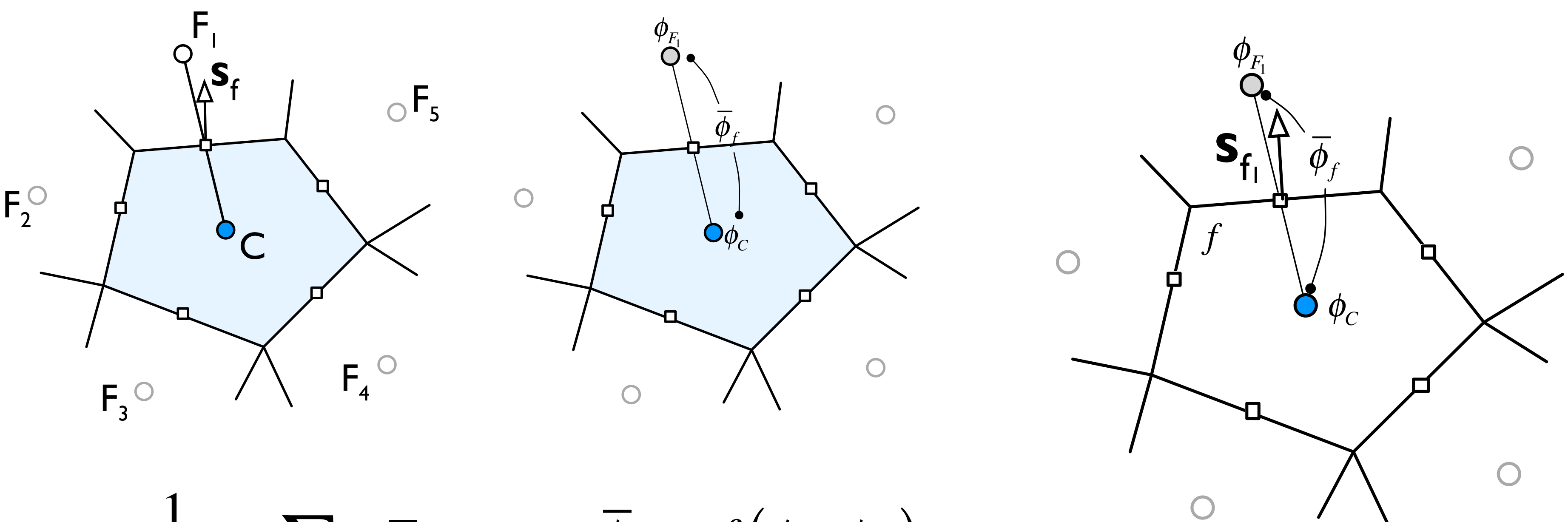
$$\overline{\nabla\phi}_C V_C = \oint_{\partial V_C} \phi \, d\mathbf{S}$$

$$\overline{\nabla\phi}_C V_C = \sum_{\partial V_C} \int \phi \, d\mathbf{S}$$

$$\overline{\nabla\phi}_C = \frac{1}{V_C} \sum_{f=nb(C)} \phi_f \, \mathbf{S}_f$$



Gradient Computation



$$\nabla\phi_{i,j} = \frac{1}{V_{i,j}} \sum_{f=nb(i,j)} \overline{\phi}_f \mathbf{S}_f$$

$$\overline{\phi}_f = f\left(\phi_C, \phi_F\right)$$

$$\overline{\phi_f} = g_F\phi_F + g_C\phi_C$$

Structured Grid

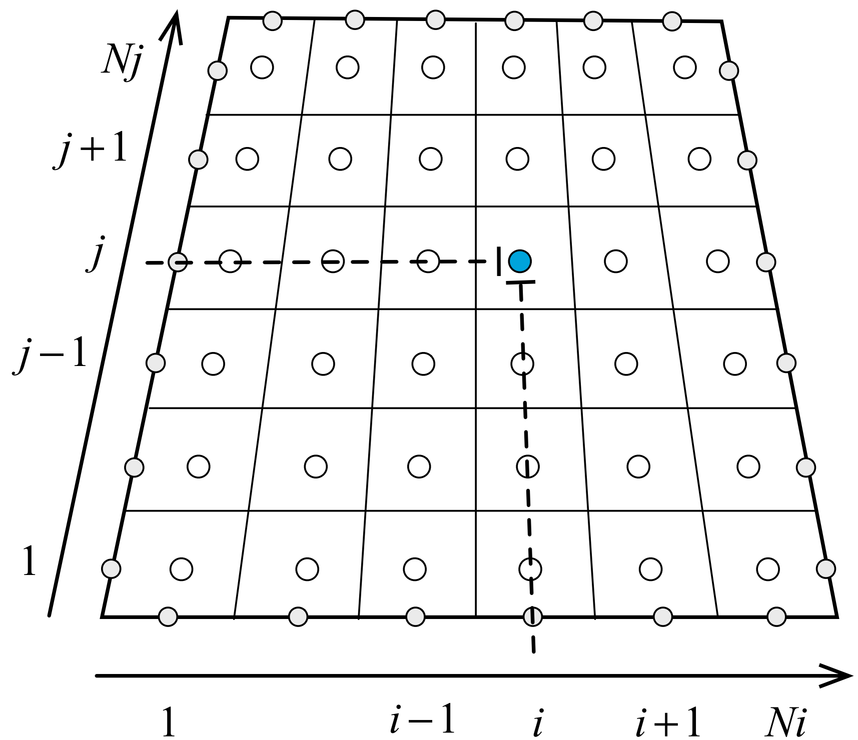


Figure 6.3 local indices and topology.

Indices

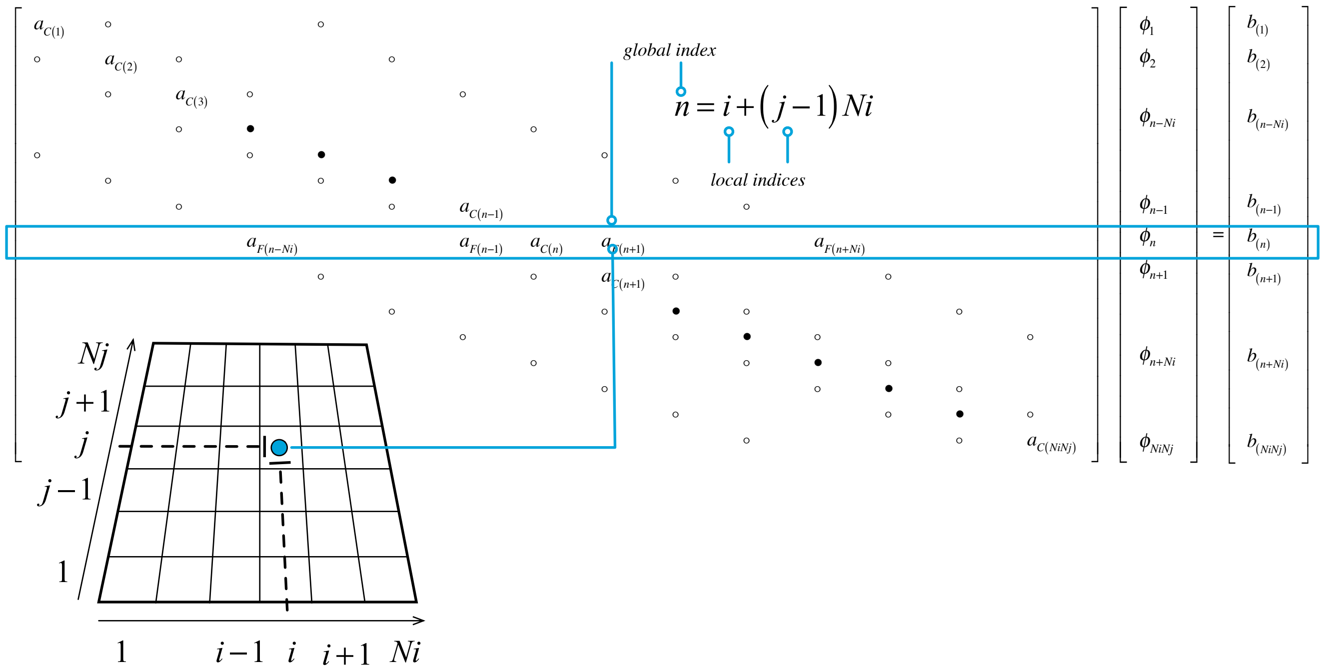


Figure 6.4 Local versus global indices.

Structured Mesh

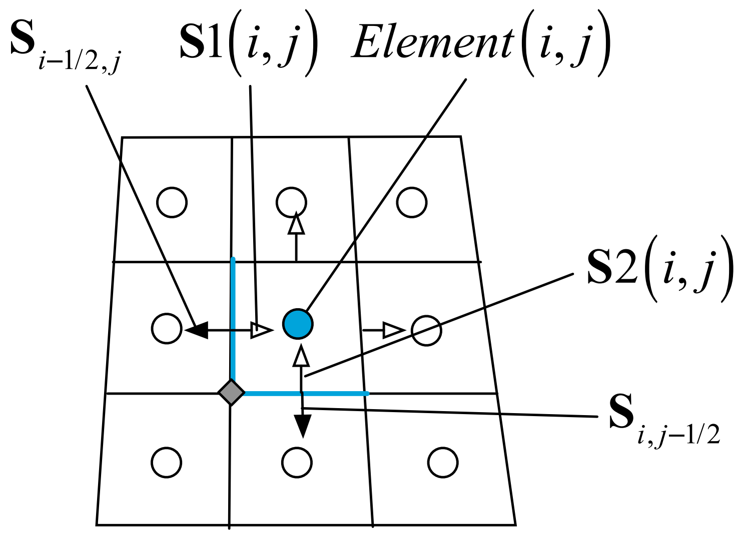


Figure 6.5 Geometric Information.

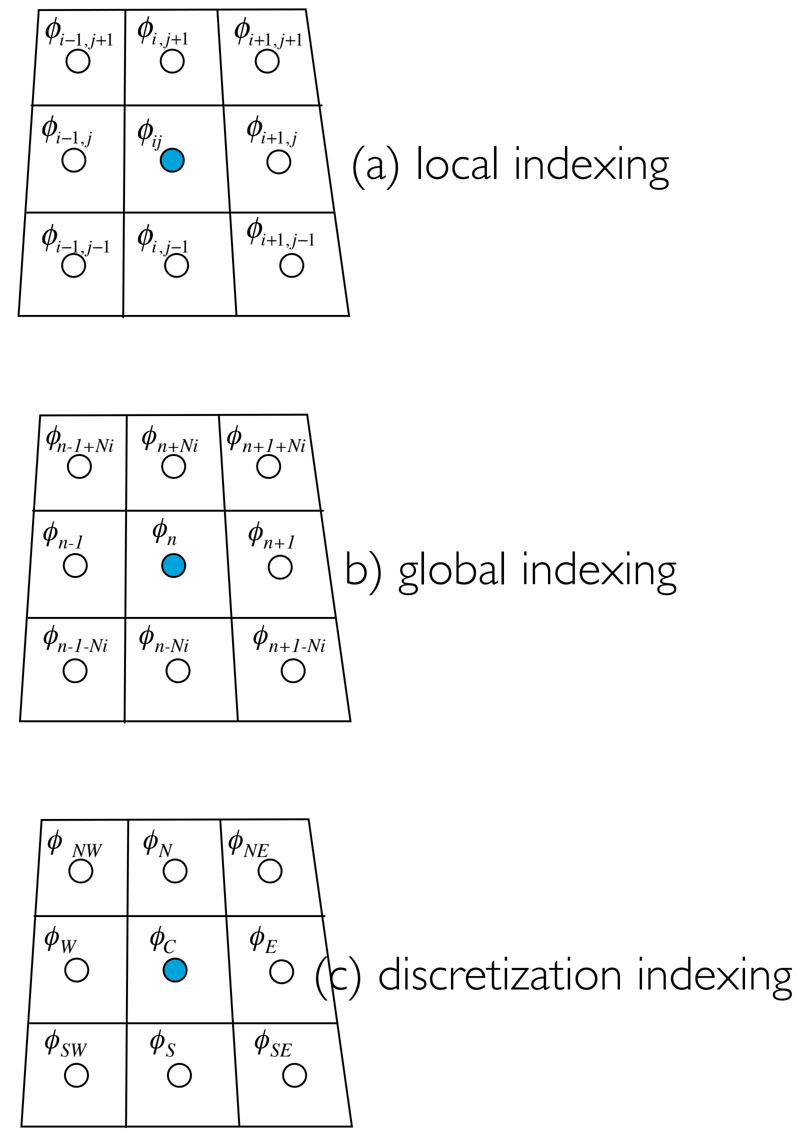
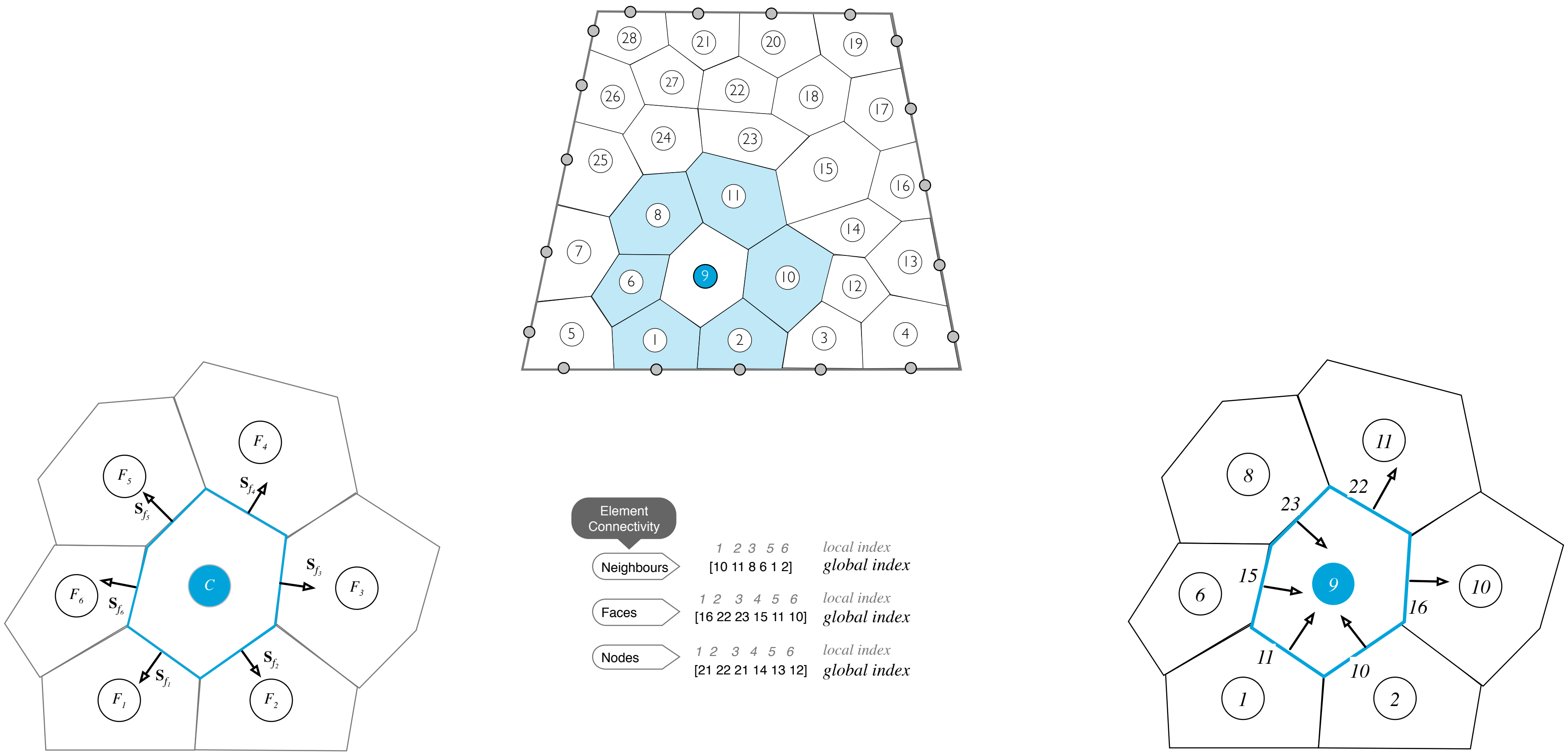


Figure 6.6 Local versus discretization versus global indices.

Unstructured Mesh indexing



(a) (b)

Figure 6.8 Element connectivity and face orientation using (a) local indices and (b) global indices.

Owners and Neighbours

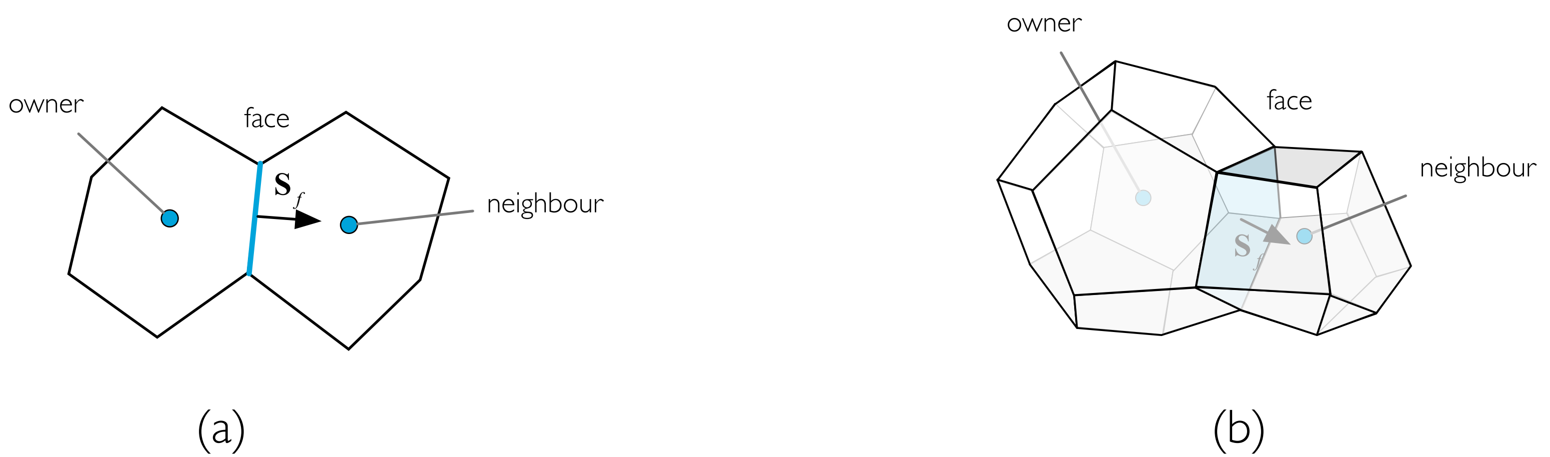


Fig. 6.9 Owners, neighbors, and faces for (a) 2D and (b) 3D elements.

Connectivities

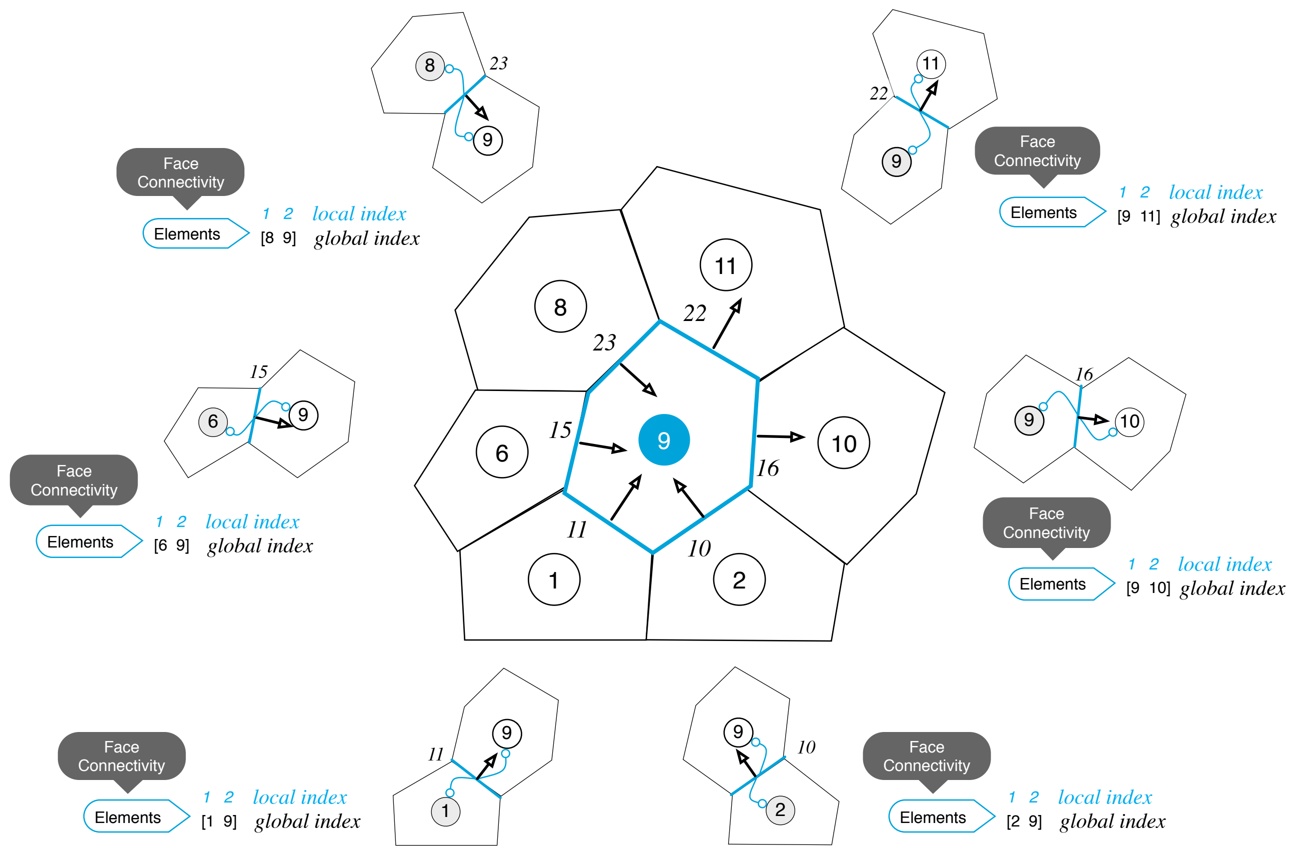


Figure 6.10 An example of face, element, and node connectivities for unstructured grids.

Unstructured Mesh

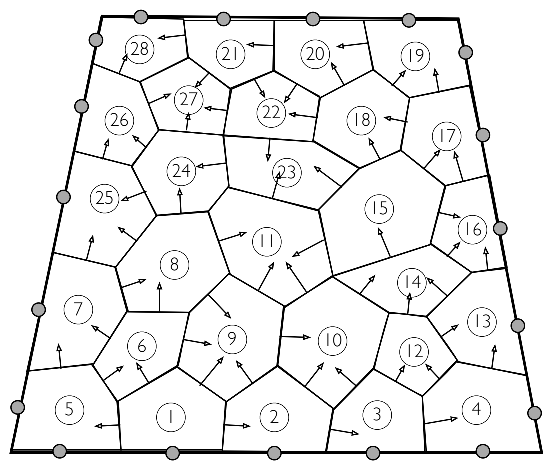


Figure 6.11 An unstructured mesh system.

Non-Orthogonality

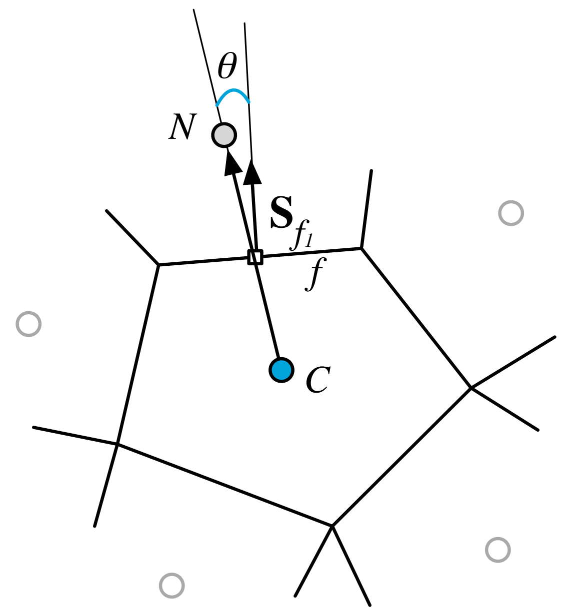


Figure 6.12 Angle between surface vector and vector joining the centroids of the owner and neighbor elements.

Elements

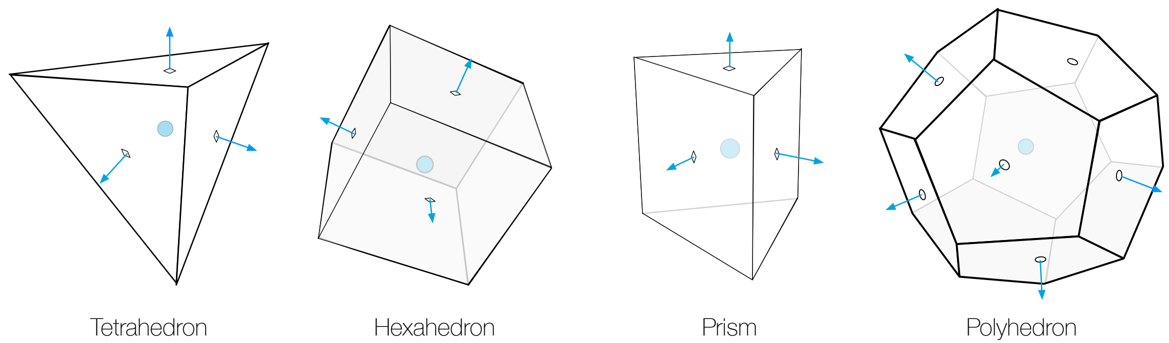


Figure 6.13 Three-dimensional element types.

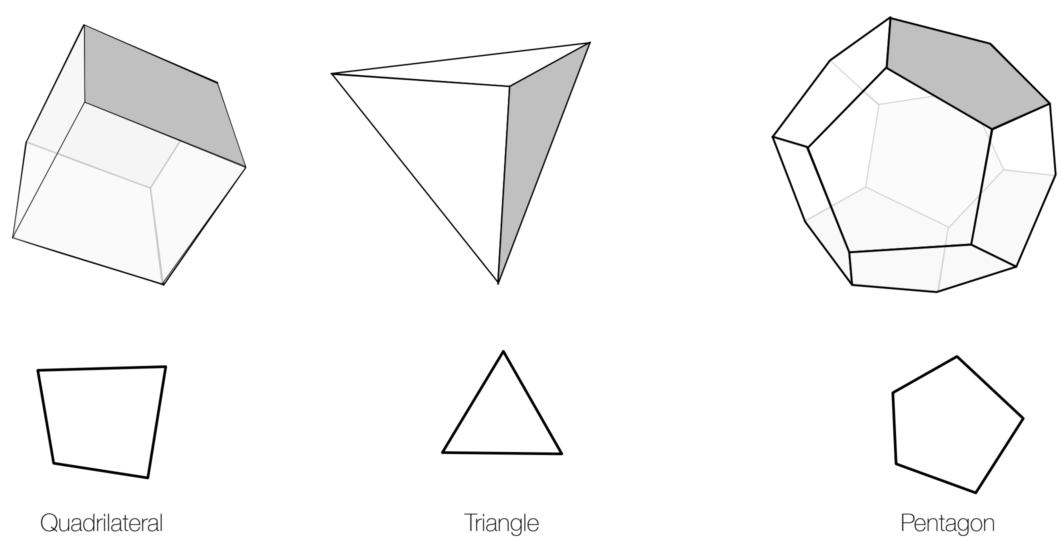


Figure 6.14 Three-dimensional face types or two-dimensional element types.

Element INformation

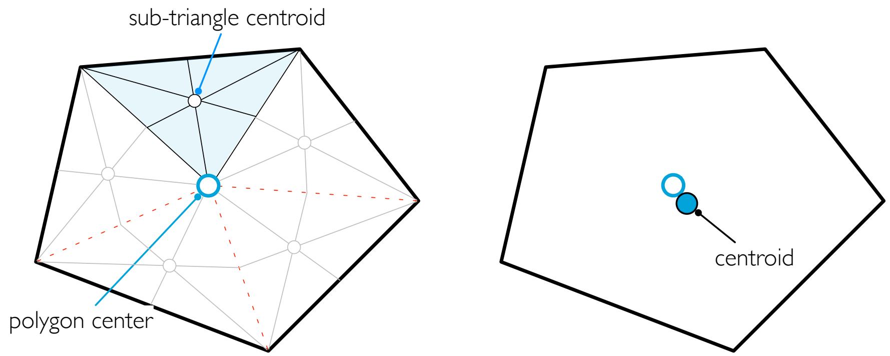


Figure 6.15 The geometric centre and centroid of a polygon.

$$\mathbf{x}_G = \frac{1}{k}\sum_{i=1}^{k}\mathbf{x}_i$$

$$S_f = \sum_{t\sim Sub-triangles(C)} S_t$$

$$\left(\mathbf{x}_{CE}\right)_f = \frac{\sum_{t\sim Sub-triangles(C)} \left(\mathbf{x}_{CE}\right)_t * S_t}{S_f}$$

$$\mathbf{S} = \frac{1}{2}\left(\mathbf{r}_2 - \mathbf{r}_1\right)\times\left(\mathbf{r}_3 - \mathbf{r}_1\right) = \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = S_x\mathbf{i} + S_y\mathbf{j} + S_z\mathbf{k}$$

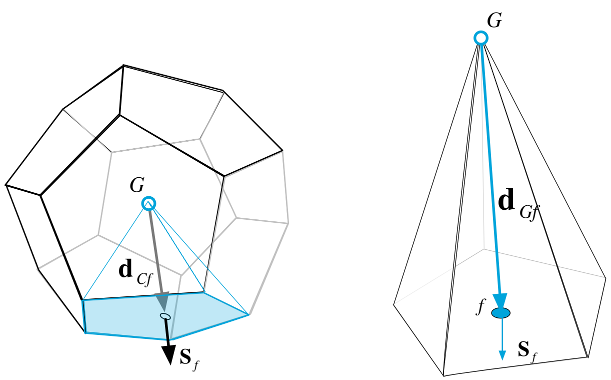


Figure 6.17 A sub-element pyramid.

$$\mathbf{x}_G = \frac{1}{k}\sum_{i=1}^{k}\mathbf{x}_i$$

$$\left(\mathbf{x}_{CE}\right)_{pyramid} = 0.75\left(\mathbf{x}_{CE}\right)_f + 0.25\left(\mathbf{x}_G\right)_{pyramid}$$

$$V_{pyramid} = \frac{\mathbf{d}_{Gf}\cdot\mathbf{S}_f}{3}$$

$$V_C = \sum_{\sim Sub-pyramids(C)} V_{pyramid}$$

$$\left(\mathbf{x}_{CE}\right)_C = \mathbf{x}_C = \frac{\sum\limits_{\sim Sub-pyramids(C)}\left(\mathbf{x}_{CE}\right)_{pyramid} V_{pyramid}}{V_C}$$

Face Weights

$$\phi_f = g_f\phi_F + \left(1 - g_f\right)\phi_C \qquad g_f = \frac{d_{Cf}}{d_{Cf} + d_{fF}}$$

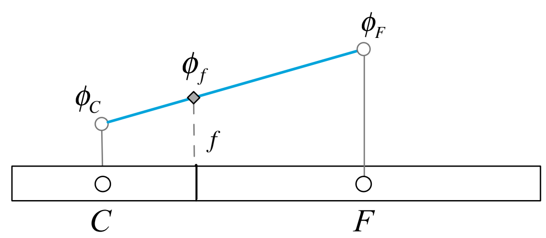


Figure 6.18 One dimensional mesh system.

$$g_f = \frac{V_C}{V_C + V_F}$$

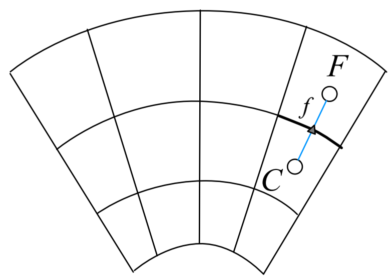


Figure 6.19 Axisymmetric grid system.

$$g_f = \frac{\mathbf{d}_{Cf} \cdot \mathbf{e}_f}{\mathbf{d}_{Cf} \cdot \mathbf{e}_f + \mathbf{d}_{fF} \cdot \mathbf{e}_f}$$

$$\mathbf{e}_f = \frac{\mathbf{S}_f}{\left\|\mathbf{S}_f\right\|}$$

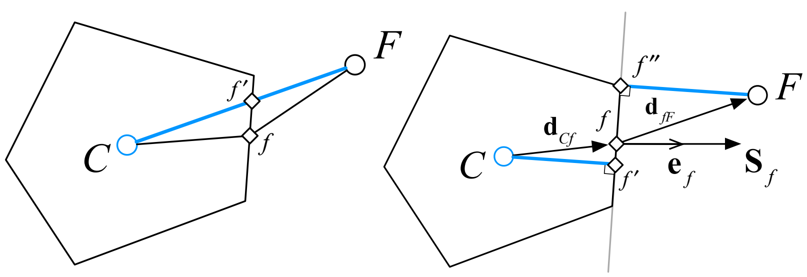


Figure 6.20 Two dimensional control volume