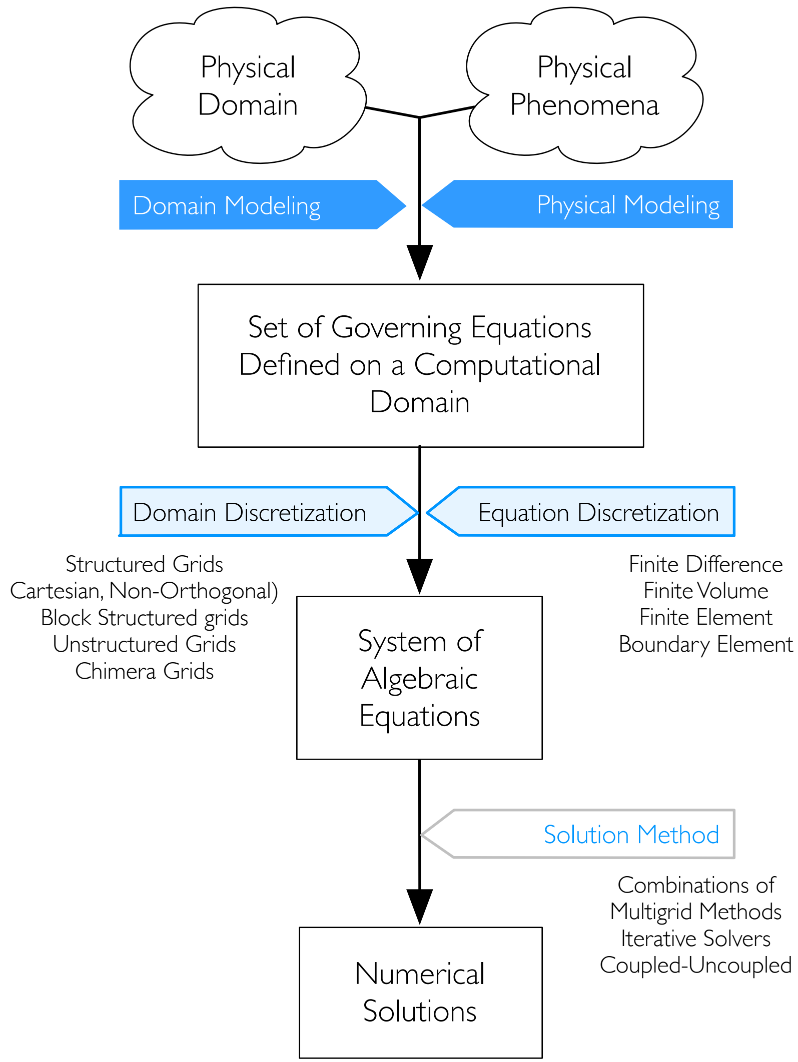
The Discretization Process

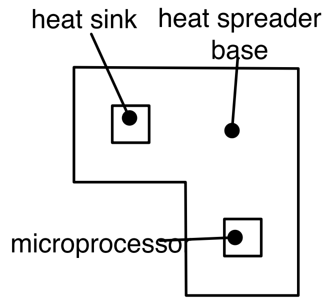
Chapter 04

The Discretization Process

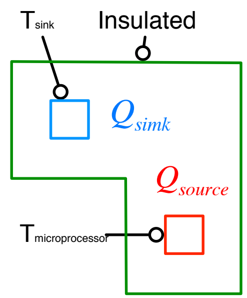
The Process


Physical Domain
Physical Phenomena
Domain Modeling
Physical Modeling
Set of Governing Equations Defined on a Computational Domain
Domain Discretization
Equation Discretization
Structured Grids
Cartesian, Non-Orthogonal)
Block Structured grids
Unstructured Grids
Chimera Grids
Finite Difference
Finite Volume
Finite Element
Boundary Element
System of Algebraic Equations
Solution Method
Combinations of
Multigrid Methods
Iterative Solvers
Coupled-Uncoupled
Numerical Solutions

Domain Modeling

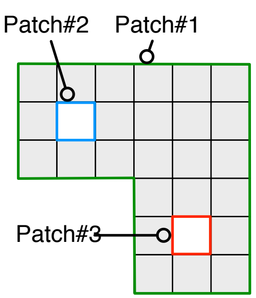


Physical Modeling

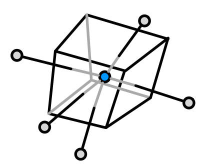


$$-\nabla\cdot\left(k\nabla T\right)=Q$$

Domain Discretization



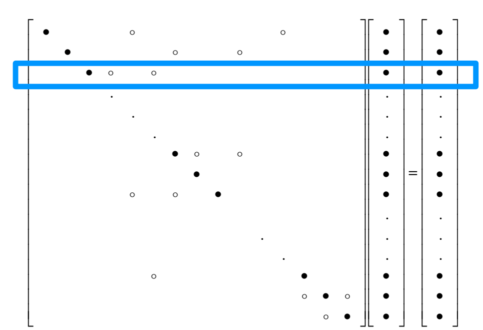
Equations Discretization



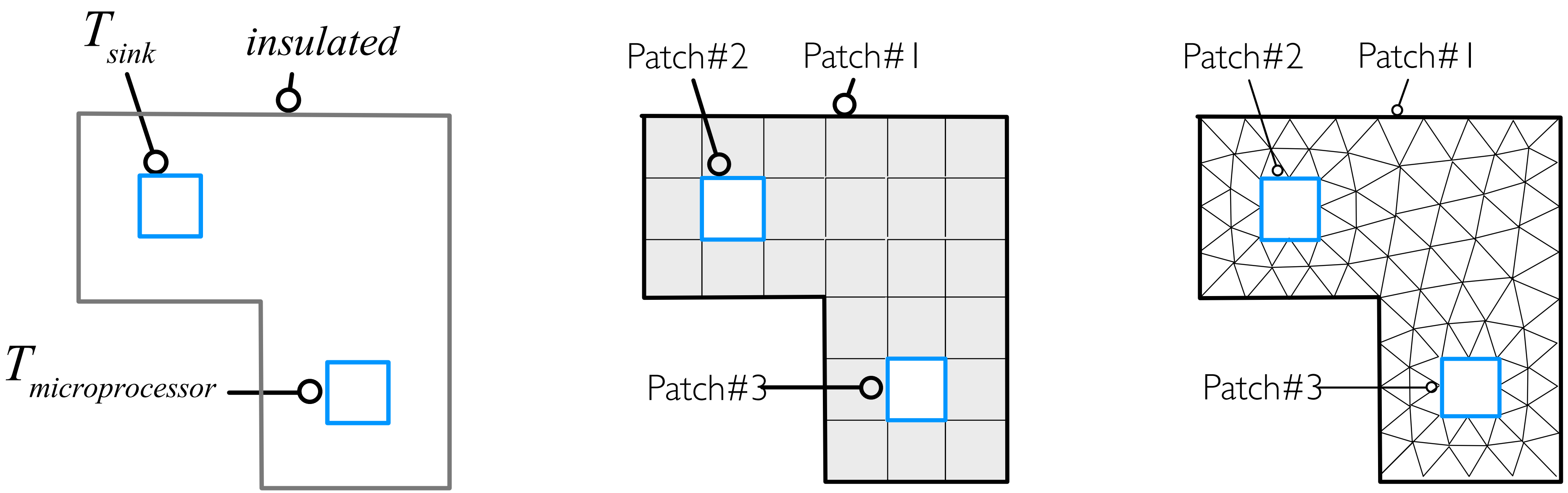
$$\underbrace{\frac{\partial(\rho\phi)}{\partial t}}_{\substack{transient\\term}}+\underbrace{\nabla\cdot(\rho\mathbf{v}\phi)}_{\substack{convection\\term}}=\underbrace{\nabla\cdot(\Gamma\nabla\phi)}_{\substack{diffusion\\term}}+\underbrace{Q}_{\substack{source\\term}}$$

$$a_C\phi_C+\sum_{NB(C)}a_{NB}\phi_{NB}=b_P$$

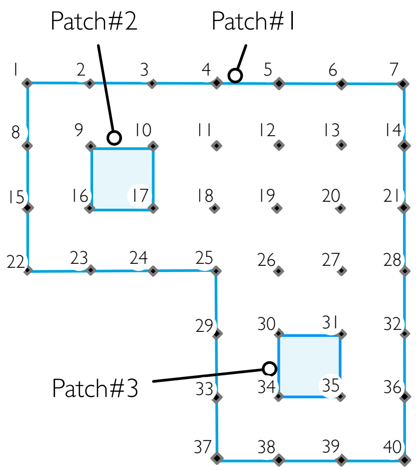
Solution Method

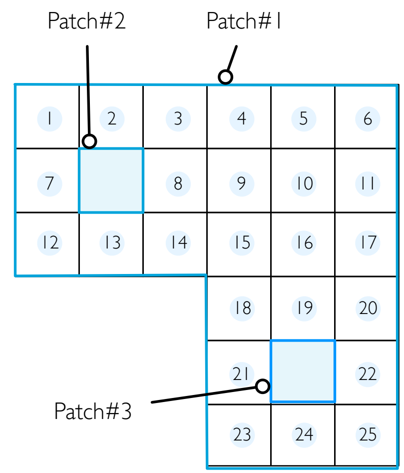


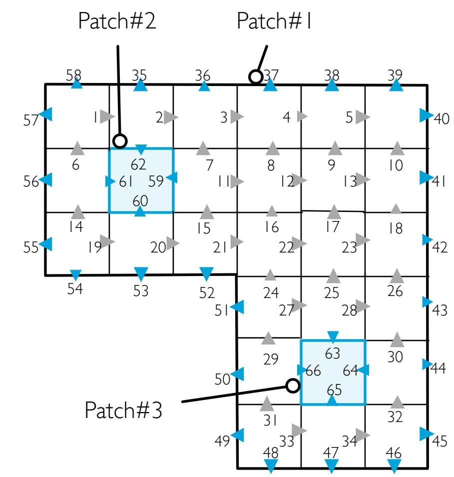
Domain Discretization

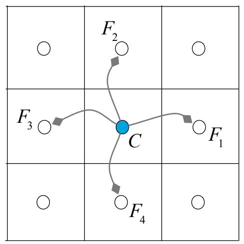


Numbering



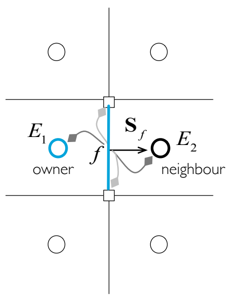






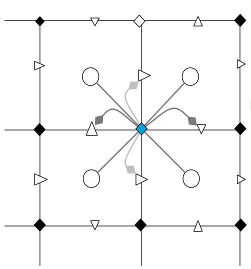
Element 9 Connectivity

Neighbours	[10 4 8 15]
Faces	[12 8 11 16]
Vertices	[19 11 12 18]



Face 12 Connectivity

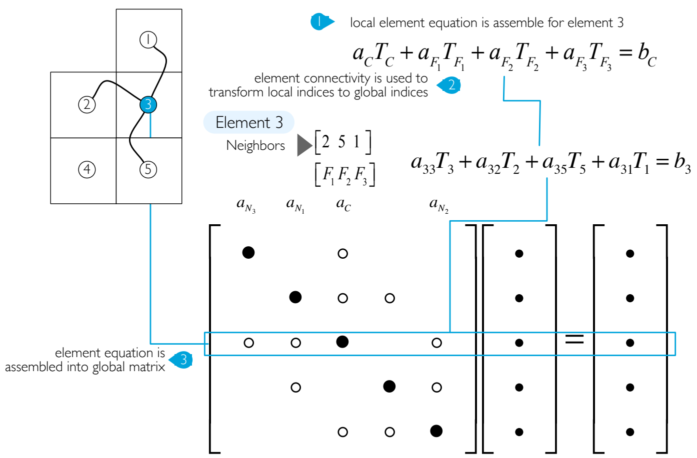
Element1	9
Element2	10
Vertices	[19 12]

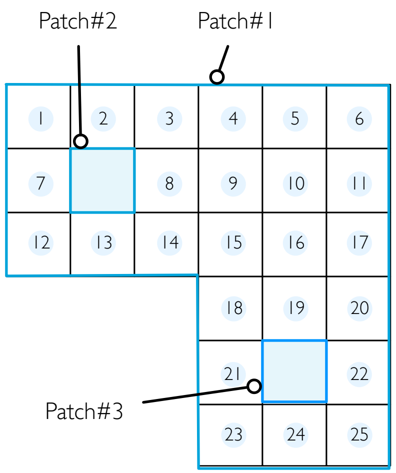


Vertex Connectivity

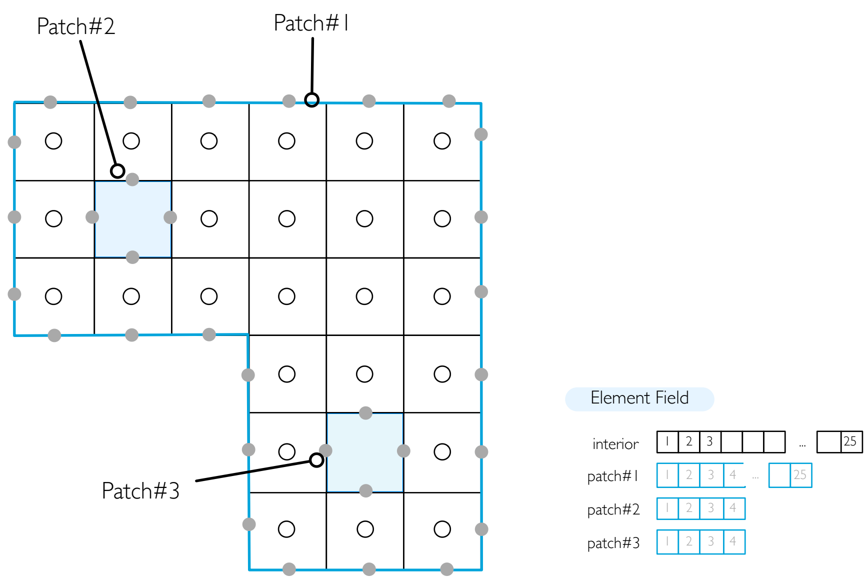
Elements	[...]
Faces	[...]

Sparse Matrix Equation

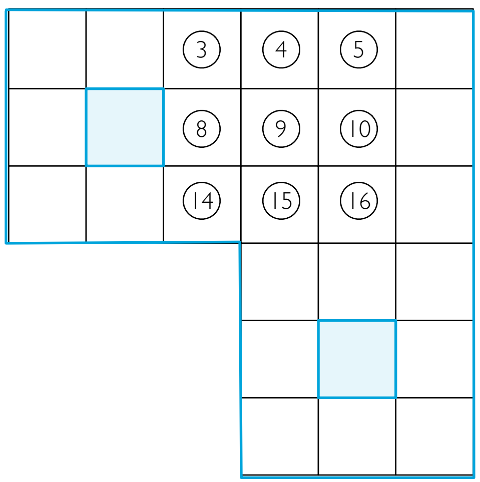


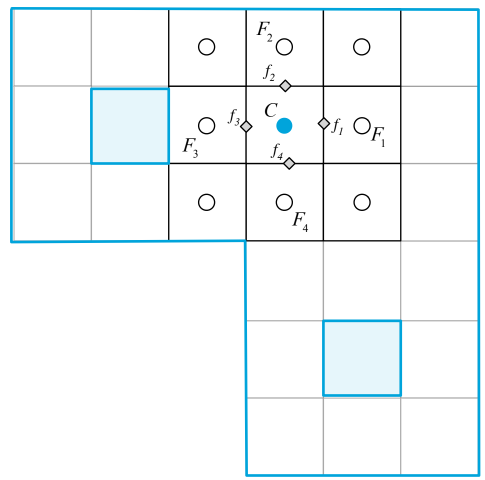


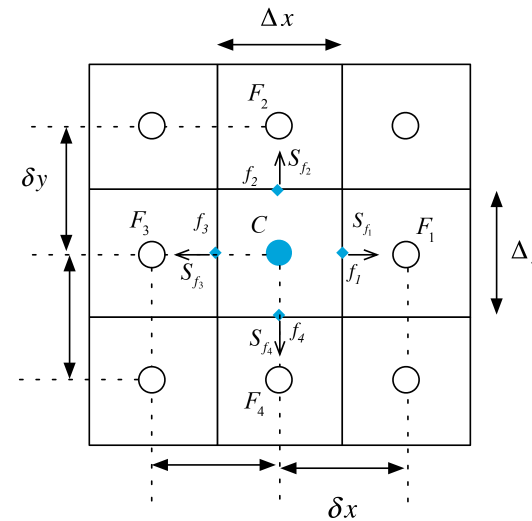
Element Fields



Equation Discretization







$$\mathbf{S}_{f_1} = \Delta y_{f_1} \mathbf{i}$$

$$\delta x_{f_1} = x_{N_1} - x_C$$

$$\nabla T_{f_1} = \frac{\partial T}{\partial x}\mathbf{i} + \frac{\partial T}{\partial y}\mathbf{j}$$

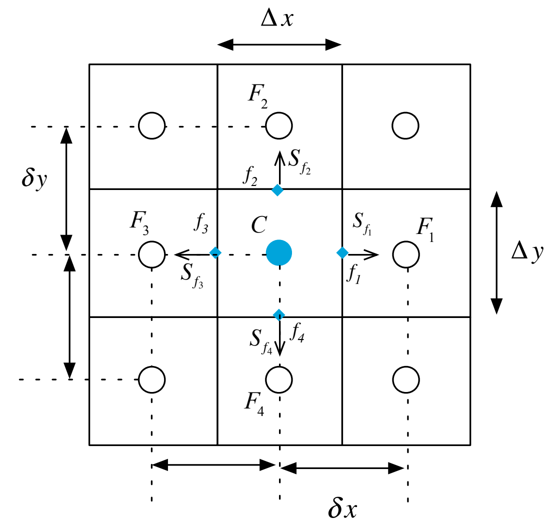
$$-\nabla \cdot \left(k \nabla T \right) = 0$$

$$-\oint_{S_C} \left(k \nabla T \right) \cdot d\mathbf{S} = 0$$

$$-\sum_{f(C)} \left(k \nabla T \right)_f \cdot \mathbf{S}_f = 0$$

$$-\left(k \nabla T \right)_{f_1} \cdot \mathbf{S}_{f_1} - \left(k \nabla T \right)_{f_2} \cdot \mathbf{S}_{f_2} - \left(k \nabla T \right)_{f_3} \cdot \mathbf{S}_{f_3} - \left(k \nabla T \right)_{f_4} \cdot \mathbf{S}_{f_4} = 0$$

Coefficients



$$\nabla T_{f_1} \cdot \mathbf{S}_{f_1} = \left(\frac{\partial T}{\partial x}\mathbf{i} + \frac{\partial T}{\partial y}\mathbf{j} \right)_{f_1} \cdot \Delta y_{f_1}\mathbf{i}$$

$$= \left(\frac{\partial T}{\partial x} \right)_{f_1} \Delta y_{f_1}$$

$$\left(\frac{\partial T}{\partial x} \right)_{f_1} = \frac{T_{N_1} - T_C}{\delta x_{f_1}}$$

$$\nabla T_{f_1} \cdot \mathbf{S}_{f_1} = \frac{T_{F_1} - T_C}{\delta x_{f_1}} \Delta y_{f_1} \qquad -\left(k \nabla T\right)_{f_1} \cdot \mathbf{S}_{f_1} = a_{F_1}\left(T_{F_1} - T_C\right)$$

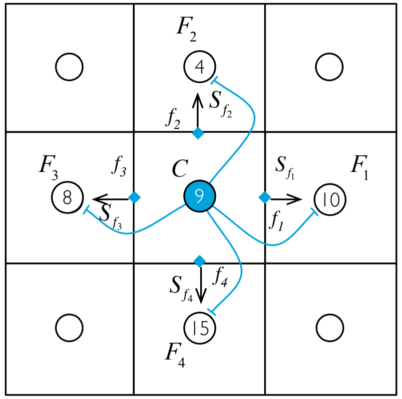
$$a_{F_1} = -k \frac{\Delta y_{f_1}}{\delta x_{f_1}}$$

Algebraic Equation

$$a_{F_2} = -k\frac{\Delta x_{f_2}}{\delta y_{f_2}}$$

$$a_{F_3} = -k\frac{\Delta y_{f_3}}{\delta x_{f_3}}$$

$$a_{F_4} = -k\frac{\Delta x_{f4}}{\delta y_{f_4}}$$



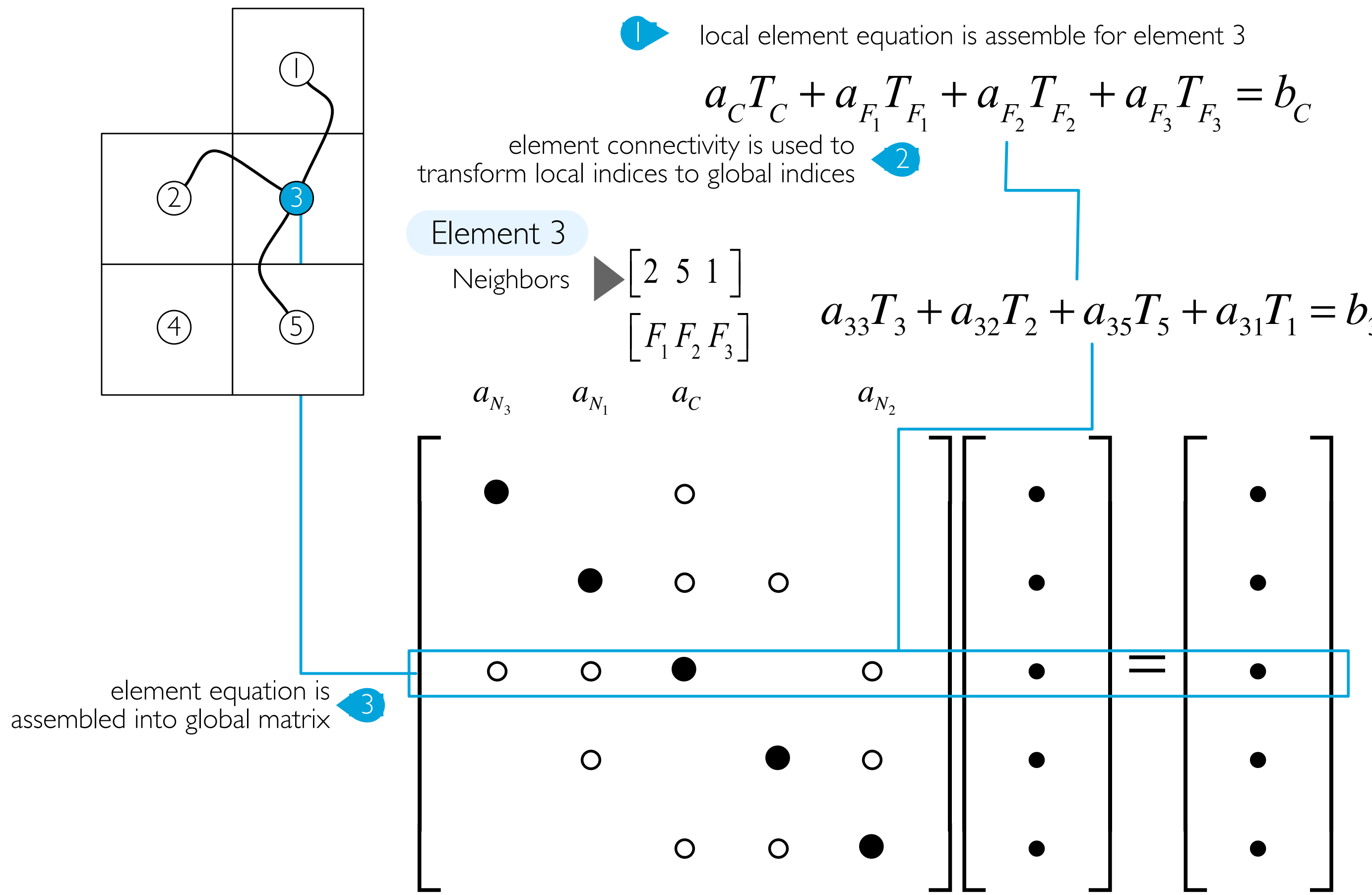
$$0 = -\sum_{f(C)} \left(k\nabla T\right)_f \cdot \mathbf{S}_f = \sum_{NB(C)} a_{NB}\left(T_{NB} - T_C\right)$$

$$= -\left(a_{F_1} + a_{F_2} + a_{F_3} + a_{F_4}\right)T_C + a_{F_1}T_{F_1} + a_{F_2}T_{F_2} + a_{F_3}T_{F_3} + a_{F_4}T_{F_4}$$

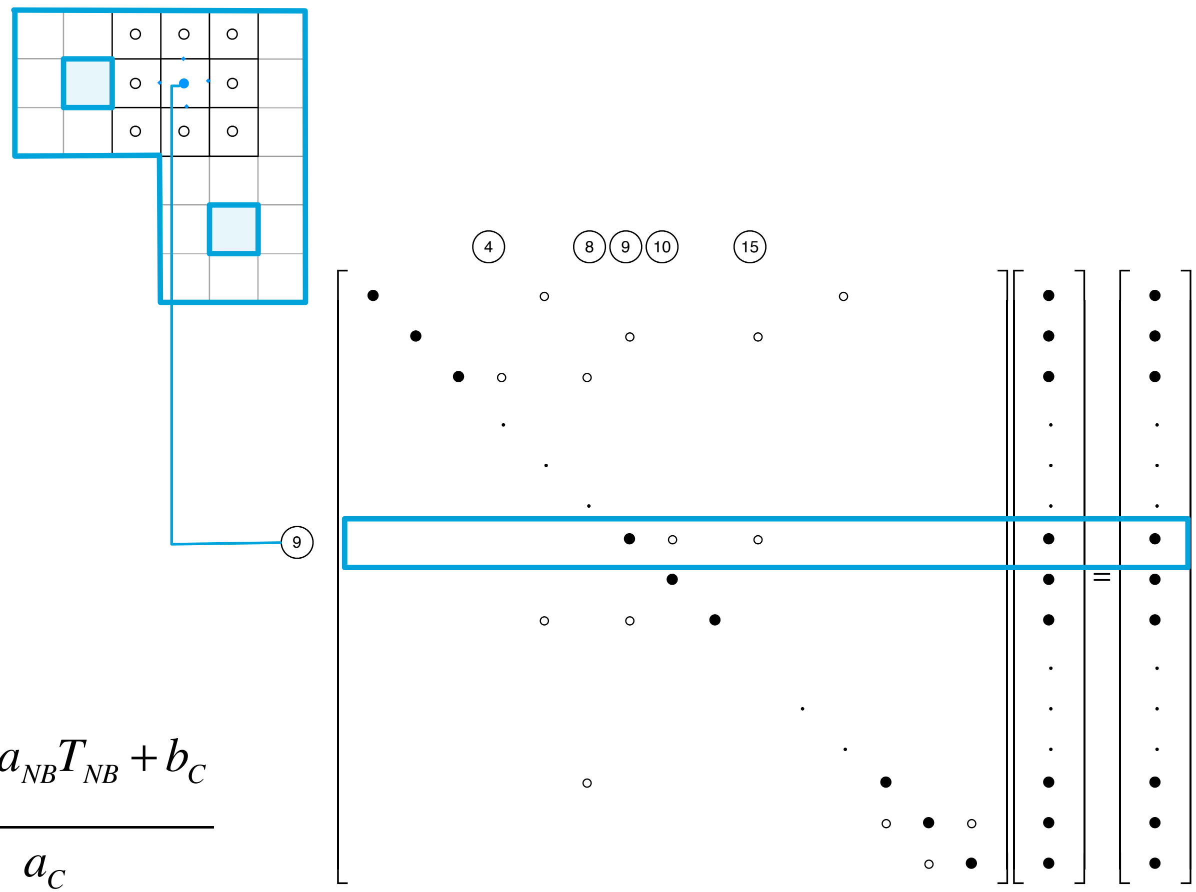
$$a_C T_C + \sum_{NB(C)} a_{NB} T_{NB} = 0$$

$$a_C = -\sum_{NB(C)} a_{NB} = -\left(a_{F_1} + a_{F_2} + a_{F_3} + a_{F_4}\right)$$

System of Equations

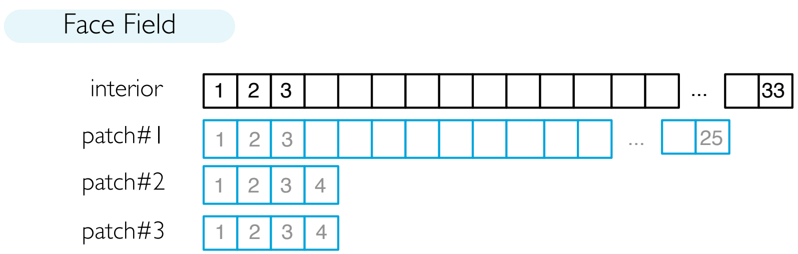


Solving the Equations



$$T_C = \frac{-\sum_{NB(C)} a_{NB} T_{NB} + b_C}{a_C}$$

Face Field

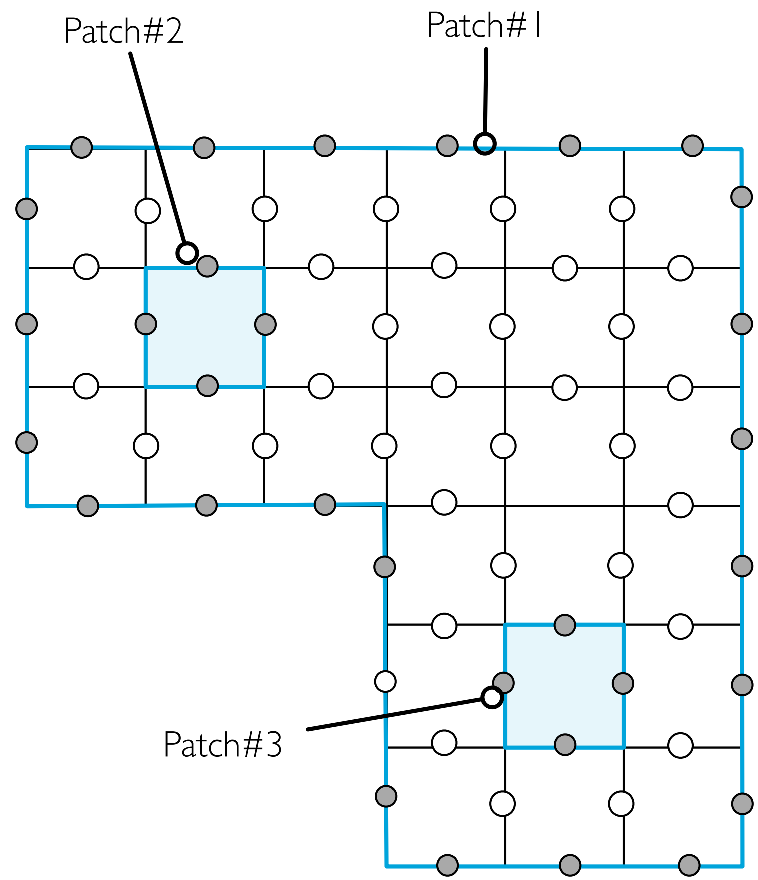


$$\mathbf{S}_f$$

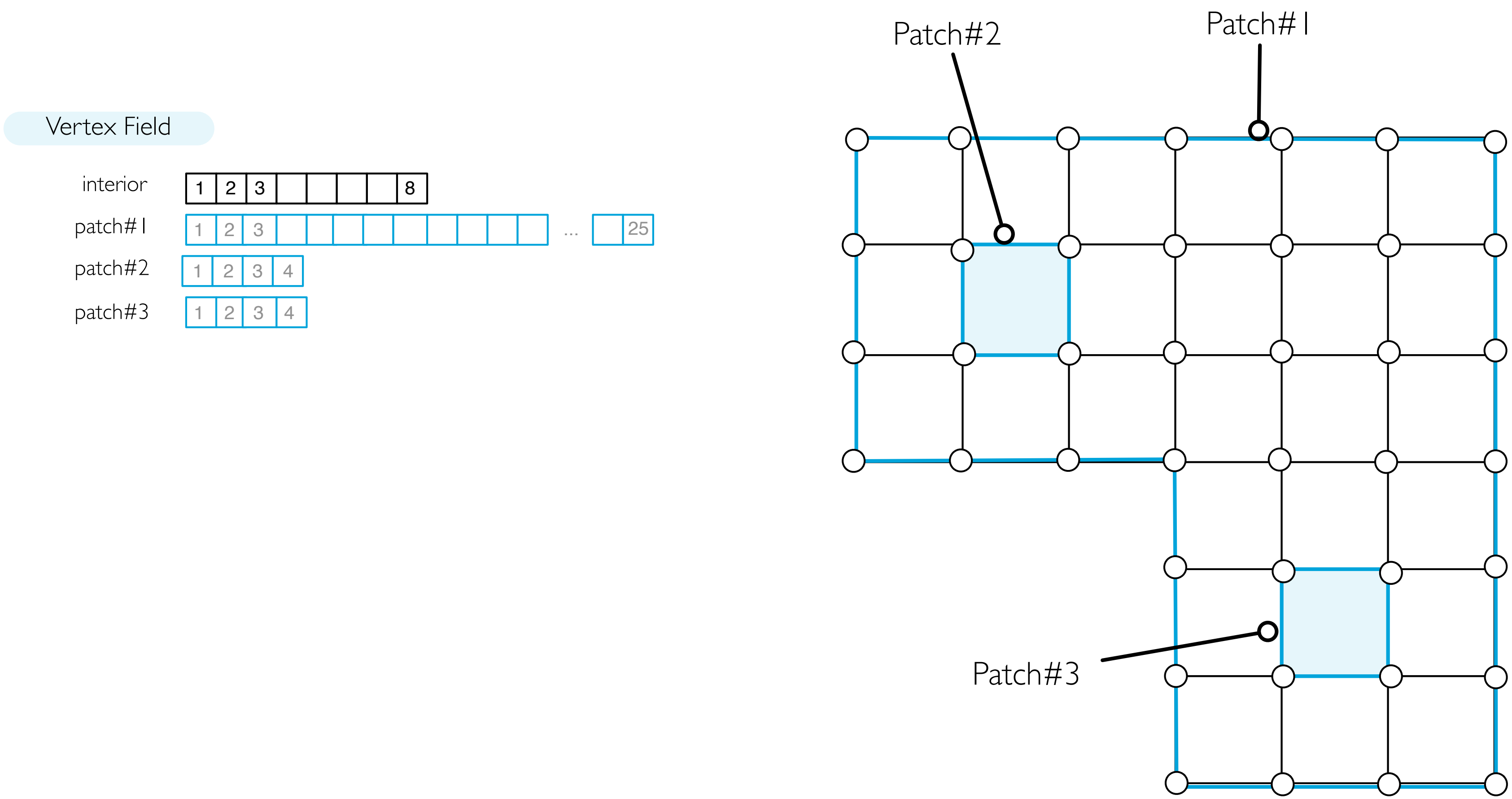
$$\mathbf{d}_{CF}$$

$$g_{CF}$$

$$\left\|\mathbf{S}_f\right\|$$



Node field



Vertex Field
interior
1 2 3 8
patch#1
1 2 3 ... 25
patch#2
1 2 3 4
patch#3
1 2 3 4
Patch#2
Patch#1
Patch#3