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Beirut Office

Lesson Study in Mathematics: Its potential for educational improvement in mathematics and for fostering deep professional learning by teachers

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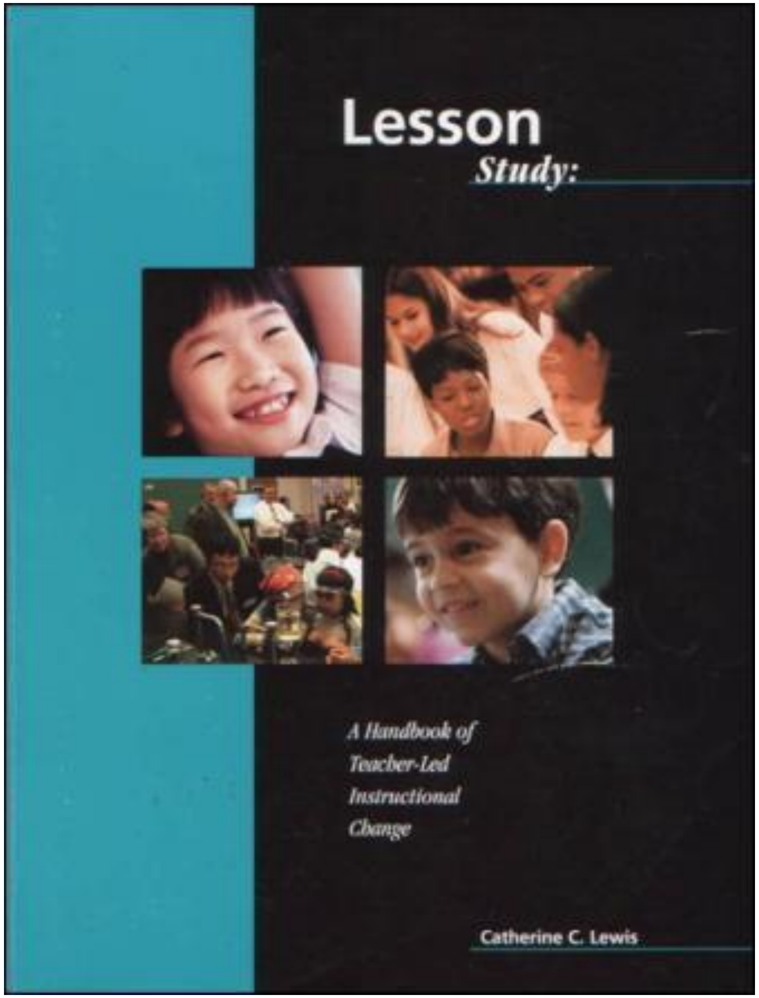
Lesson Study in Japan

- Lesson study needs to be viewed as a feature teacher professional learning across the whole-school
- It needs to be supported at all levels of the school and by educational agencies beyond the school
- It has a direct relationship to the National Course of Study

Lesson Study in Japan

- Lesson study is a proving ground for all teachers
- Lesson Study is about building teacher capacity – in the long-term
- It is not a hobby for a few teachers, or an optional extra
- Its focus is on the improvement of teaching and learning



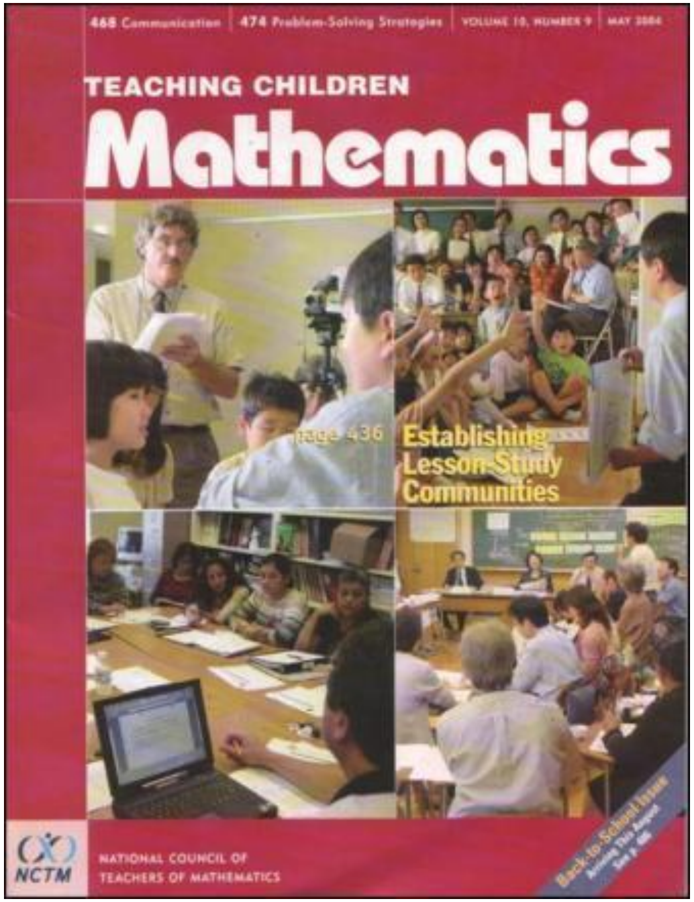
Lesson Study:

A Handbook of Teacher-Led Instructional Change

Catherine Lewis (2002)

Research for

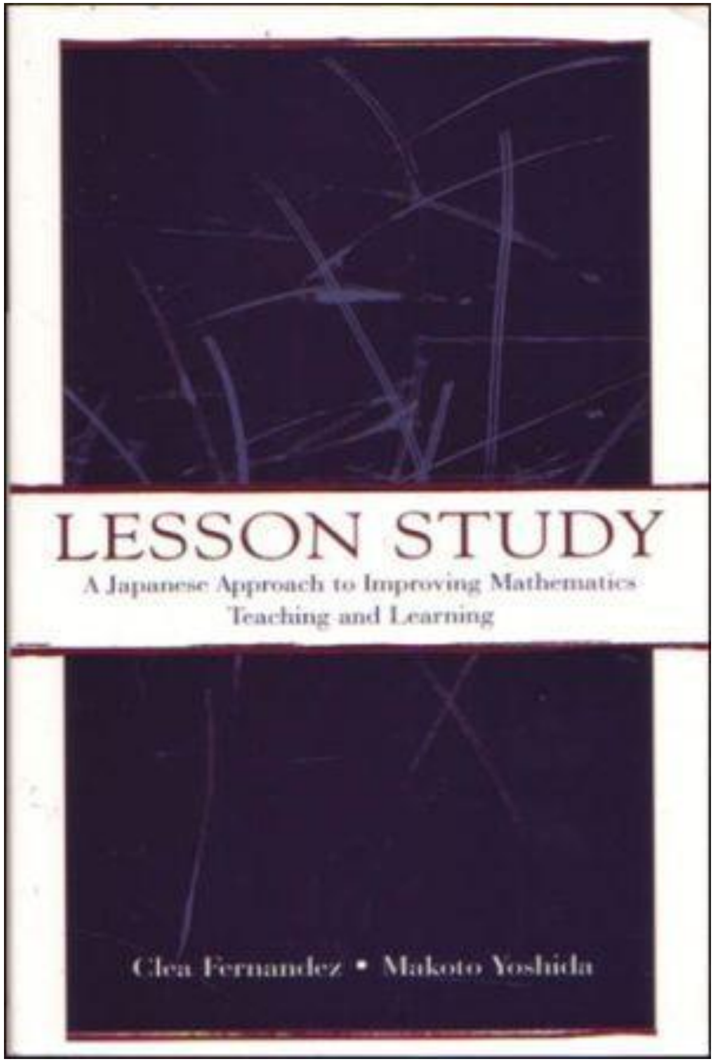
Better Schools



"Ideas for Establishing Lesson Study Communities"

Takahashi & Yoshida

Teaching Children Mathematics, May, 2004 (NCTM)

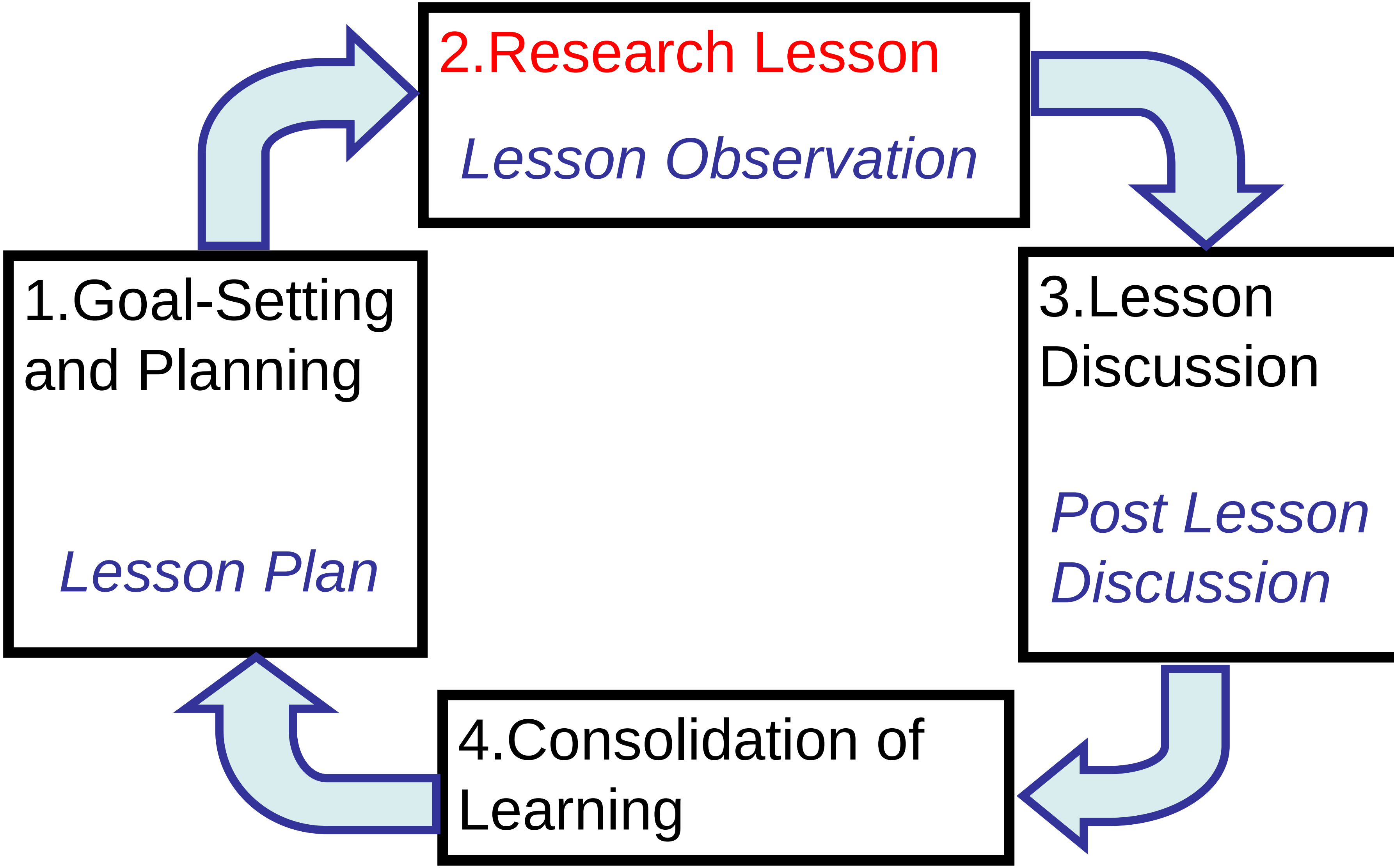


Lesson Study: A Japanese Approach to Improving Mathematics Teaching and Learning

Fernandez & Yoshida (2004)

Lawrence Erlbaum Associates, Publishers

Lesson Study Cycle (Lewis (2002)

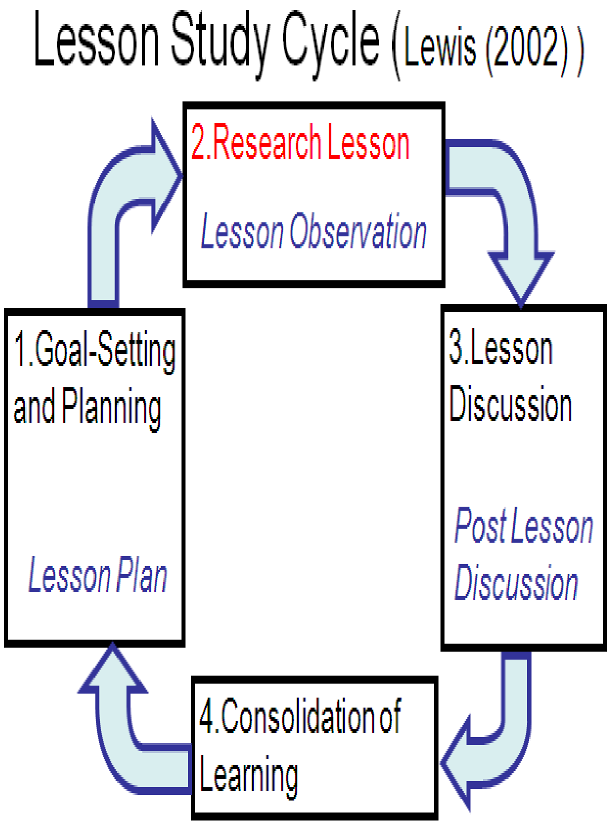


Lesson Study Cycle

- Lesson study is not just about improving a single lesson
- It is about building pathways for improvement of instruction
- It contributes to a culture of teacher-initiated research and to teachers' collective knowledge
- It focus is always improving children's mathematical learning and understanding

(Lewis, 2004, p. 18)

Lesson Study Cycle



Planning : making a detailed lesson plan

How do teachers in Japan work together to create a plan for a research lesson?

1 単元の目標

Goals of this unit

○分数に整数をかける乗法や，分数を整数でわる除法の意味とその計算のしかたについて理解し，それを用いる能力を伸ばす。

〔関心・意欲・態度〕・分数×整数，分数÷整数の計算のしかたを，既習の分数の性質，計算と関連づけて考えようとする。

〔数学的な考え方〕・分数×整数，分数÷整数の計算のしかたは，既習の分数の性質，計算をもとにして考えればよいことに気づく。

〔表現・処理〕・分数×整数，分数÷整数の計算ができる。

〔知識・理解〕・分数×整数，分数÷整数の計算の意味やその計算のしかたが分かる。

2 教材の関連と発展

Related Units in previous and following grades

5年

第10単元
- 同じ大きさを表す分数
- 約分，通分の意味とその方法
- 異分母分数の加減計算

↓

第11単元
- 整数の除法の商は，分数を用いて表せること
- 分数倍の意味
- 整数，小数，分数の関係

6年

第1単元
- 分数に整数をかける計算
- 分数を整数でわる計算

第2単元
- 分数をかけることの意味とその計算
- 基準量×分数倍＝比較量

第3単元
- 分数でわることの意味とその計算
- 逆数の意味
- 整数と小数の混合計算を，分数の形にして計算すること

中学

- $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
- $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

の形が，有理数(負の数も含む)へ拡張しても保存されていること

4. 展開

1. 題意をとらえる。 ●教科書の絵は何をしている場面ですか。 ●分かっていることは何ですか。また，求めることは何ですか。	◎さし絵を見ながら，1を読み，題意をつかむ。 ○$\frac{3}{4}$dℓで，板を$\frac{2}{5}$ m²ぬれるペンキがあるとき，1 dℓでは何 m²ぬれるか。 ◎求めるのは1 dℓでぬれる面積であることを理解する。 ○分かっていること…$\frac{3}{4}$dℓでぬれる面積は$\frac{2}{5}$ m² ○求めること…………1 dℓでぬれる面積	＊本時の課題である単位量あたりの面積を求める場合について，関心を向けるような発問を工夫する。 ＊問題文を板書するか，紙に書いて提示する。 ＊問題の解決に必要な数値にアンダーラインをひかせるなどして，条件と求答事項を明確にする。
2. 1 dℓでぬれる面積を求める式を考える。 (自力解決) ●どんな式を書けばよいですか。その理由も考えましょう。	◎既習事項をもとに自力で立式を考える。 ◎数直線図や言葉の式をもとに考え，整数の場合と同じ構造であることに気づく。	関分数の除法の意味を数直線図などを用いて考えようとする。 ○吹き出しを手がかりにして$\frac{3}{4}$を整数（例，2など）にして考えるように助言する。
3. たてた式とその根拠を発表し，検討する。 ●$\frac{2}{5}\div\frac{3}{4}$の式でよいわけを説明しましょう	◎自分で考えた立式の根拠を説明する。 ㋐数直線が整数，小数と同じ形だから。 ㋑言葉の式にあてはめると，$\frac{2}{5}\div\frac{3}{4}$になるから。 ㋒$\square\times\frac{3}{4}=\frac{2}{5}$より，$\square=\frac{2}{5}\div\frac{3}{4}$	関数直線図や言葉の式をもとに立式の根拠を明らかにしようとする。（発言・ノート） ＊㋒は，第2小単元の倍とわり算で活用するアイディアである。
4. 立式の根拠と分数でわる除法の意味を理解する。	◎わり算は，1つ分の数量を求める計算であることに気づき，除数が分数であってもわり算の式がたてられることを理解する。	知分数でわることの意味が分かる。 ○言葉の式のみを根拠として立式した児童には，数直線図で÷整数と÷分数が同じ形になっていることに気づくように支援する。
5. 分数でわる計算のしかたを考える。 (自力解決) ●$\frac{2}{5}\div\frac{3}{4}$の商は，どのように求めればよいでしょうか。分数でわる計算のしかたを考えましょう。また，いろいろな計算のしかたを考えましょう。	◎既習事項をもとに自力で考える。また，一つだけでなく，多様な方法も模索する。 ㋐図等を手がかりに分数の意味（$\frac{3}{4}$は$\frac{1}{4}$が3つ）に戻って考えた。 $\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\div3\times4=\frac{2\times4}{5\times3}$ ㋑小数のわり算で用いたわり算の計算のきまりを活用した。 $\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\times4\div3=\frac{2\times4}{5\times3}$	＊教科書は閉じるように指示する 考分数の除法の計算のしかたを筋道立てて説明することができる。（ノート・発言） ＊見通しがつかない児童には，教科書の図をもとに考えるように助言する。 ＊教科書の面積図をもとに指導する場合には，色の濃淡の違いに気をつけたい。
6. それぞれの考えを発表し，検討する。 ●友達の考えの中で同じところや似ているところ，あるいは違っているところを見つけましょう。	◎集団解決でそれぞれの考えを発表する。 ◎疑問点などを質問したり，似ているところを見つけたりしながら，それぞれの方法を検討する。	＊自分の考えを発表する際には，面積図や数直線図を積極的に用いるように指示する。 関友達の考えに関心をもち，それらの共通点や相違点を見つけ，よさを認めようとする。（発言・学習感想）
7. 真分数÷真分数の計算のしかたをまとめ，適用問題に取り組む。	◎教科書の面積図をもとに，$\frac{1}{4}$dℓでぬれる面積を求めた後に，1 dℓでぬれる面積を求める式変形を確認する。 ◎計算のしかたをまとめる。	＊児童の実態によっては，他の真分数でも説明するなどの活動をとおして，真分数÷真分数の計算のしかた一般化まで高めるように配慮したい。
8. 学習感想を書く。	◎自分の言葉でまとめる。	

Key items and questions to ask

Anticipated students' responses

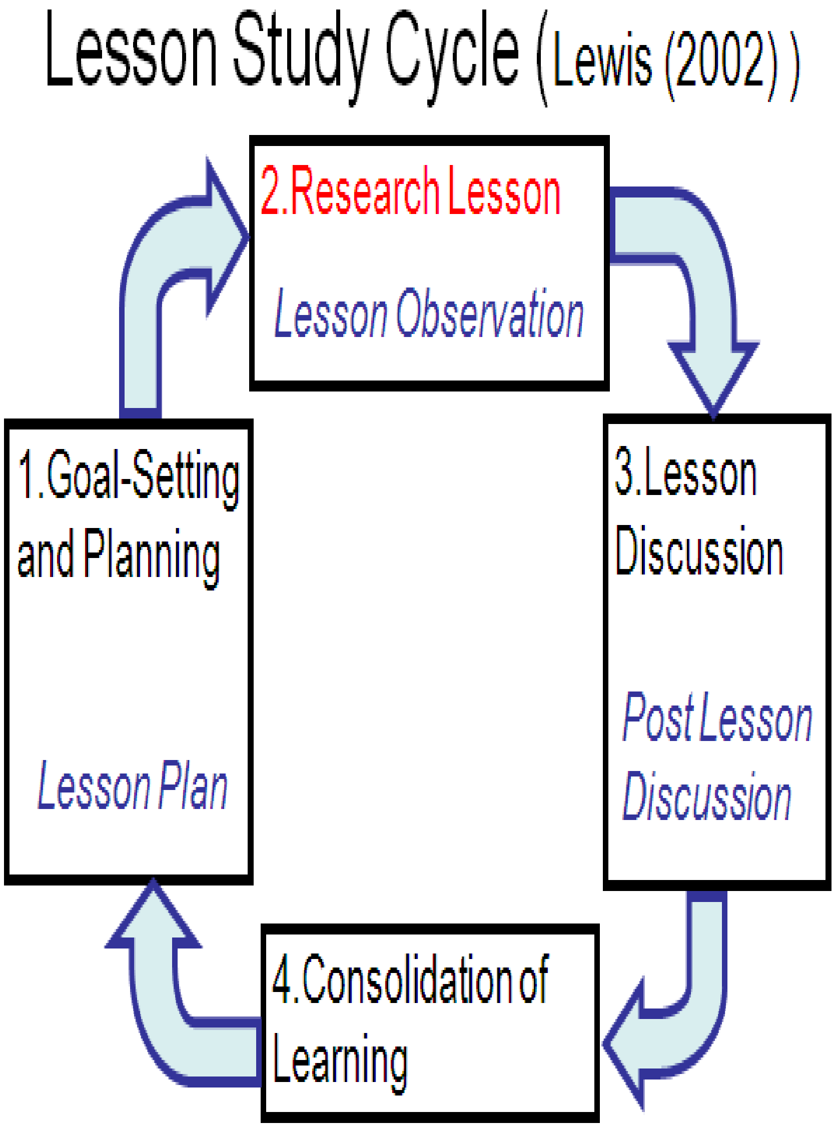
Teacher's notes:

how to evaluate

how to use tools,

what to emphasize

Lesson Study Cycle



The Lesson is a Problem solving oriented lesson

Shulman(1987) on Teacher's knowledge

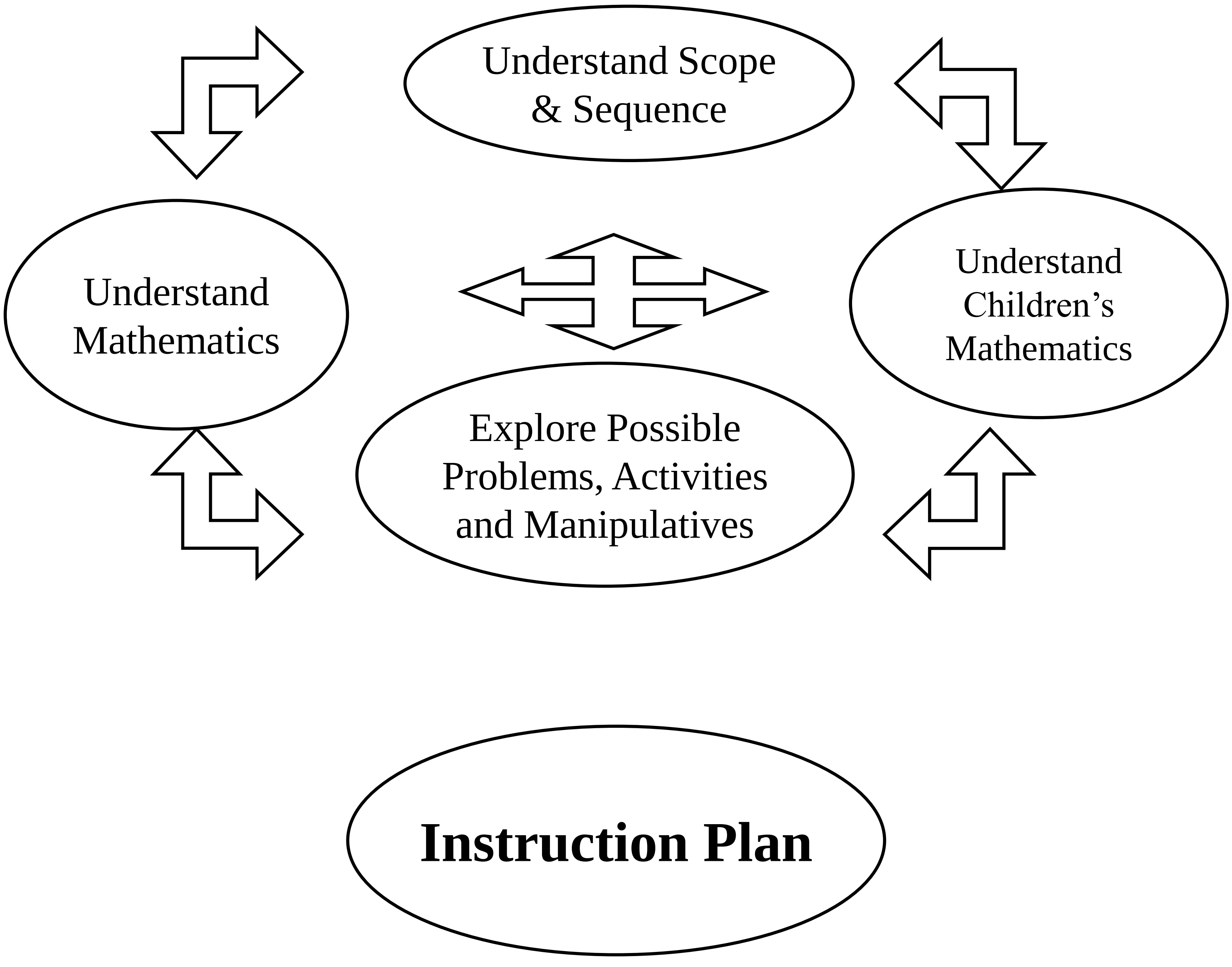
1) content knowledge
2) general pedagogical knowledge
3) curriculum knowledge
4) pedagogical content knowledge
5) knowledge of learners and their characteristics
6) knowledge of educational contexts
7) knowledge of educational ends, purposes, and values, and their philosophical and historical grounds.

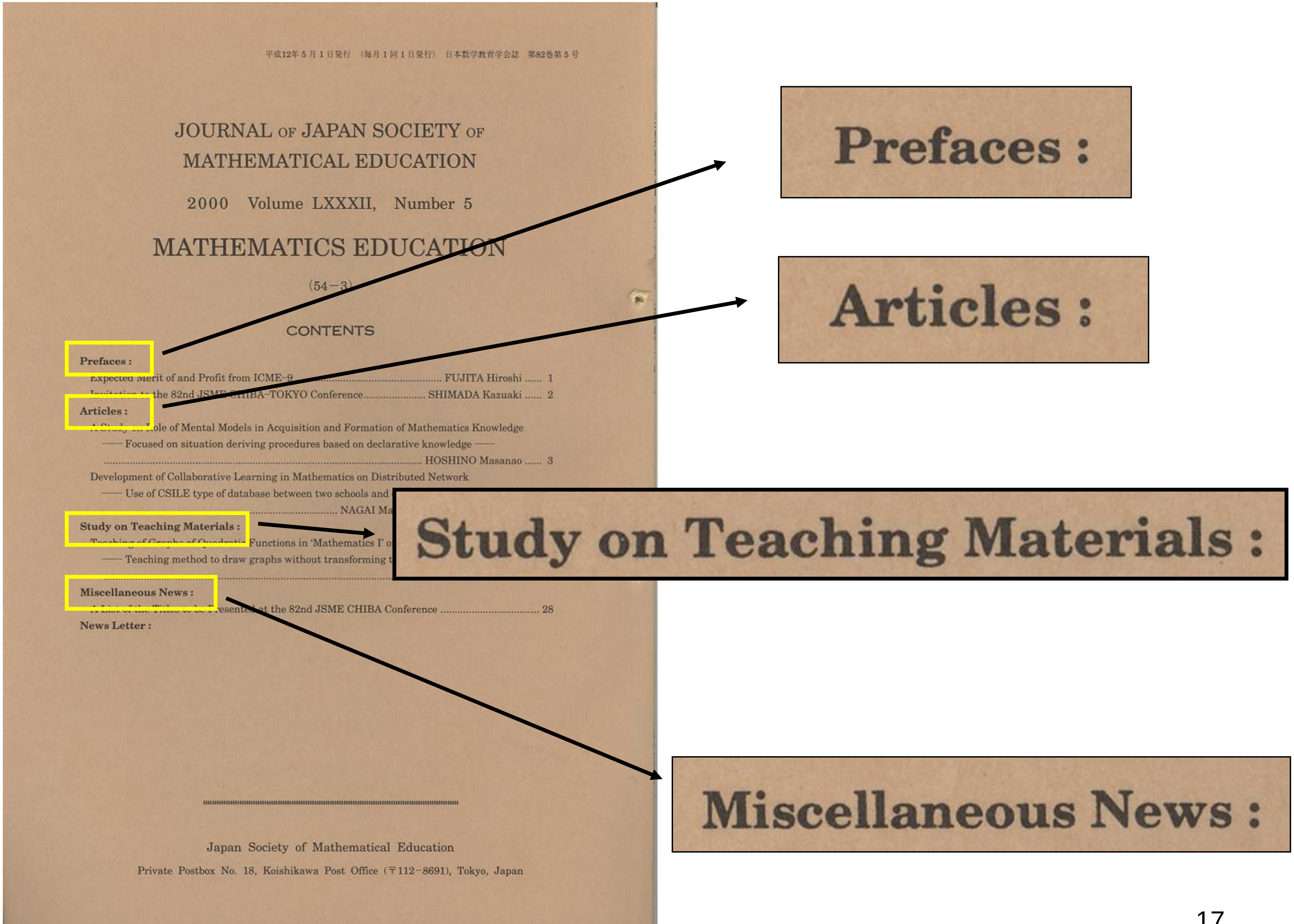
Knowledge for Teaching: three additional categories

- Knowing how to organize and plan problem solving oriented lessons.
- Knowing how to evaluate and research teaching materials
- Knowledge of the lesson study as a continuing system for building teacher capacity

In lesson study, research on teaching materials is a key element

- Research on teaching materials involves viewing the materials with the aim of building Knowledge for Teaching
- Knowledge for Teaching is knowledge-in-action
- Knowledge for Teaching requires:
  - A mathematical point of view
  - An educational point of view
  - And from the students' point of view


Understand Scope
& Sequence
Understand
Mathematics
Understand
Children's
Mathematics
Explore Possible
Problems, Activities
and Manipulatives
Instruction Plan



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CONTENTS

Japan Society of Mathematical Education
Private Postbox No. 18, Koishikawa Post Office (〒112−8691), Tokyo, Japan

Organization of Japanese Math Lesson

- **Presenting the problem for the day**
- **Problem solving by students**
- **Comparing and discussing**
- **Summing up by teacher**

Presenting the problem for the day

Stigler & Hiebert (1999) comment that

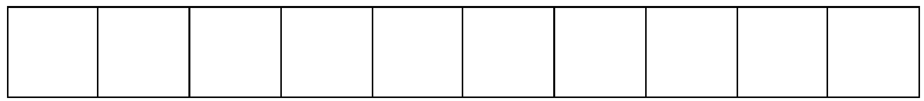
- “the (Japanese) teacher presents a problem to the students without first demonstrating how to solve the problem.”
- “ U.S. teachers almost never do this….the teacher almost always demonstrates a procedure for solving problems before assigning them to students.”
- Japanese teachers therefore have to ensure that students understand the context in which the task is embedded and the mathematical conditions required for its solution

An example of “Presenting a problem”

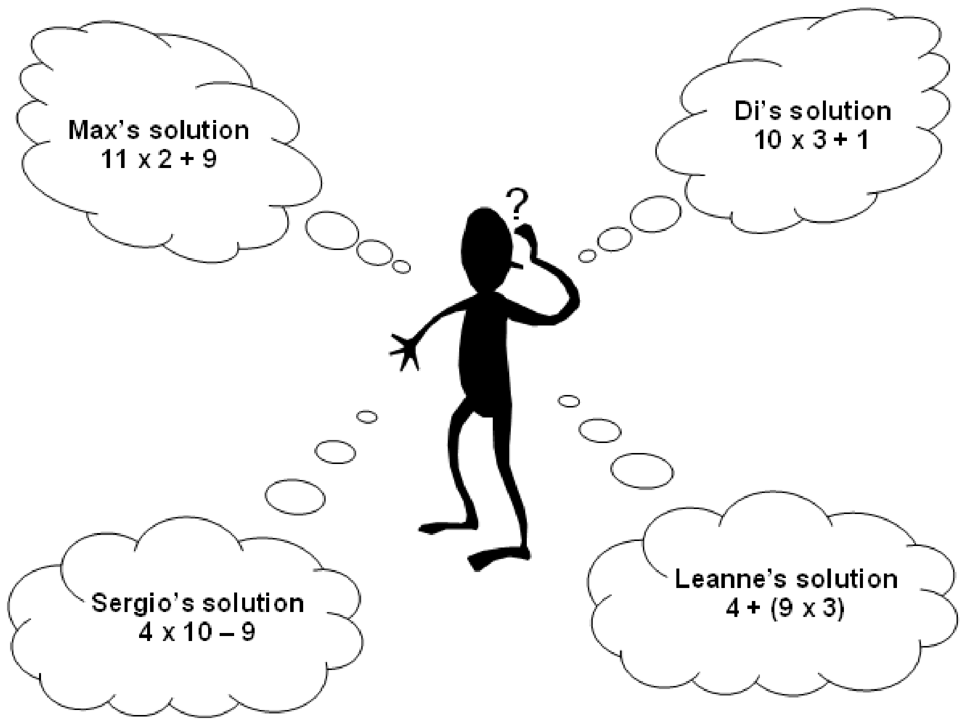
- Curriculum-free task: Match Sticks Problems.
- Used in the US–Japan cross cultural research project (4th and 6th graders) (T.Miwa,1992)
- At that time (1992) ,the task was unfamiliar for both countries but after that appeared in textbooks in Japan and it is well known even internationally
- In Australia it is part of a series of rich assessment tasks for upper primary and junior secondary students (Stephens, 2008)

A Mathematically Rich Task

Without counting, can you work out how many matchsticks were needed to make 10 cells?



Four students gave different solutions which are shown below.



A Mathematically Rich Task

Part A

Do these four strategies give a correct result?

Part B

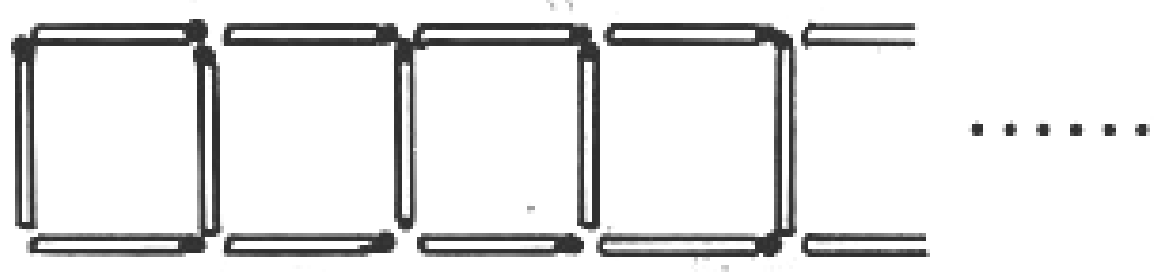
How many matchsticks would be needed to make 5 cells, 12 cells, 27 cells? Explain your thinking.

Part C

Choose 2 of the above strategies. How do you think the person arrived at his or her strategy? Explain the thinking involved.

Number of Matchsticks (Grade 4, 6)

- Squares are made by using matchsticks as shown in the picture. When the number of squares is five, how many matchsticks are used?



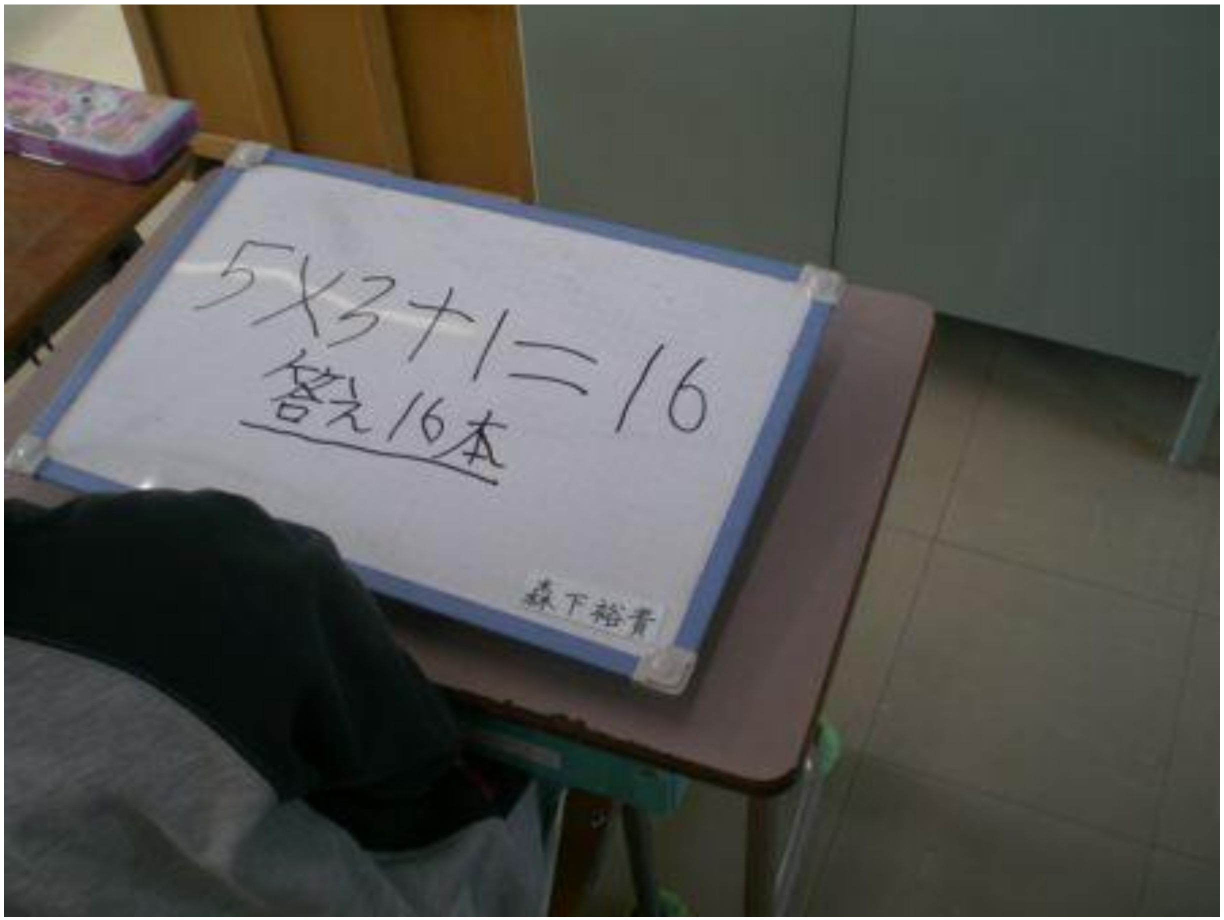
(1)Write your way of solution and the answer.

(2)Now make up your own problems like the one above and write them down.

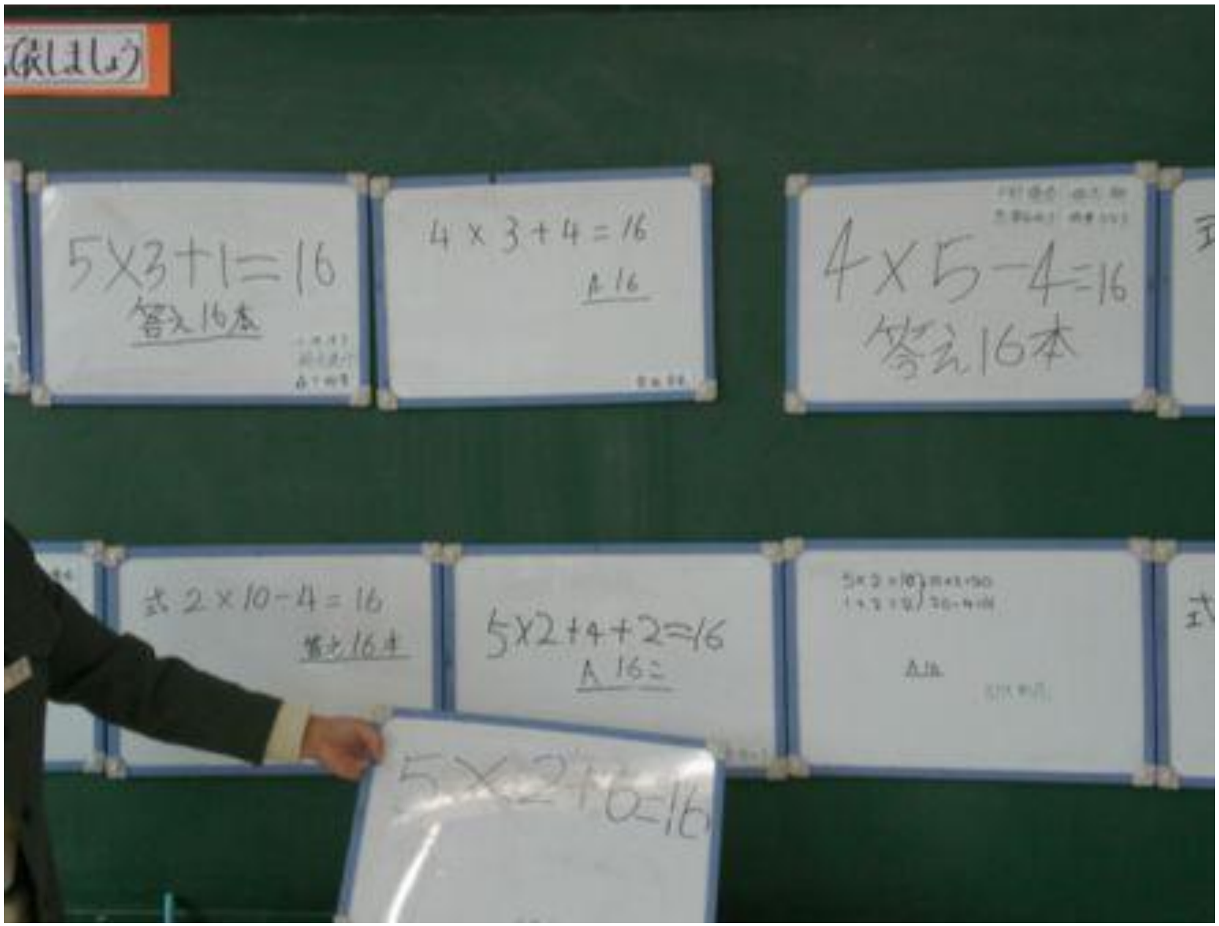
Lesson Study – Grade 4

- In this class, the teacher presented the children with five cells, and asked them to find the number of match-sticks required to make this number of cells
- They were then asked to think about a rule that they could use for this number of cells, and for any other number
- Children developing and explaining their rules are the focus of the lesson

Students work is written on magnetic boards that are easy to display for the whole class



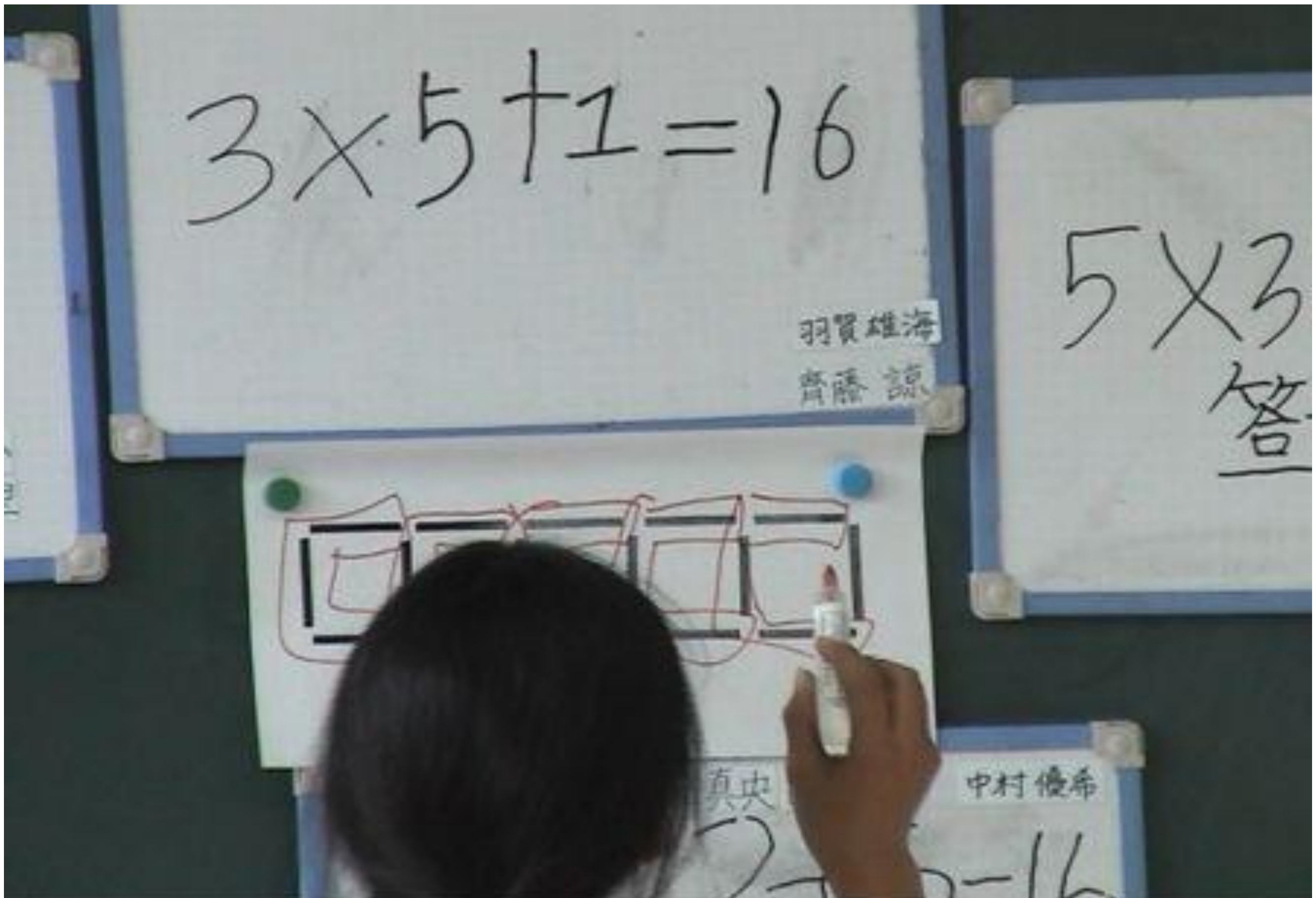
Teacher has carefully selected children's solutions for whole class discussion

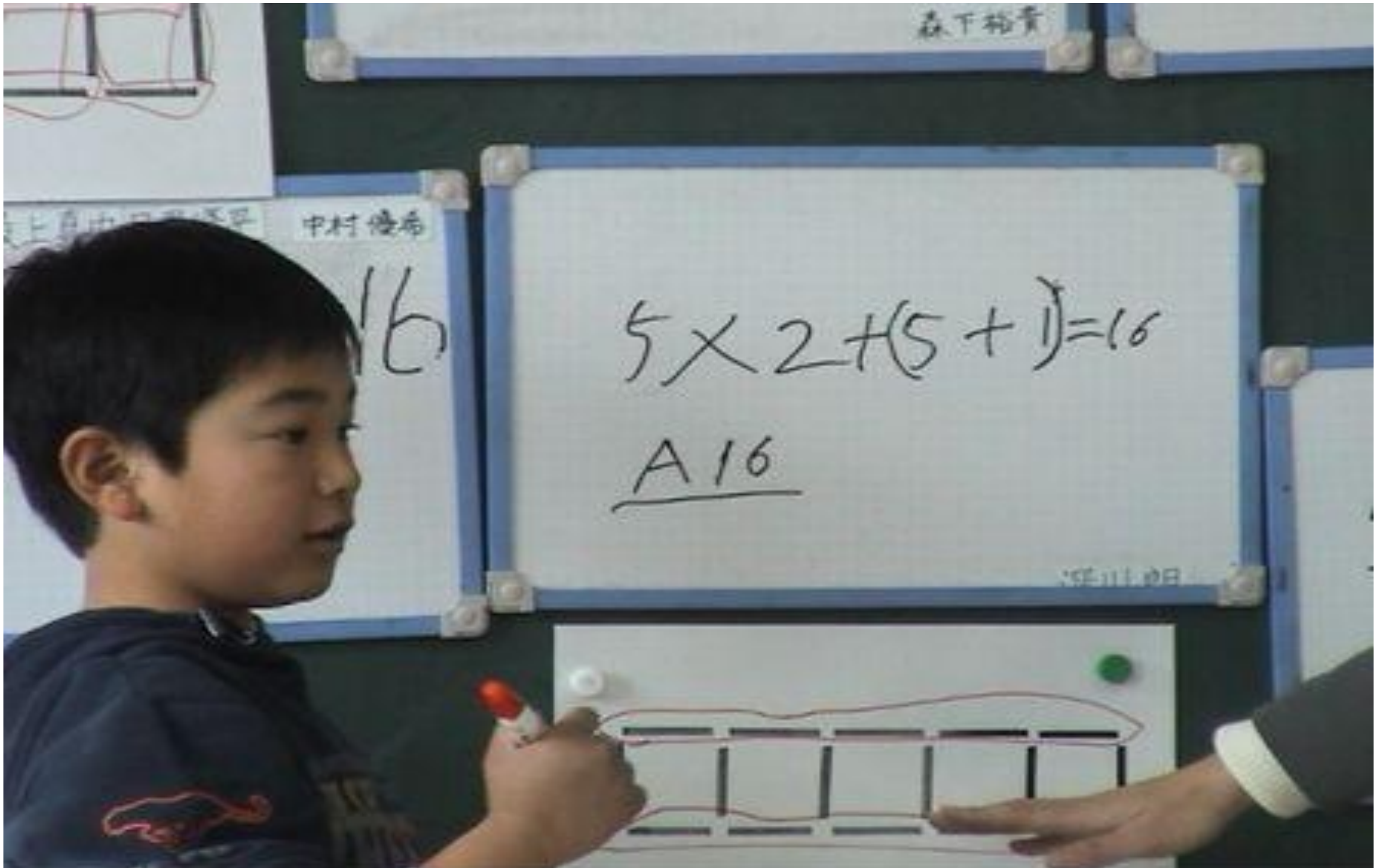


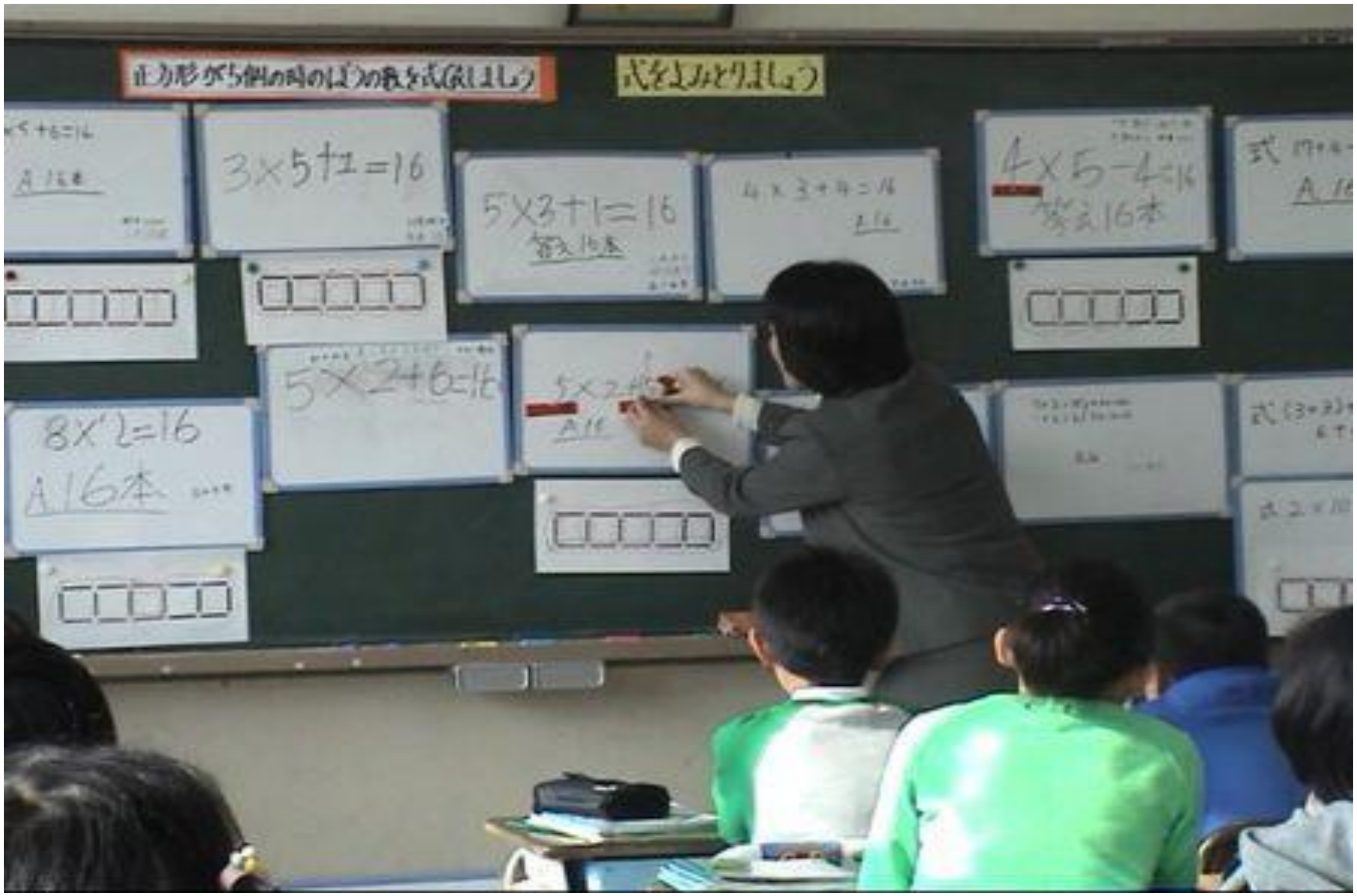
Observers have the teacher's detailed lesson plan and are looking at how children and teacher are moving ahead according to the plan

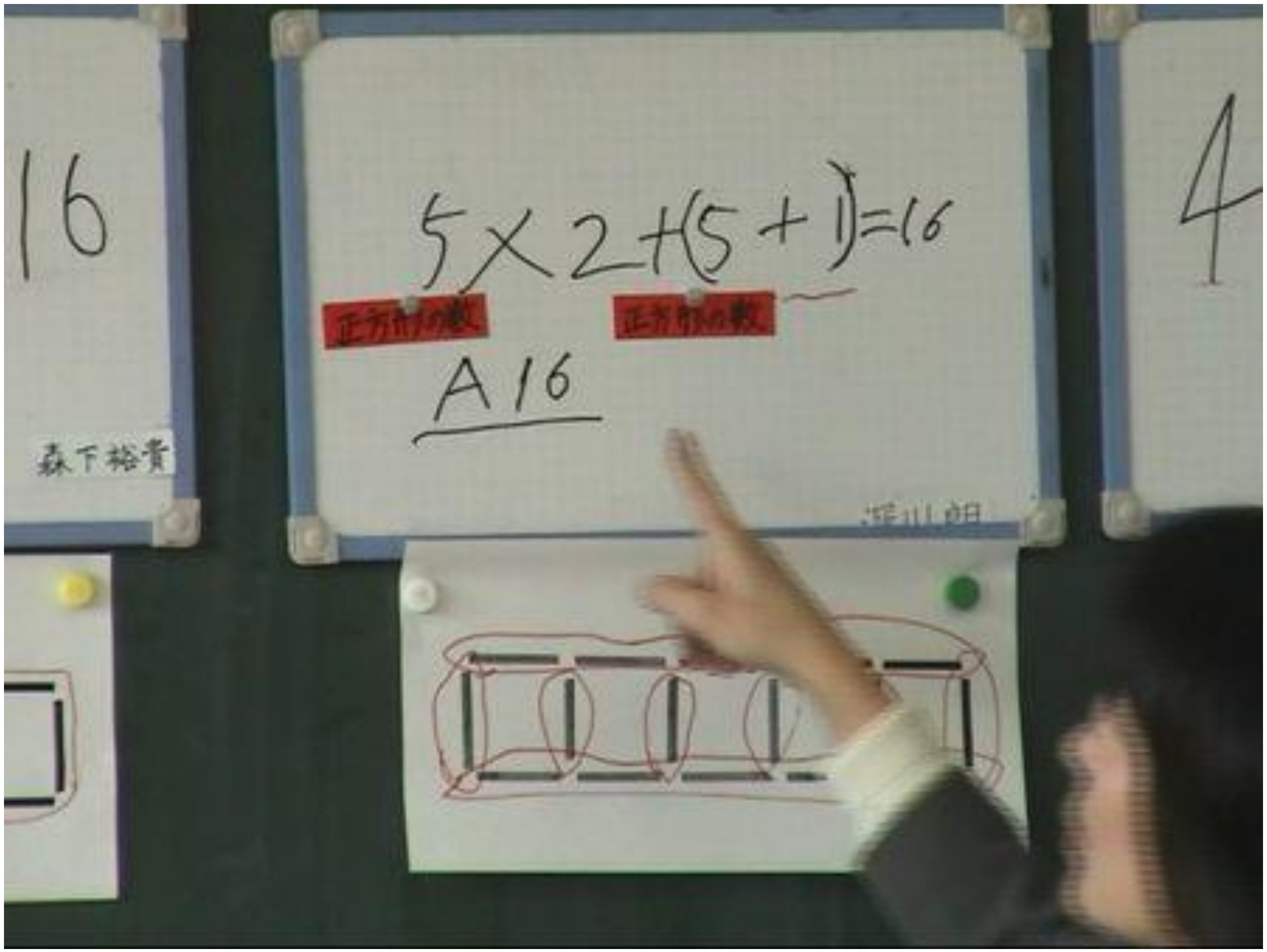
The teacher asked student to explain the work of another student using geometrical figures

This student is explaining her visual thinking that supports her generalisation


3×5+1=16
羽賀雄海
齊藤 諒
5×3
答
中村優希

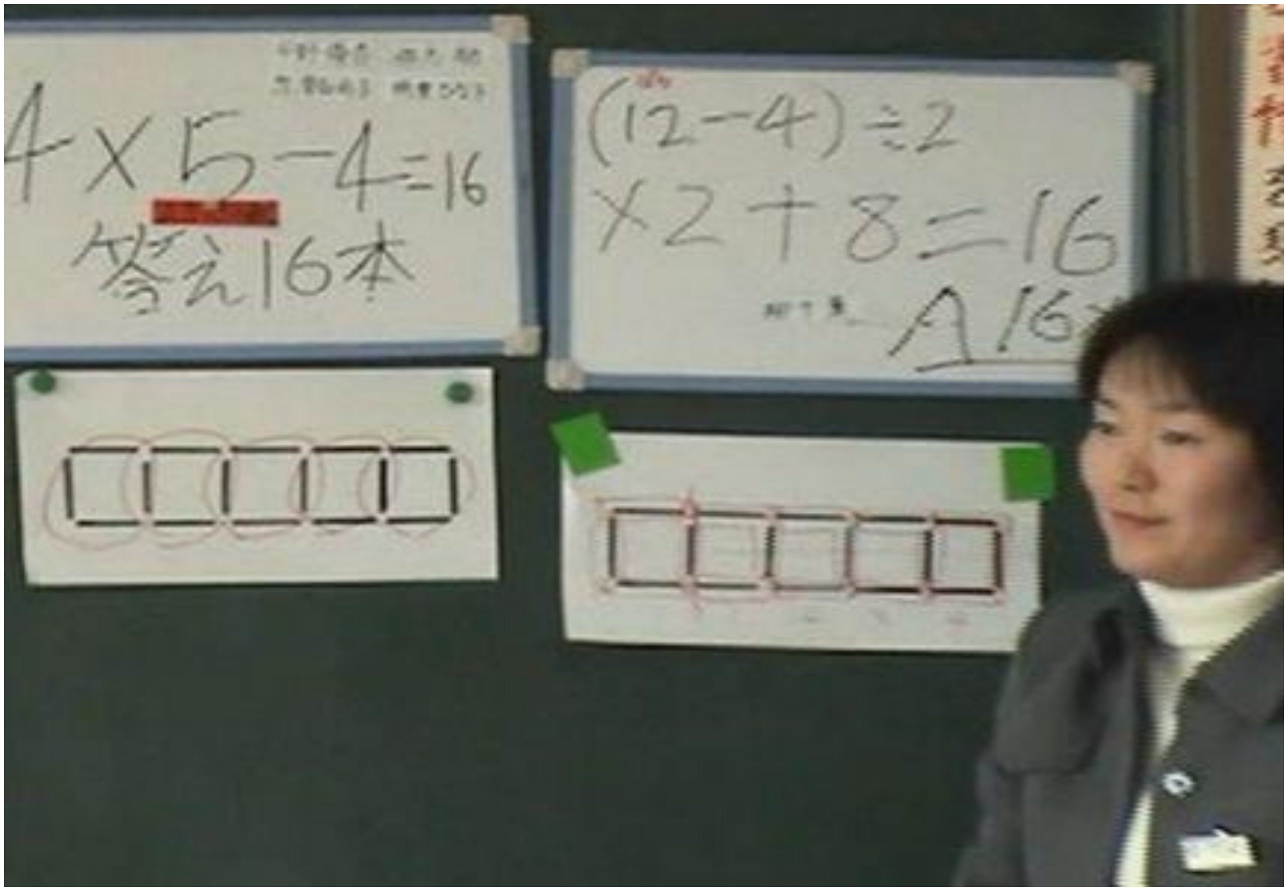

5×2+(5+1)=16
A16


正方形が5個の時のぼうの数を式にしましょう
式をよみとりましょう
5×3+1=16
8×2=16
A16本

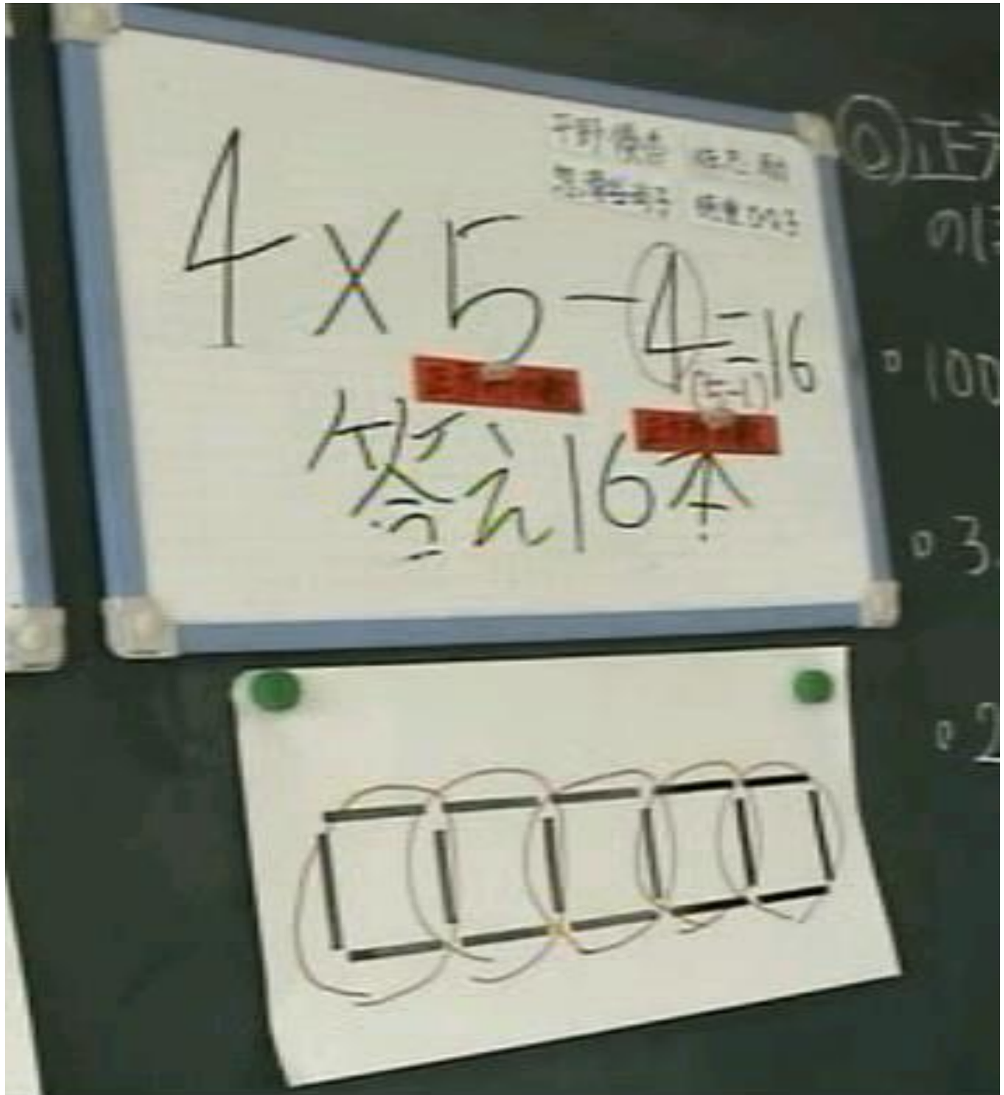

16
5×2+(5+1)=16
A16
4

Why is the teacher highlighting some numbers?

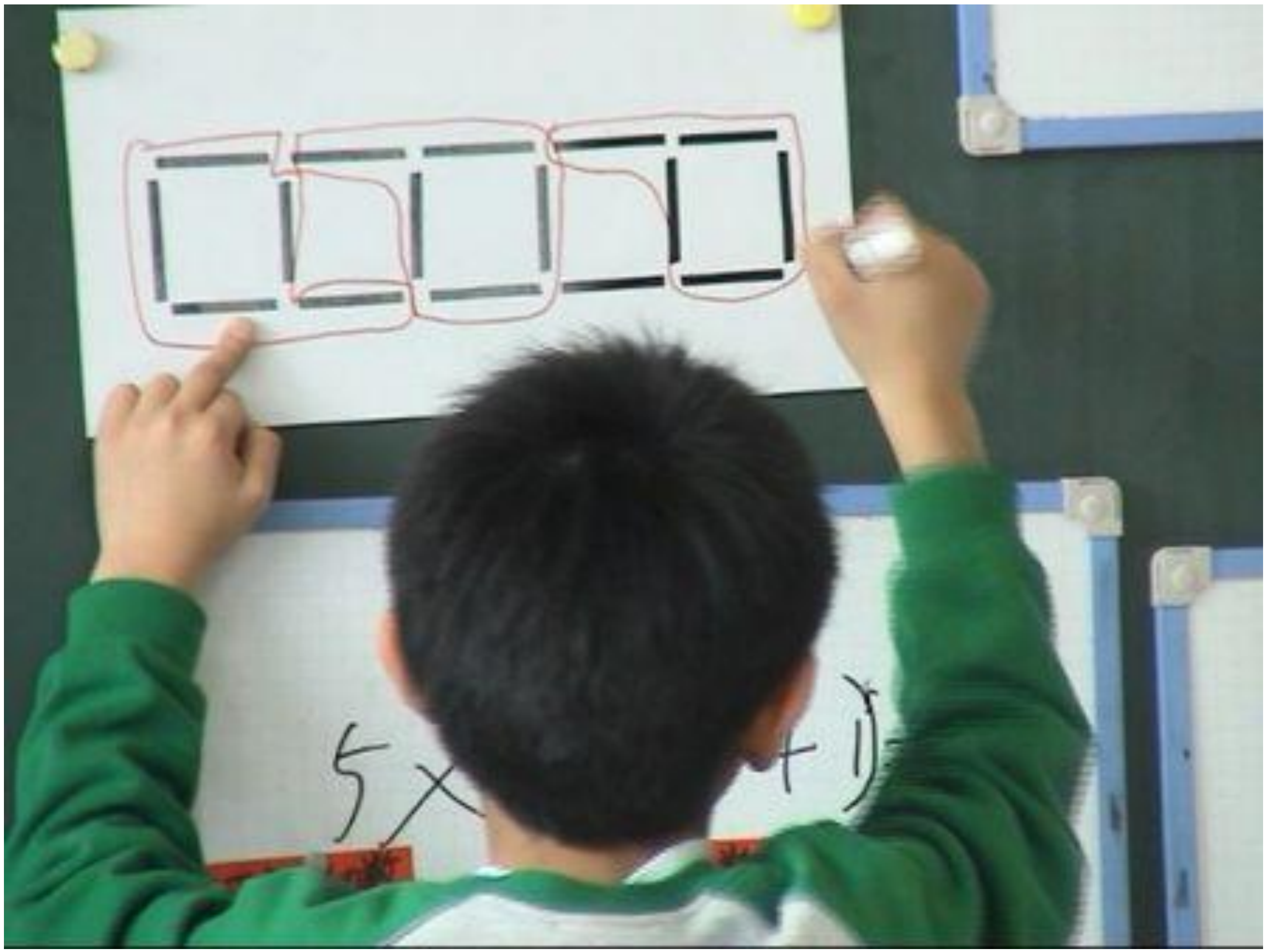
- This was done by the teacher to give emphasis to the idea that each highlighted number is an instance of a general pattern – not a number for calculation.
- She wants the children to see concrete numbers as generalizable numbers.
- This knowledge-in-action is the result of the deep research on teaching materials



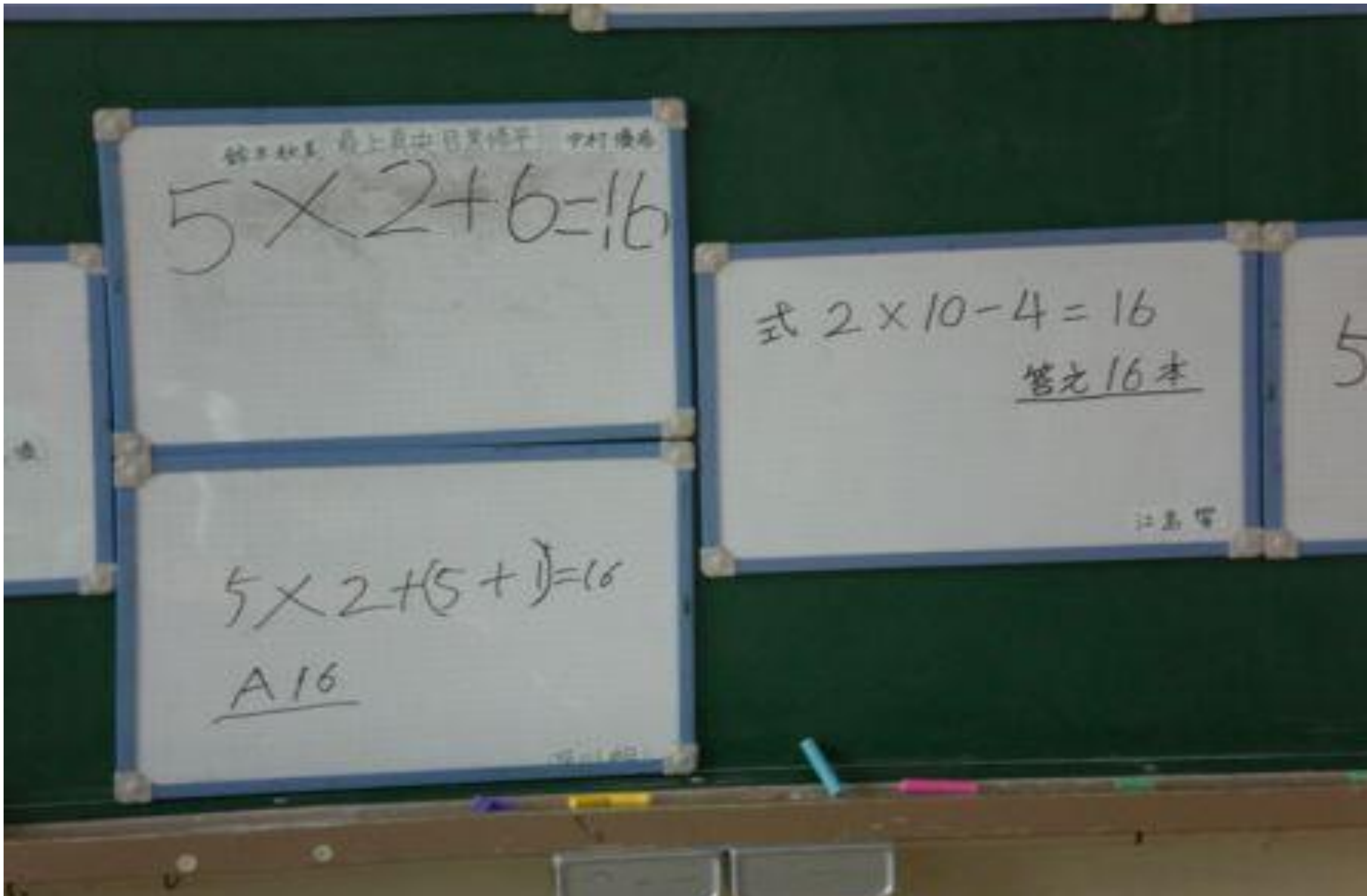
4×5−4=16
答え16本
(12−4)÷2
×2+8=16
A16


4×5−4=16
答え16本

This student presents a solution that looks interesting – but does it generalise?

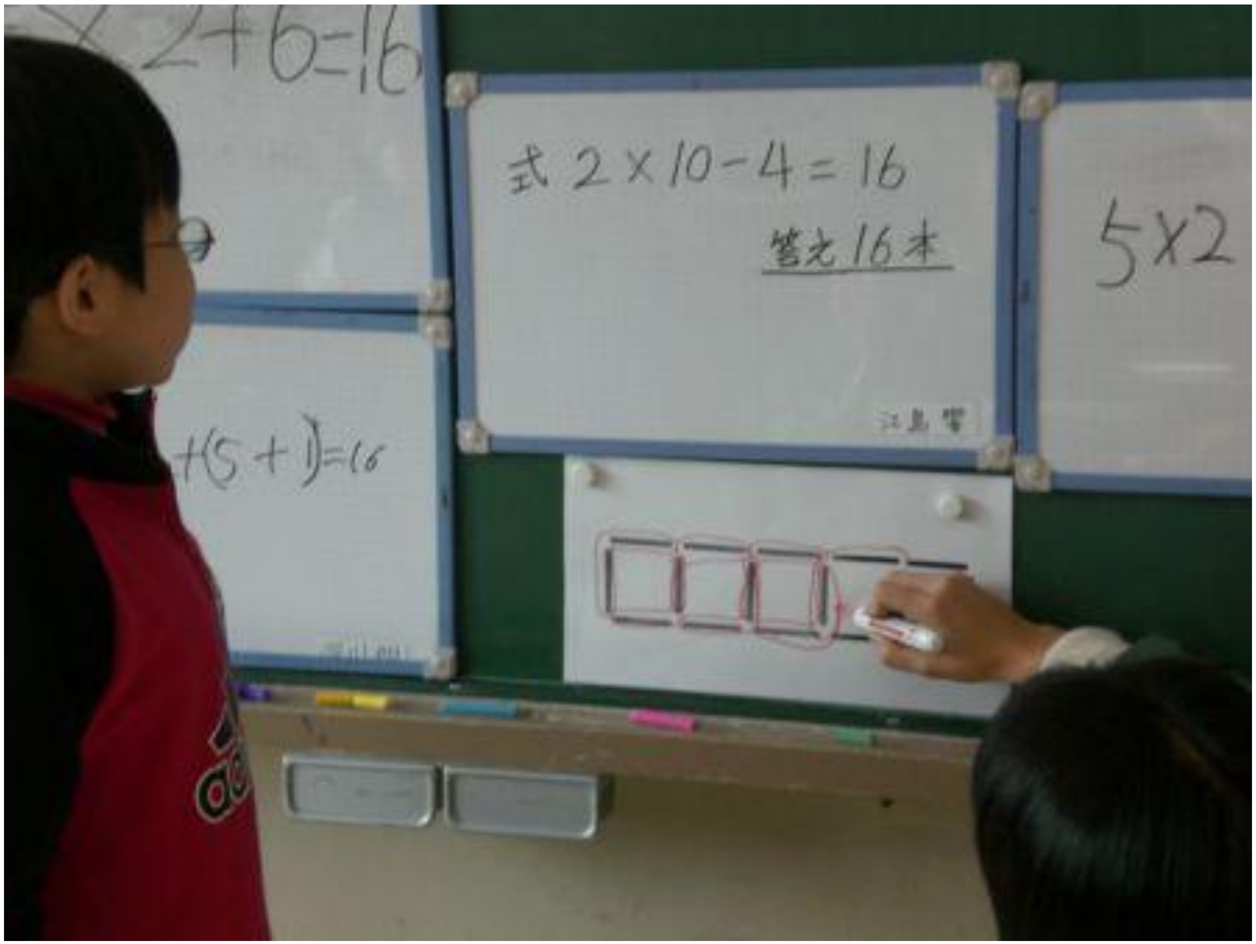


Here, two versions of the same rule are being compared. The teacher asks “Which one is easier to follow?”



Teacher is asking students to think about the visual thinking behind 5×2 + (5+1)

This student explains his visual thinking behind 5×4 – 4, or is it 5×4 – (5 – 1)?



What is the purpose of having children come to the front and to explain their thinking?

- Sometimes this comparison-discussion activity may appear to be "show-and-tell" (Takahashi,2008) but in reality that is not the case.
- Different student responses have been anticipated in the lesson plan and are carefully selected by the teacher to promote deep mathematical thinking.

$2 \times 5 + (5+1) = 16$

$20 - 4 = 16$

$2 \times 10 - 4 = 16$

$3 \times 5 + 1 = 16$

$5 \times 2 + 4 + 2 = 16$

$4 \times 5 = 20$

$20 - 4 = 16$

$4 \times 5 - (5 - 1) = 16$

$5 \times 3 + 1 = 16$

$\langle 3+3 \rangle + 2 \times 3 + 4 = 6 + 6 + 4 = 16$

$17 + 4 - 5 = 16$

$4 \times 3 + 4 = 16$

$8 \times 2 = 16$

$5 \times 2 + 6$

$5 \times 2 + (5+1) = 16$

$(12 - 4) \div 2 \times 2 + 8 = 16$

Some examples of actual students' work as observed by the teachers in this research lesson (before whole class discussion)
Those that contain the red markers show evidence of generalising (my red markings)

Post lesson discussion (Professor Fujii is chairing the meeting, three teachers who taught the lesson are on his left, all observers are present as is school principal)

At the post-lesson discussion

- Professor Fujii – the external facilitator – introduced the discussion drawing attention to the planning phase and to the goals for these particular lessons – fostering mathematical thinking, visualisation and generalisation
- The principal and her deputy talked about how these lessons meshed in with some over-arching goals of the school
  - listening and learning from others
  - promoting deep thinking
  - fostering communication
- Observers, who were other teachers in the school, had been released from regular classes in order to participate in lesson study
- All teachers were expected to attend the discussion which lasted for about 90 minutes

At the post-lesson discussion

- Observers asked teachers about particular points where they had departed from their lesson plan
- Observers asked teachers about specific responses by students
- Teachers brought magnetic boards to refer to and to illustrate particular students' thinking
- Teachers explained where they thought the lesson had succeeded and where it might be improved next time

Knowledge for Teaching always includes Mathematical Values

In this lesson, we can note that: Mathematical values are crystallized, such as

- Mathematical thinking needs to be flexible.
- Mathematical expression can also be flexible.
- Seeing concrete numbers as generalizable numbers is important.
- Making a generality visible is important

Knowledge for Teaching always includes Pedagogical Values

In this lesson, we can note that certain Classroom culture values are crystallized, such as

- Moving beyond seeing answers simply as “wrong” or “correct”
- Listening carefully to friends’ talk
- Express ideas clearly to friends
- Avoid underestimating friends’ ideas

Knowledge for Teaching always includes Human Values

In this lesson, we can note that certain Human values are crystallized, such as

- Using previous knowledge and experience is often needed to solve a new problem
- Learning from errors is important
- In order to clarify A, knowing and being able to think about non-A is important

Sometimes a professor teaches a research lesson: Why?

Mr Hosomizu’s Grade 5 Lesson

- The lesson we will now see is another “problem oriented lesson”
- Notice how the lesson follows a similar format as the one we discussed:
  - Presenting problem for the day
  - Problem solving by students
  - Comparing and discussing
  - Summing up by teacher

Your thinking about the lesson

- If you had to pick out one or two really important things mathematical from the lesson, what would they be?
- Please share your thinking with the person next to you.
- Are these features what you expect to see in typical lessons here in Lebanon?

Some comments on the lesson

- Mr Hosomizu’s summation is important: “If we know the result of an *expression*, we can use it to get the result of another *expression*”
- Students are expected to deal with mathematical expressions as objects for thinking – not simply as calculations
- These are related to the big ideas of the elementary school curriculum

Some comments on the lesson

- You can work with one problem for a long time provided you don’t focus on the results of the problem but on processes that led to that result
- Students basically used three approaches to simplifying $5.4 \div 3$
- These are all related to important ideas about equivalence in the elementary school curriculum

Three mathematical procedures

- Enlarge 5.4 to 54, then do 54 ÷ 3, but you have to remember that when you get an answer it will be necessary to ÷ by 10
- Change 5.4 ÷ 3 to 54 ÷ 30 in order to get a result without having to adjust the answer. Some students did not think this made the problem easier, but …
- Think of 5.4 as 5.4 metres and so 540 cm, the convert the answer of 180 cm back to metres

Extending mathematical thinking

- Considering $2.7 \div 3$, some students repeated one of the three procedures used for $5.4 \div 3$
- Mr Hosomizu is happy to accept this, but
- Other students were able to connect this new problem with the original problem.
- "Knowing the result, *and way of calculating*, of an expression is important because we can use it for other expressions"

Extending mathematical thinking

Finally, students are asked to consider what other numbers could be used in

$$\square \div 3$$

where they can use the result of $5.4 \div 3$ to find the result of this new expression

Some of the numbers suggested are:

Extending mathematical thinking

- If you know that $5.4 \div 3 = 1.8$, you can also reason that $2.7 \div 3 = 0.9$
- Mr Hosomizu asks: If you know these two results, what number can go in the blue box: [ ] $\div 3$ such that one of the above results can be used to give the new answer?
- Children suggest: 15.12, 0.35, 410.8, 1.35, 8.1, 3.24, 1.8, 21.6 and 7.1

Extending mathematical thinking

- If you know that $5.4 \div 3 = 1.8$, you can also reason that $2.7 \div 3 = 0.9$
- Children suggest: 15.12, 0.35, 410.8, 1.35, 8.1, 3.24, 1.8, 21.6 and 7.1
- Mr Hosomizu concludes the lesson by saying that he can understand why students said 8.1, 1.8, 21.6, 1.35
- To be discussed in the next lesson

For the next lesson

- If you know that $5.4 \div 3 = 1.8$, you can also reason that $2.7 \div 3 = 0.9$
- What about $8.1 \div 3 = ?$

  $1.8 \div 3 = ?$

  $21.6 \div 3 = ?$

  $1.35 \div 3 = ?$
- "Knowing the result of an expression is important because we can use it for other expressions"

For the next lesson

- If you know that $5.4 \div 3 = 1.8$, you can also reason that $2.7 \div 3 = 0.9$
- What about
  $8.1 \div 3 = 2.7$ ($8.1 = 3 \times 2.7$)
  $1.8 \div 3 = 0.6$ ($1.8 = 5.4 \div 3$)
  $21.6 \div 3 = 7.2$ ($21.6 = 5.4 \times 4$)
  $1.35 \div 3 = 0.45$ ($1.35 = 2.7 \div 2$)
- "Knowing the result of an expression is important because we can use it for other expressions"

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