Thinking, Doing, and Talking Mathematically: Planning Instruction for Diverse Learners

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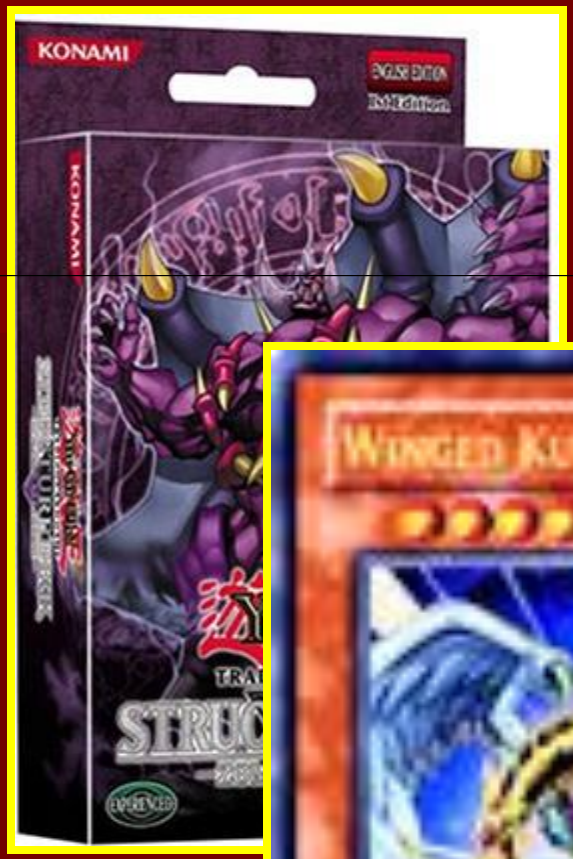


ATK/ 704 DEF/ 304

Predicting Risk of Heart Attack

- Researchers have reported that 'waist-to-hip' ratio is a better way to predict heart attack than body mass index.

- A ratio that exceeds .85 puts a woman at risk of heart attack. If a woman's hip measurement is 94 cm, what waist measurement would put her at risk of heart attack?

Students with Learning Difficulties

- More than 60% of struggling learners evidence difficulties in mathematics (Light & DeFries, 1995).

- Struggling learners at the elementary level have persistent difficulties at the secondary level, because the curriculum is increasingly sophisticated and abstract.

What Does Research Say Are Effective Instructional Practices For Struggling Students?

- Explicit teacher modeling.
- Student verbal rehearsal of strategy steps during problem solving.
- Using physical or visual representations (or models) to solve problems is beneficial.
- Student achievement data as well as suggestions to improve teaching practices.

Fuchs & Fuchs (2001); Gersten, Chard, & Baker (in review)

What Does Research Say Are Effective Instructional Practices For Struggling Students?

- Cross age tutoring can be beneficial only when tutors are well-trained.
- Goal setting is insufficient to promote mathematics competence
- Providing students with elaborative feedback as well as feedback on their effort is effective (and often underutilized).

Fuchs & Fuchs (2001); Gersten, Chard, & Baker (in review)

Mathematical Proficiency

1. *Conceptual understanding* – comprehension of mathematical concepts, operations, and relations

2. *Procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

3. *Strategic competence* – ability to formulate, represent, and solve mathematical problems

4. *Adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification

5. *Productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

(U. S. National Research Council, 2001, p. 5)

Common Difficulty Areas for Struggling Learners

Memory and Conceptual Difficulties	**Background Knowledge Deficits**
Linguistic and Vocabulary Difficulties	**Strategy Knowledge and Use**

Memory and Conceptual Difficulties

Students experience problems:

- Remembering key principles;
- Understanding critical features of a concept;
- Because they attend to irrelevant features of a concept or problem.

Addressing Diverse Learners Through Core Instruction

Memory and Conceptual Difficulties	Thoroughly develop concepts, principles, and strategies using multiple representations. Gradually develop knowledge and skills that move from simple to complex. Include non-examples to teach students to focus on relevant features. Include a planful system of review.

Primary

Big Idea - Number

Plan and design instruction that:

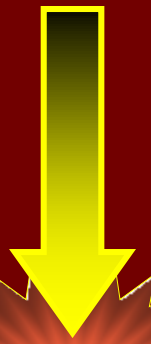
- Develops student understanding from concrete to conceptual,
- Scaffolds support from teacher ⇨ peer ⇨ independent application.

Primary Sequencing Skills and Strategies

- Adding w/ manipulatives/fingers
- Adding w/ semi-concrete objects
- Adding using a number line
- Min strategy
- Missing addend addition
- Addition number family facts
- Mental addition (+1, +2, +0)
- Addition fact memorization

Concrete/ conceptual

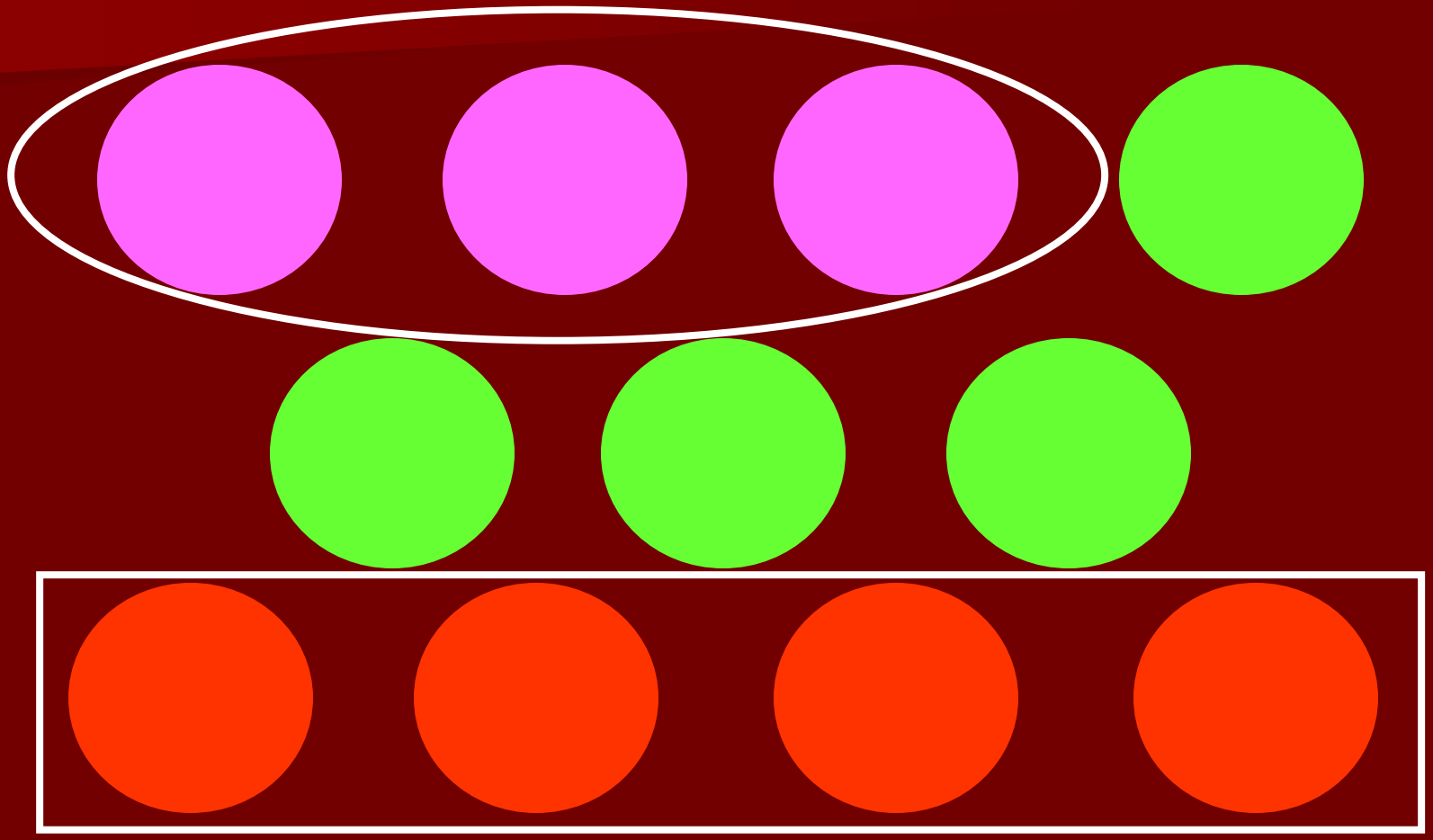
Semi-concrete/ representational



Abstract

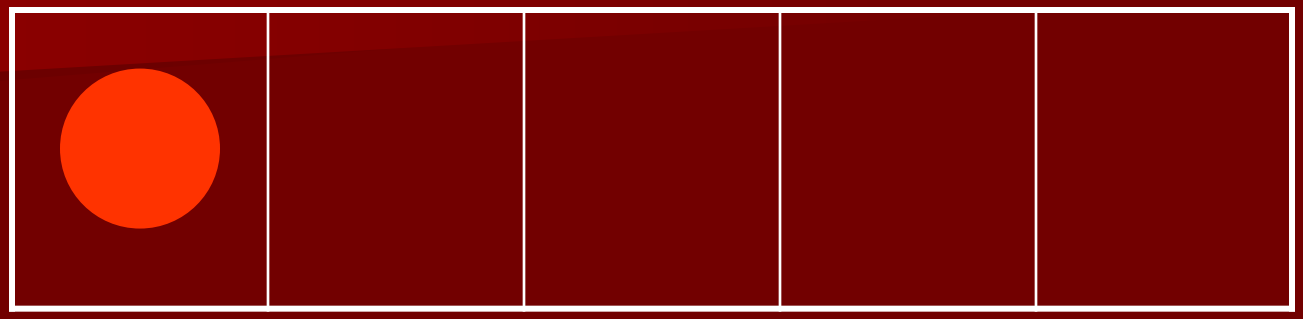
Intermediate

Rational Numbers



Intermediate

Rational Numbers

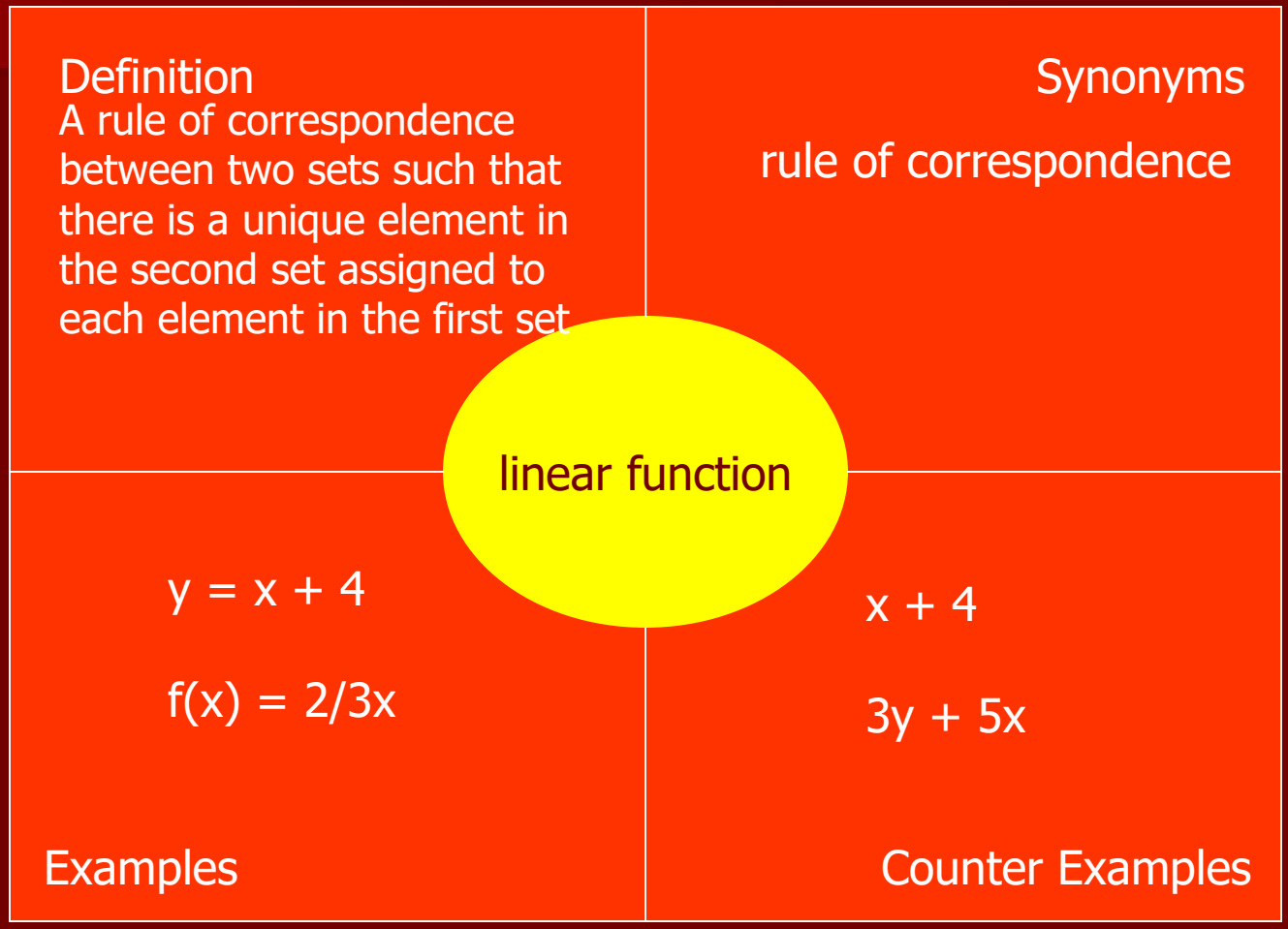


What rational number represents the filled spaces?

What rational number represents the empty spaces?

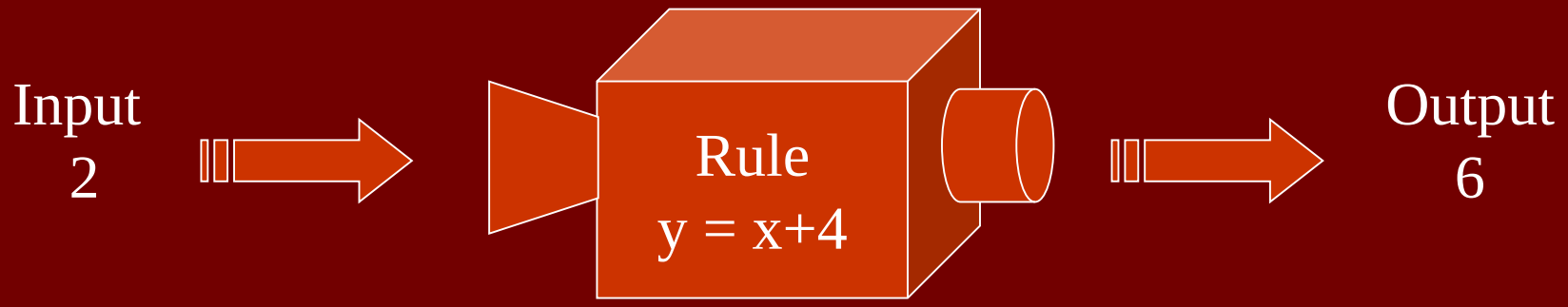
What is the relationship between the filled and empty spaces?

Presenting Rational Numbers Conceptually



Secondary

Introduction to the Concept of Linear Functions



Functions with increasingly complex operations

$y = x$

$y = 3x+12$

$f(x) = 2.3x-7$

Functions to Ordered Pairs → Ordered Pairs to Functions

$y = 3x$ →

x	1	2	3	4	5	10
y	?	?	?	?	?	?

x	3	4	5	6	7	10
y	7	9	11	13	15	?

→ y is 2 times x plus 1

$y = 2x + 1$

$y = 2(10) + 1$

$y = 20 + 1 = 21$

	Primary	Intermediate	Secondary
Concept Development			
Practice Opportunities			
Key Vocabulary			
Problem Solving Strategy			

Background Knowledge Deficits

Students experience problems:

- With a lack of early number sense;
- Due to inadequate instruction in key concepts, skills, and strategies;
- Due to a lack of fluency with key skills.

For many students struggling with mathematics, mastery of key procedures is dependent on having adequate practice to build fluency.

Addressing Diverse Learners Through Core Instruction

Background Knowledge Deficits	Identify and preteach prerequisite knowledge.
	Assess background knowledge.
	Differentiate practice and scaffolding.

Number Families

$4 + 3 = 7$ $7 - 4 = 3$

$3 + 4 = 7$ $7 - 3 = 4$

Fact Memorization

- $\begin{array}{r} 5 \\ +2 \\ \hline \end{array}$ $\qquad$ $\begin{array}{r} 3 \\ +6 \\ \hline \end{array}$

  $1 + 8 =$ $\qquad$ $4 + 3 =$

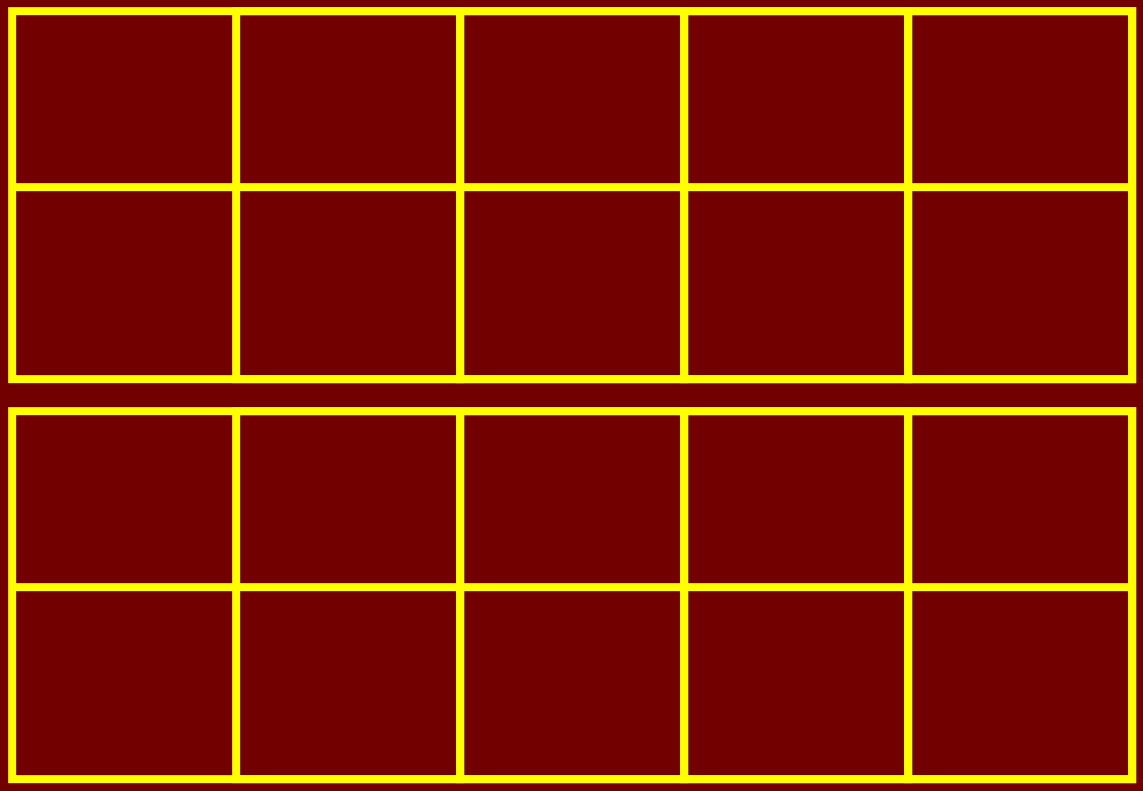
  $\begin{array}{r} 4 \\ +4 \\ \hline \end{array}$ $\qquad$ $\begin{array}{r} 2 \\ +7 \\ \hline \end{array}$

- $5 + 2 =$ $\qquad$ $6 + 0 =$

13 - 5 = ☐

+10 +3 -3 -2

"Manipulative Mode"



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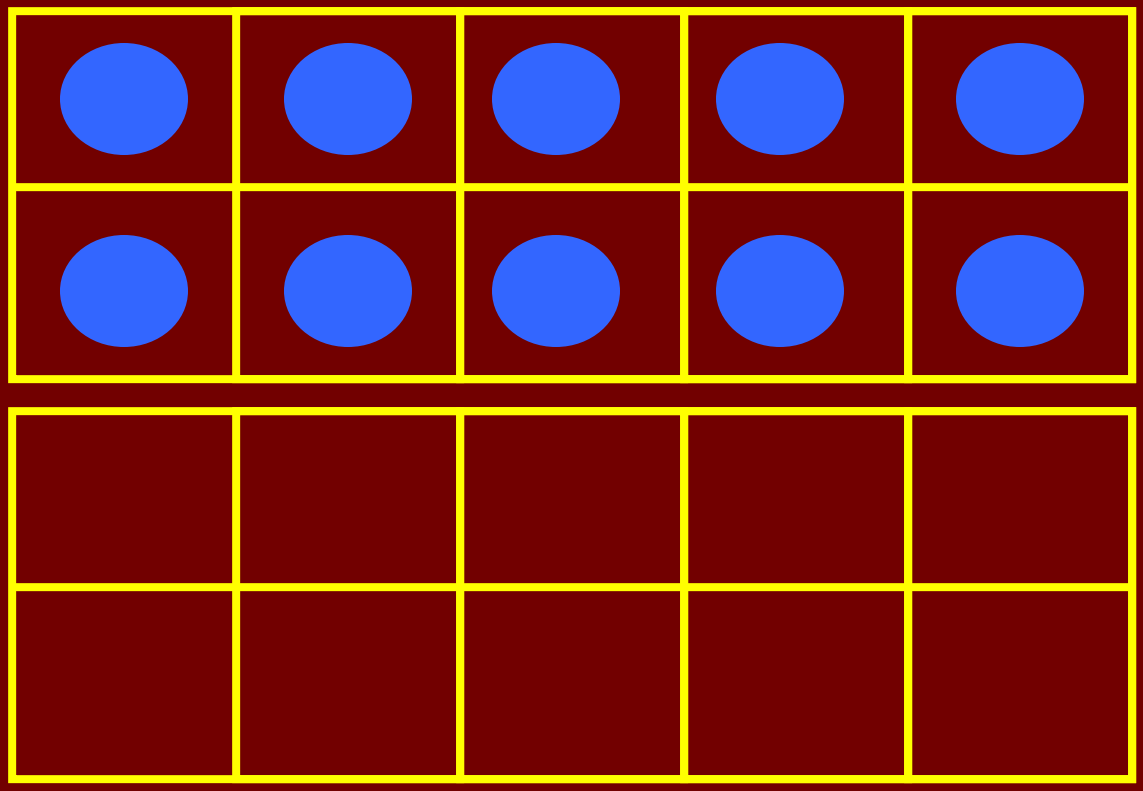
+10 +3 -3 -2



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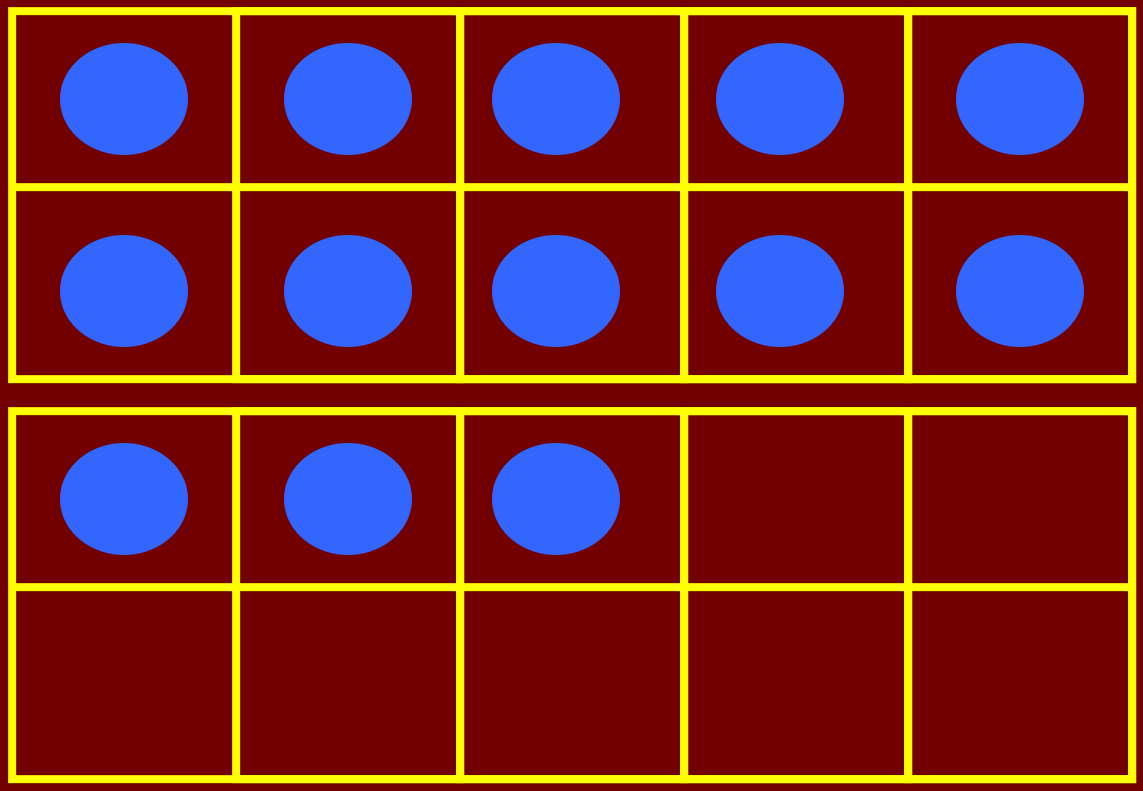
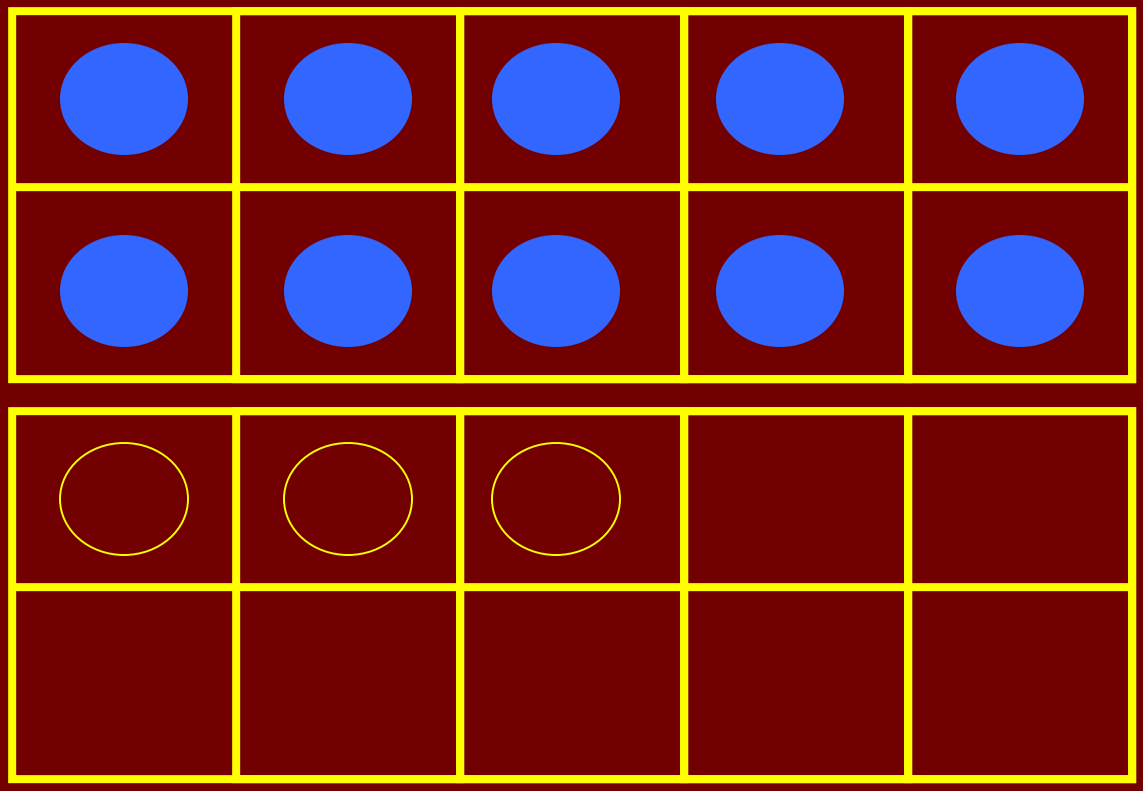


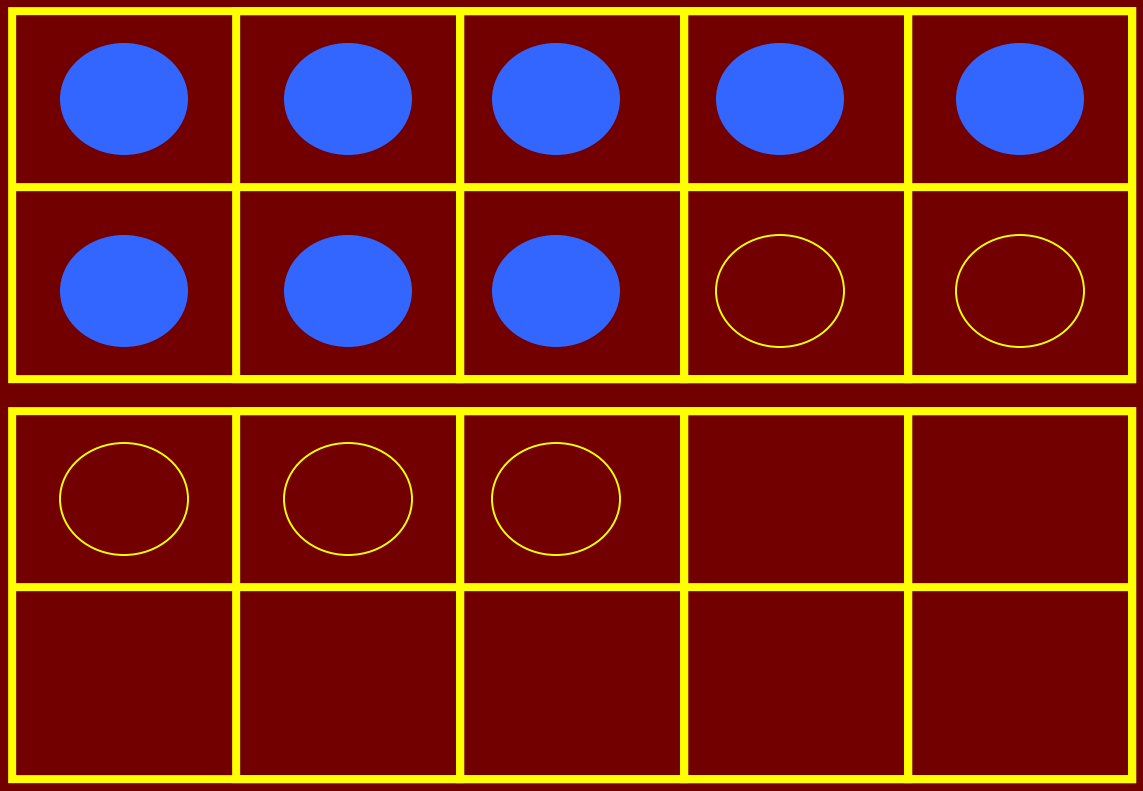
13 - 5 =

+10 +3 -3 -2



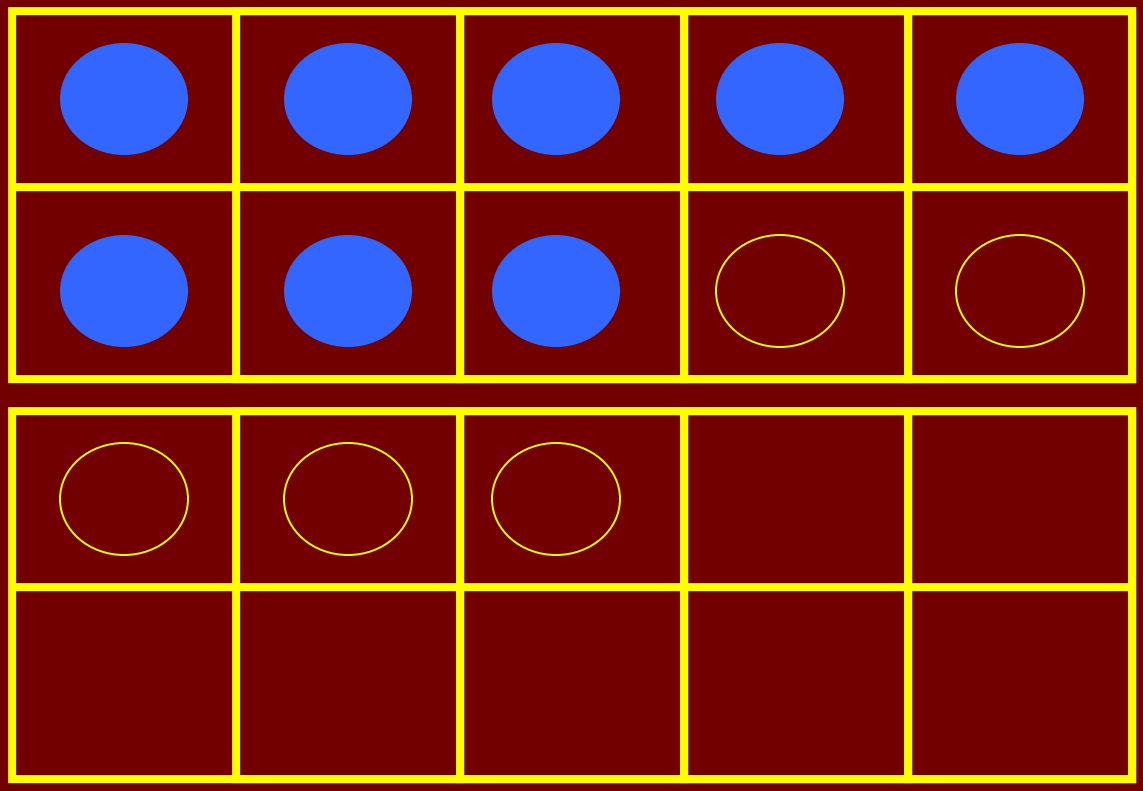
13 - 5 =

+10 +3 -3 -2



13 - 5 = 8

+10 +3 -3 -2



Linguistic and Vocabulary Difficulties

Students experience problems:

- Distinguishing important symbols;
- With foundation and domain specific vocabulary;
- With independent word recognition.

A Plan for Vocabulary in Mathematics

1. Assess students' current knowledge.
2. Teach new vocabulary directly before and during reading of domain specific texts.
3. Focus on a small number of critical words.
4. Provide multiple exposures (e.g., conversation, texts, graphic organizers).
5. Engage students in opportunities to practice using new vocabulary in meaningful contexts.

(Baker, Gersten, & Marks, 1998; Bauman, Kame'enui, & Ash, 2003; Beck & McKeown, 1999; Nagy & Anderson, 1991; Templeton, 1997)

Check Your Vocabulary Knowledge

1. 1, 2/3, .35, 0, -14, and 32/100 are _____.
2. In the number 3/8, the 8 is called the _____.
3. In the number .50, the _____ is 5.
4. $\frac{3}{4}$ and 9/12 are examples of _____ fractions.

numerator	equivalent
denominator	rational

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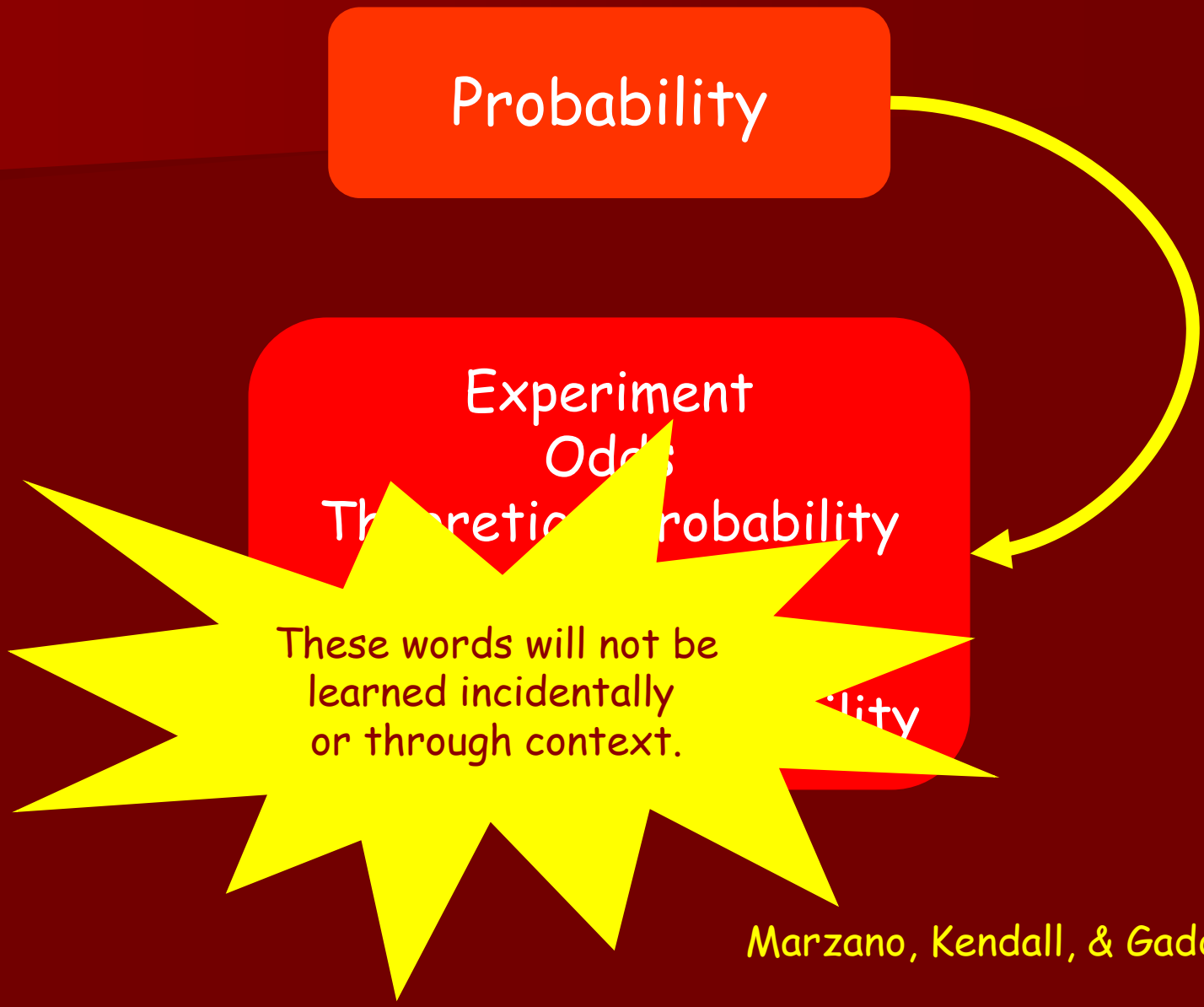
Recommended Procedures for Vocabulary Instruction

- **Modeling** - when difficult/impossible to use language to define word (e.g., triangular prism)
- **Synonyms** - when new vocabulary equates to a familiar word (e.g., sphere)
- **Definitions** - when more words are needed to define the vocabulary word (e.g., equivalent fractions)



Marzano, Kendall, & Gaddy (1999)

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Selection Criteria for Instructional Vocabulary

	Tier 1	Tier 2	Tier 3
Description	Basic words that many children understand before entering school	Words that appear frequently in texts which students need for conceptual understanding	Uncommon words associated with a specific domain
Math examples	clock, count, square	perimeter, capacity, measure	subtrahend, asymptote

(Beck, McKeown, Kucan, 2002)

Tier 3
Uncommon words associated with a specific domain
subtrahend, asymptote, symmetry, hypotenuse

Teaching children subject matter words (Tier 3) can double their comprehension of subject matter texts.

The effect size for teaching subject matter words is .97 (Stahl & Fairbanks, 1986)

Vocabulary

Word Identification Strategies

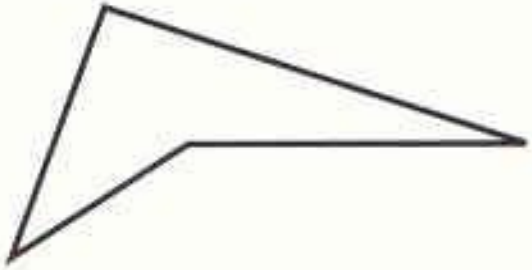
- Teach the meanings of affixes; they carry clues about word meanings (e.g., -meter, -gram, pent-, etc.)

- Teach specific glossary and dictionary skills

GLOSSARY

Q

quadrilateral A polygon with four sides.



quotient The answer in a division problem.

Example: 35 ÷ 7= 5

quotient

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PLUTO

Scientists know quite a bit about most of the planets in the solar system. And then there is Pluto.

Even powerful telescopes don't reveal much about Pluto except that it is a distant, barely visible cold point of light. Pluto remains the only planet not yet visited by an Earth-launched spacecraft.

If Congress approves a $546 million mission, NASA will build a spacecraft called *New Horizons*. NASA hopes to launch the robotic probe in January 2006. Even if the launch goes smoothly and according to plan, *New Horizons* won't reach Pluto until 2016.

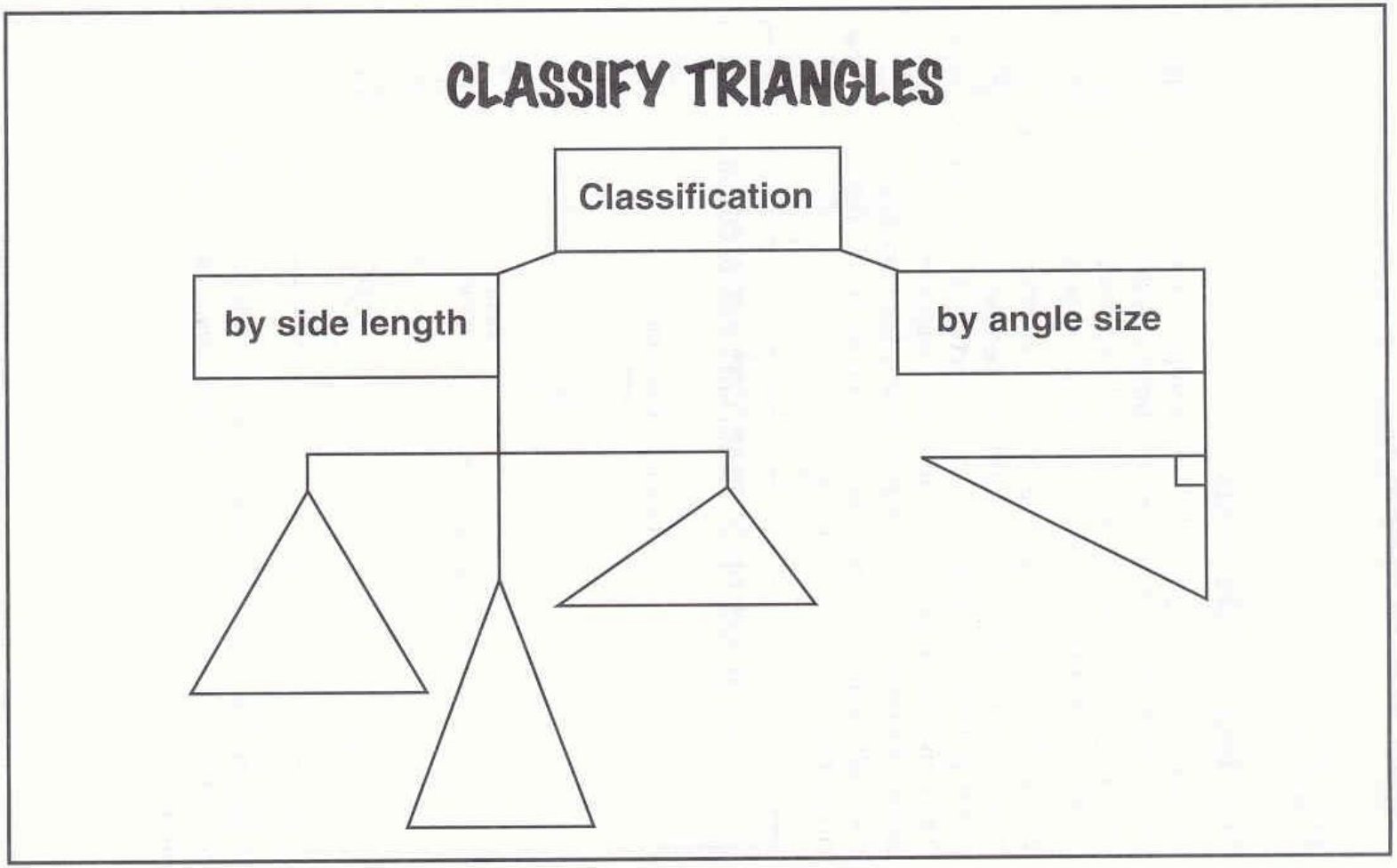
Timing Is Everything

Pluto takes almost 248 Earth-years to orbit the sun. If the *New Horizons* mission is delayed by only a year, scientists said, their next best hope to gather valuable information about Pluto might not be until after 2200!

After snapping pictures of Pluto during a six-month period, *New Horizons* would travel for ten more years deep into the Kuiper Belt, a ring of icy rocks beyond Neptune's orbit. Many scientists believe the Kuiper Belt is where many comets originate. Comets are dirty snowballs of ice that hurtle through the solar system.

Some astronomers believe Pluto should be counted as a Kuiper Belt object rather than as the ninth planet. *New Horizons* could settle that issue once and for all. Then again, if Congress scraps the mission, Pluto—and astronomers—will be left out in the cold.

Carefully Selected Graphic Organizers



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"…students must have a way to participate in the mathematical practices of the classroom community. In a very real sense, students who cannot participate in these practices are no longer members of the community from a mathematical point of view."

Cobb (1999)

Extending mathematical knowledge through conversations

Discuss the following ideas about rational numbers.

1. Describe how you know that ¾ and .75 are equivalent.

2. Explain how you can simplify a rational number like 6/36.

Encourage Interactions with Words

- Questions, Reasons, Examples:
  - If two planes are landing on *intersecting* landing strips, they must be cautious. Why?
  - Which one of these things might be *symmetrical*? Why or why not?
    - A car?
    - A water bottle?
    - A tree?

- Relating Word
  - Would you rather play catch with a *sphere* or a *rectangular prism*? Why?

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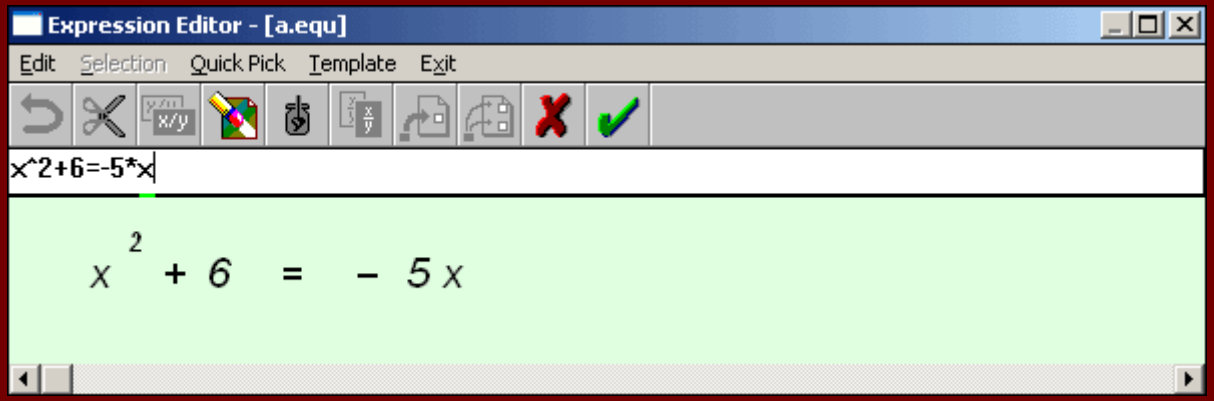
Strategy Knowledge and Use

Students experience problems:

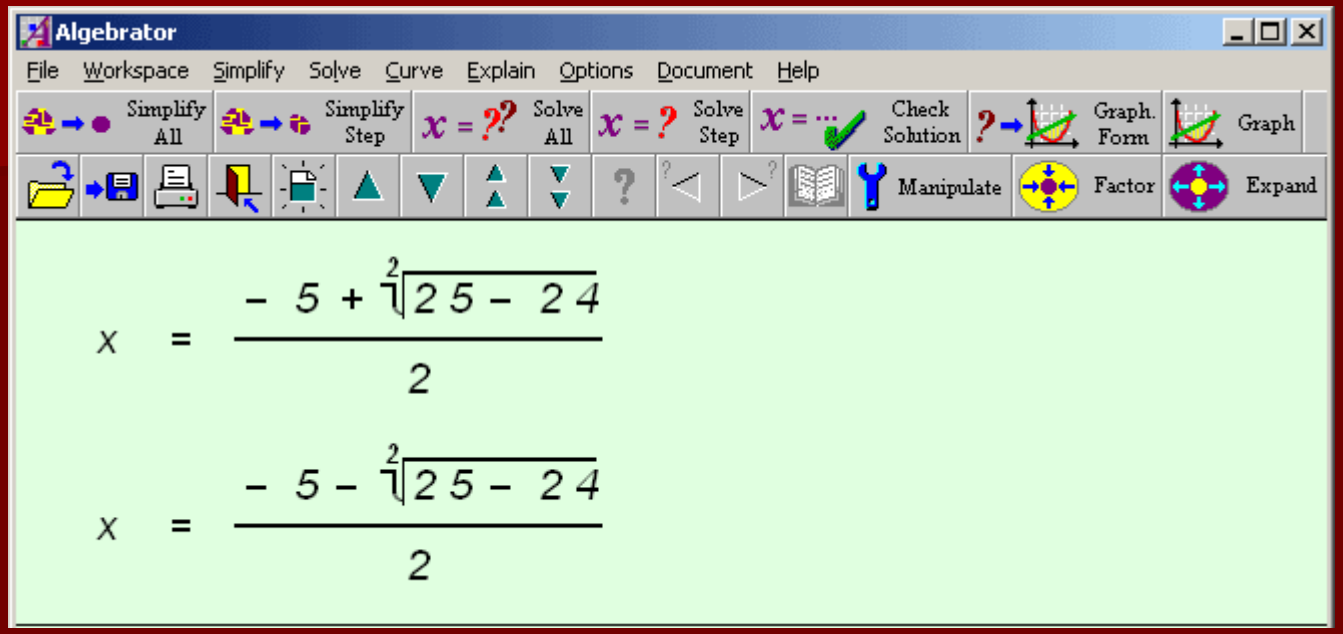
- Remembering steps in a strategy;
- Developing self-questioning skills;
- Selecting an appropriate strategy to fit a particular problem.

You could use the ‘Algebrator” . . .

Step 1. Enter the equation into the window.



Step 2. Let the Algebrator solve it.



Step 3. Stop Thinking!!!

. . . What would you be missing?

Thank You