# Preventing Mathematics Difficulties in Young Children

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# Objectives

- Briefly review the research literature on effective mathematics instruction for students with learning disabilities and low achievement.
- Define the big idea of number and operations.
- Describe instructional scaffolding that promotes access to number and operations.
- Examine models of early number development.
- Discuss trends in assessment and instruction that are currently under study.

# Building a Culture of Competence

<u>Intensive, Individual Interventions</u>

1-5%

- Individual Students
- Assessment-based
- High Intensity

5-10%

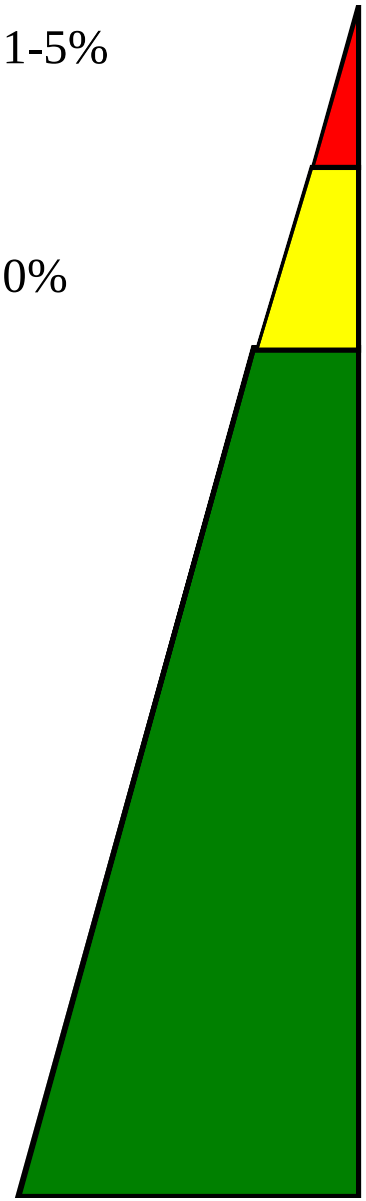
<u>Targeted Group Interventions</u>

- Some students (at-risk)
- High efficiency
- Rapid response

<u>Universal Interventions</u>

80-90%

- All students
- Preventive, proactive



## Quantity Discrimination

| | | |
|---|---|---|
| 3 7 | 5 2 | 6 8 |
| 1 8 | 3 6 | 9 5 |
| 4 2 | 1 7 | 6 2 |
| 8 3 | 7 5 | 3 4 |

## Missing Number

| | | |
|---|---|---|
| 3 4 _ | 5 6 _ | 6 7 _ |
| 1 2 _ | 2 3 _ | 7 8 _ |
| 4 5 6 | 2 3 _ | 6 7 _ |
| 7 8 _ | 5 6 _ | 3 4 _ |

# Mathematics proficiency includes:

Conceptual understanding: comprehension of mathematical concepts, operations, and relations

Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

Strategic competence: ability to formulate, represent, and solve mathematical problems

Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification

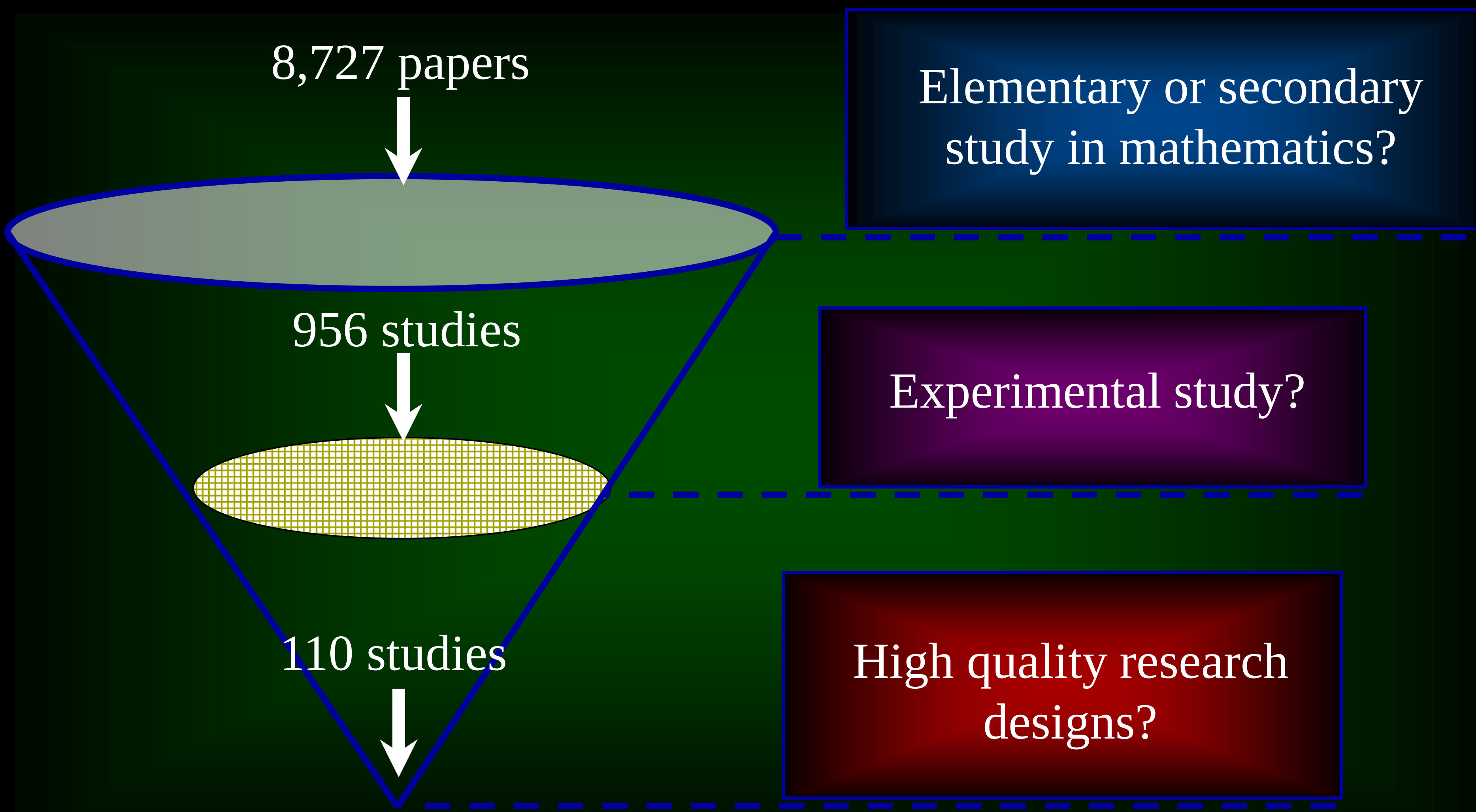
Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

(MLSC, p. 5, 2002)

# Initial Comments about Mathematics Research

- The knowledge base on documented effective instructional practices in mathematics is less developed than reading.
- Mathematics instruction has been a concern to U.S. educators since the 1950s, however, research has lagged behind that of reading.
- Efforts to study mathematics and mathematics disabilities has enjoyed increased interests recently.

# Identifying High Quality Instructional Research



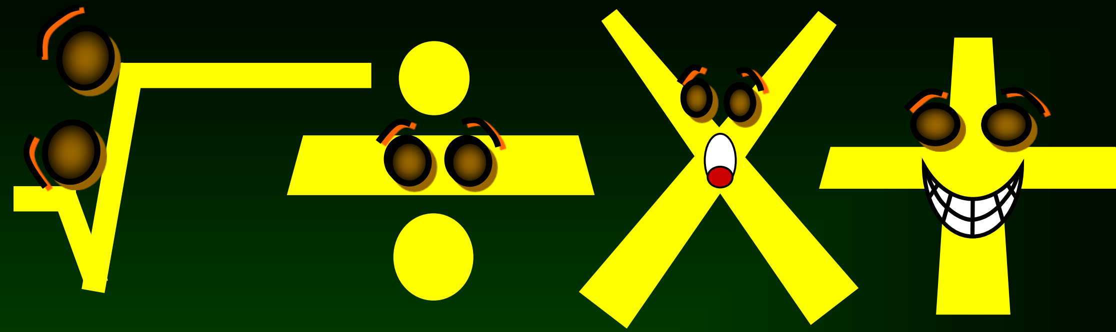
# RESULTS

## *Feedback to Teachers on Student Performance*

1. Seems much more effective for special educators than general educators, though there is less research for general educators.

2. May be that general education curriculum is often too hard.

# *Overview of Findings*



- Teacher modeling and student verbal rehearsal remains phenomenally promising and tends to be effective.
- Feedback on effort is underutilized and the effects are underestimated.
- Cross-age tutoring seems to hold a lot of promise as long as tutors are well trained.
- Teaching students how to use visuals to solve problems is beneficial.
- Suggesting multiple representations would be good.

# Translating Research to Practice

At your table respond to the following questions:

1. How does the research support/confirm what I'm already doing in my classroom in mathematics?

2. How might I apply what I've learned to change what I'm doing in mathematics instruction?

# Architecture for Learning

Big Ideas

Scaffolding

Review/Reteaching

Integration

Conspicuous Strategies

# Big Idea – Number Operations

Plan and design instruction that:

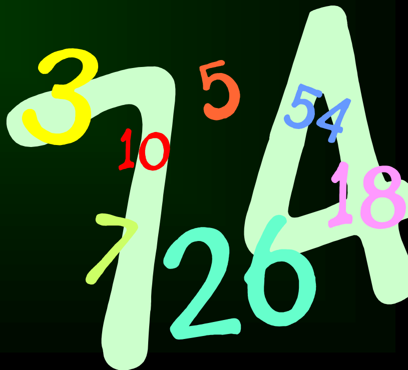
- Develops student understanding from concrete to conceptual,
- Applies the research in effective math instruction, and
- Scaffolds support from teacher ⇨ peer ⇨ independent application.

# Number and Operations

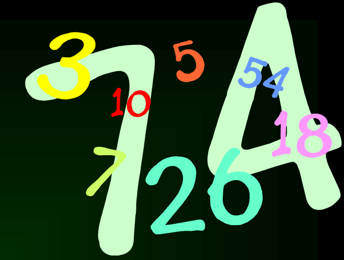
## Standard

Instructional programs from pre-Kindergarten through grade 12 should enable all students to:

1. Understand numbers, ways of representing numbers, relationship among numbers, and number systems
2. Understand meanings of operations and how they relate to one another.
3. Compute fluently and make reasonable estimates.



# Number and Operations



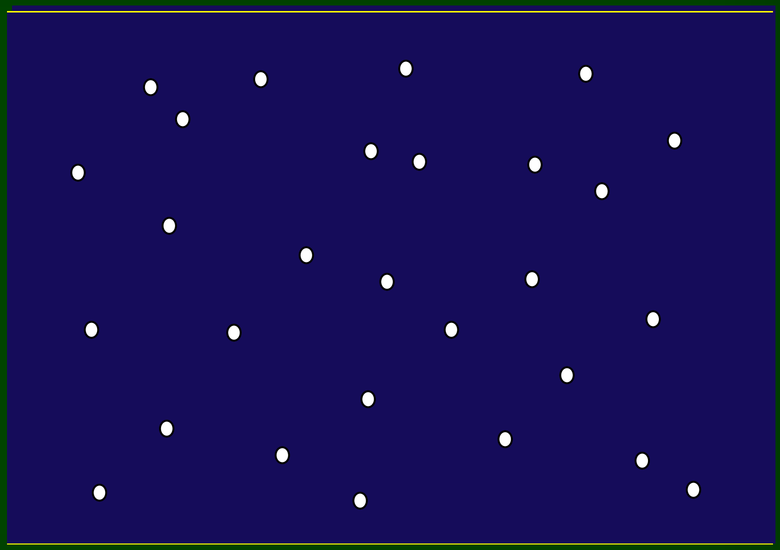

## Pre K-2 Expectation:

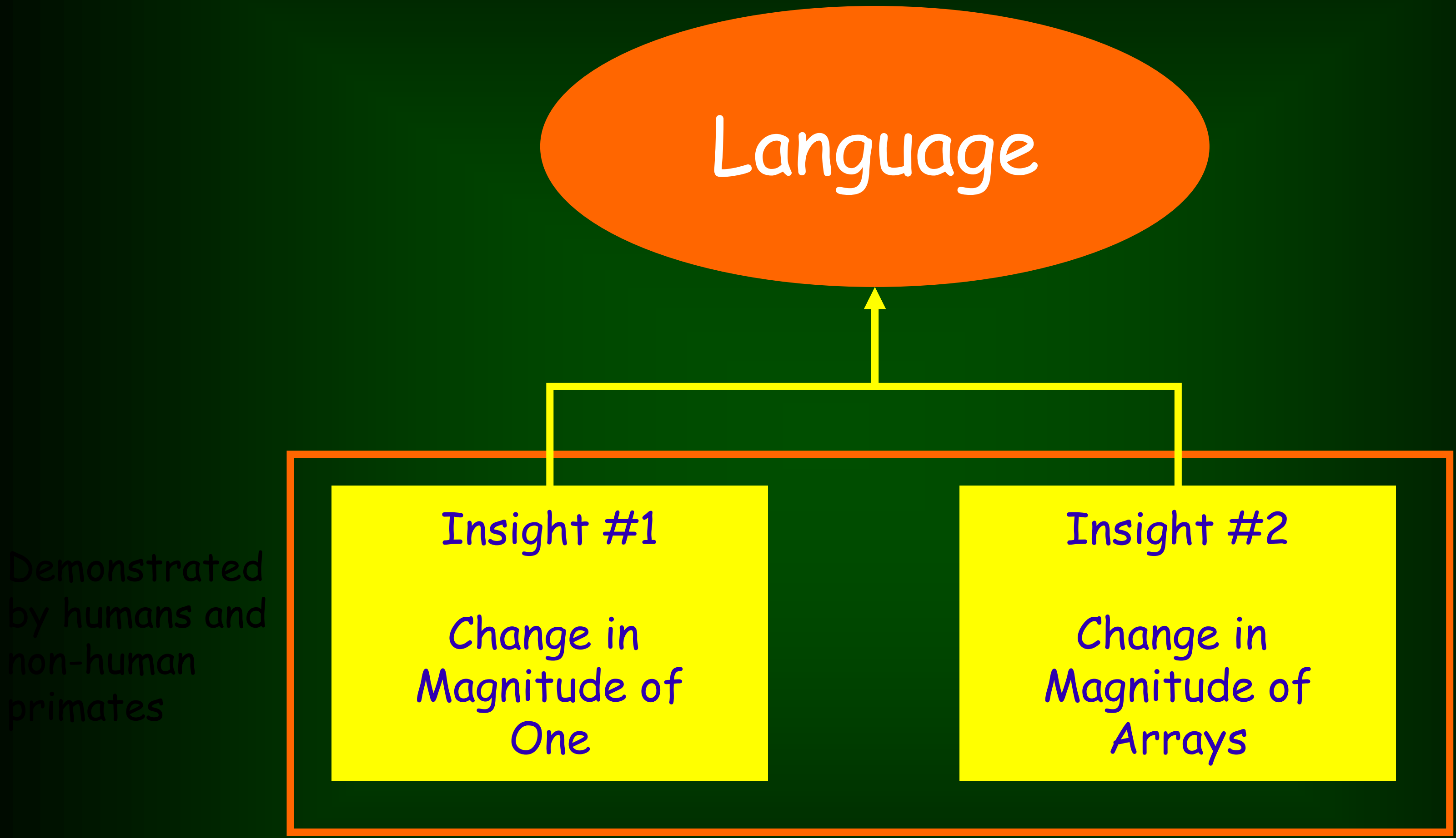
- **Count with understanding and recognize "how many" in sets of objects**
- Use multiple models to develop initial understanding of place value and the base-ten number system
- Develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections
- **Develop a sense of whole numbers and represent them in flexible ways, including relating, composing, and decomposing numbers**
- Connect number words and numerals to the quantities they represent, using physical models and representations; understand and represent commonly used fractions, such as 1/4, 1/3, and1/2.

# Sometime Later

Insight #1

Change in Magnitude of One




Language
Insight #1
Change in Magnitude of One
Insight #2
Change in Magnitude of Arrays
Demonstrated by humans and non-human primates

# Early Number Concepts and Number Sense

Number sense refers to a child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons (Case, 1998; Gersten & Chard, 1999).

# BIG IDEA #1: Counting

To make counting meaningful, it is important to promote knowledge and awareness of patterns, classification, cardinality principle, and conservation of number.

Reys, Lindquist, Lambdin, Smith, & Suydam (2003); Van de Walle (2004)

# Summary: Counting

- **Informal Strategies** (Gersten & Chard, 1999)
  - Parents can help children develop early number sense using various activities such as asking them to:
    - ascend and count four steps and then count and descend two steps
    - count forks and knives when setting the table.
- **Formal Strategies**
  - Rational counting (Stein, Silbert, Carnine, 1997)
  - “Min” strategy: Starting with the larger number and counting on when trying to find the answer to either 3+8 or 8+3, whether using one’s fingers, manipulatives, or stick figures (Gersten & Chard, 1999).

# Counting Sets Activity

Have children count several sets where the number of objects is the same but the objects are very different in size.

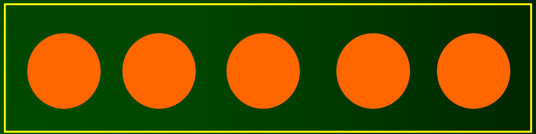
Discuss how they are alike or different. “Were you surprised that they were the same amount? Why or why not?

What insight are we hoping all children will develop?

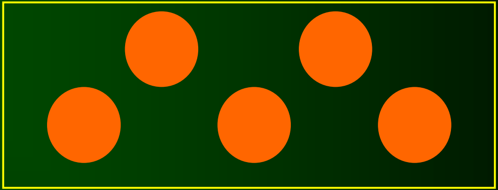
**(Van de Walle, 2004)**

# Count and Rearrange Activity

Have children count a set. Then rearrange the set and ask, “ How many now?”



If they see no need to count over, you can infer that they have connected the cardinality to the set regardless of its arrangement.



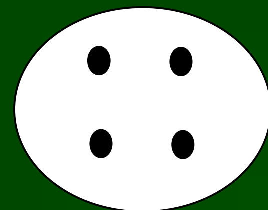
**nt again, discuss why they**
**same.**

What insight are we hoping all children will develop?


**(Van de Walle, 2004)**

# Learning Patterns Activity

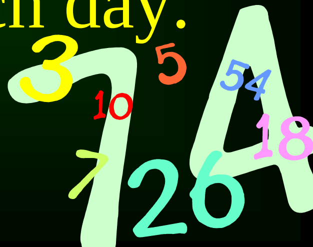
To introduce the patterns, provide each student with about 10 counters and a piece of construction paper as a mat. Hold up a dot plate for about 3 seconds.



**“Make the pattern you saw using the counters on the mat. How many dots did you see? How did you see them?**

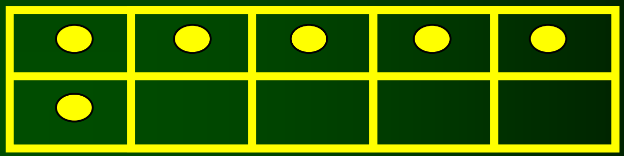
Discuss the configuration of the patterns and how many dots. Do this with a few new patterns each day.

**(Van de Walle, 2004)**



# Ten-Frame Flash Activity

- Flash ten-frame cards to the class or group, and see how fast the children can tell how many dots are shown.



- Variations:
  - Saying the number of spaces on the card instead of the number of dots
  - Saying one more than the number of dots (or two more, and also less than)
  - Saying the "ten fact" – for example, "Six and four make ten"

**(Van de Walle, 2004)**

# Part-Part-Whole:
# Two out of Three Activity

Make lists of three numbers, two of which total the whole.
Here is an example list for the number 5.

2-3-4

5-0-2

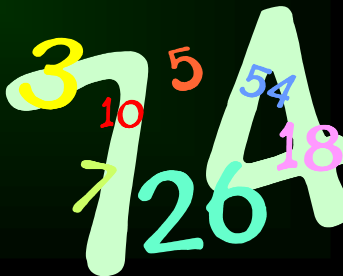
1-3-2

3-1-4

2-2-3

4-3-1

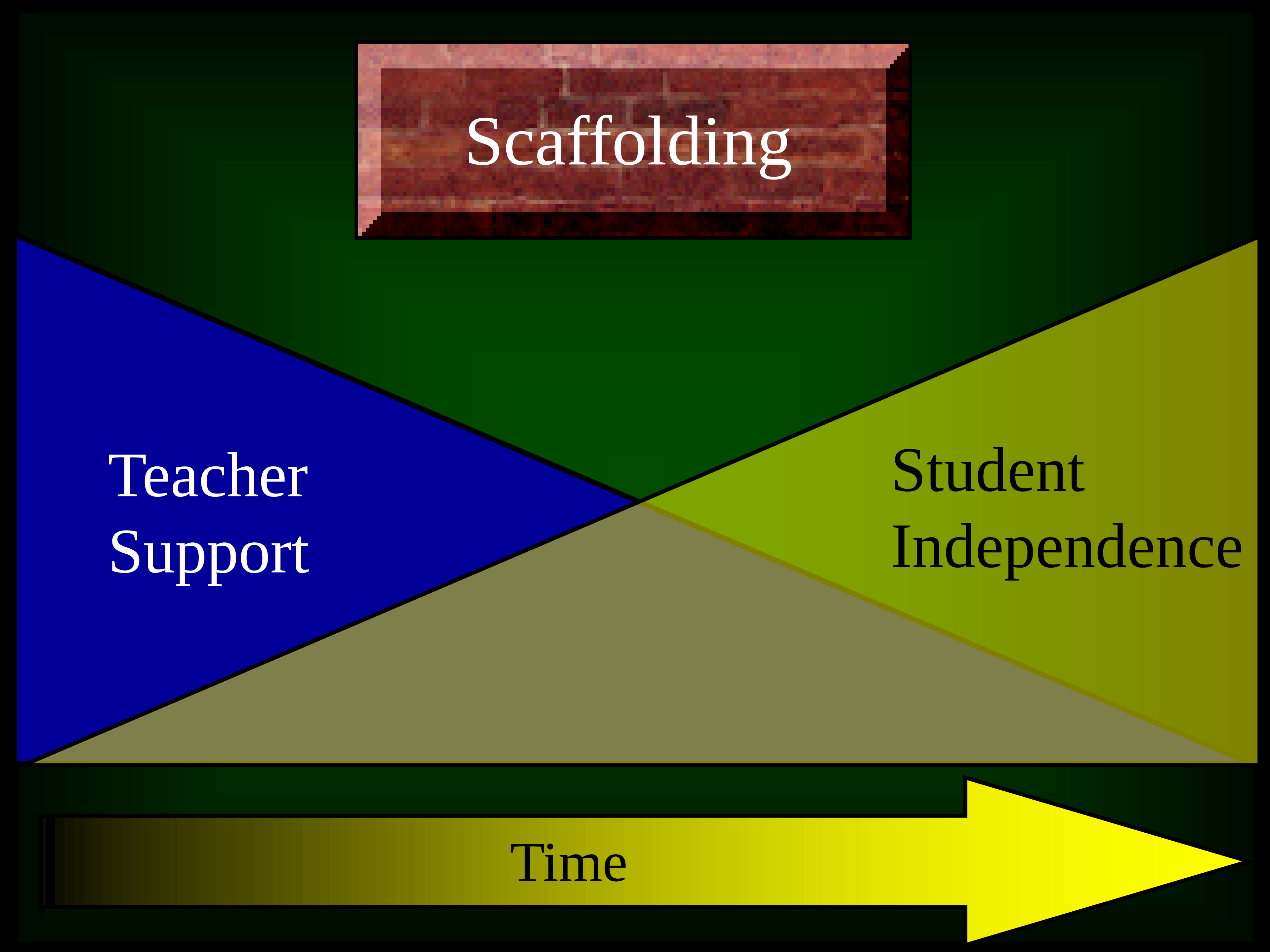
1. Write the list on the board or overhead.
2. Have children take turns selecting the two numbers that make the whole.
3. Challenge children to justify their answers.



**(Van de Walle, 2004)**

# Summary: Teaching Number Sense

- Math instruction that emphasizes memorization and repeated drill is limited (Gersten & Chard, 1999).
- Strategies that support mathematical understanding are more generalizable.
- Teaching number sense shifts the focus from computation to mathematical understanding and better helps students with learning difficulties.


Scaffolding
Teacher
Support
Student
Independence
Time

# Instructional Scaffolding includes:

Sequencing instruction to avoid confusion of similar concepts.

Carefully selecting and sequencing of examples.

Pre-teaching prerequisite knowledge.

Providing specific feedback on students' efforts.

Offering ample opportunities for students' to discuss their approaches to problem solving.

# Sequencing Skills and Strategies

Adding w/ manipulatives/fingers

Adding w/ semi-concrete objects (lines or dots)

Adding using a number line

Min strategy

Missing addend addition

Addition number family facts

Mental addition (+1, +2, +0)

Addition fact memorization

Concrete/
conceptual

Semi-concrete/
representational

Abstract

# Teach prerequisite skills thoroughly.

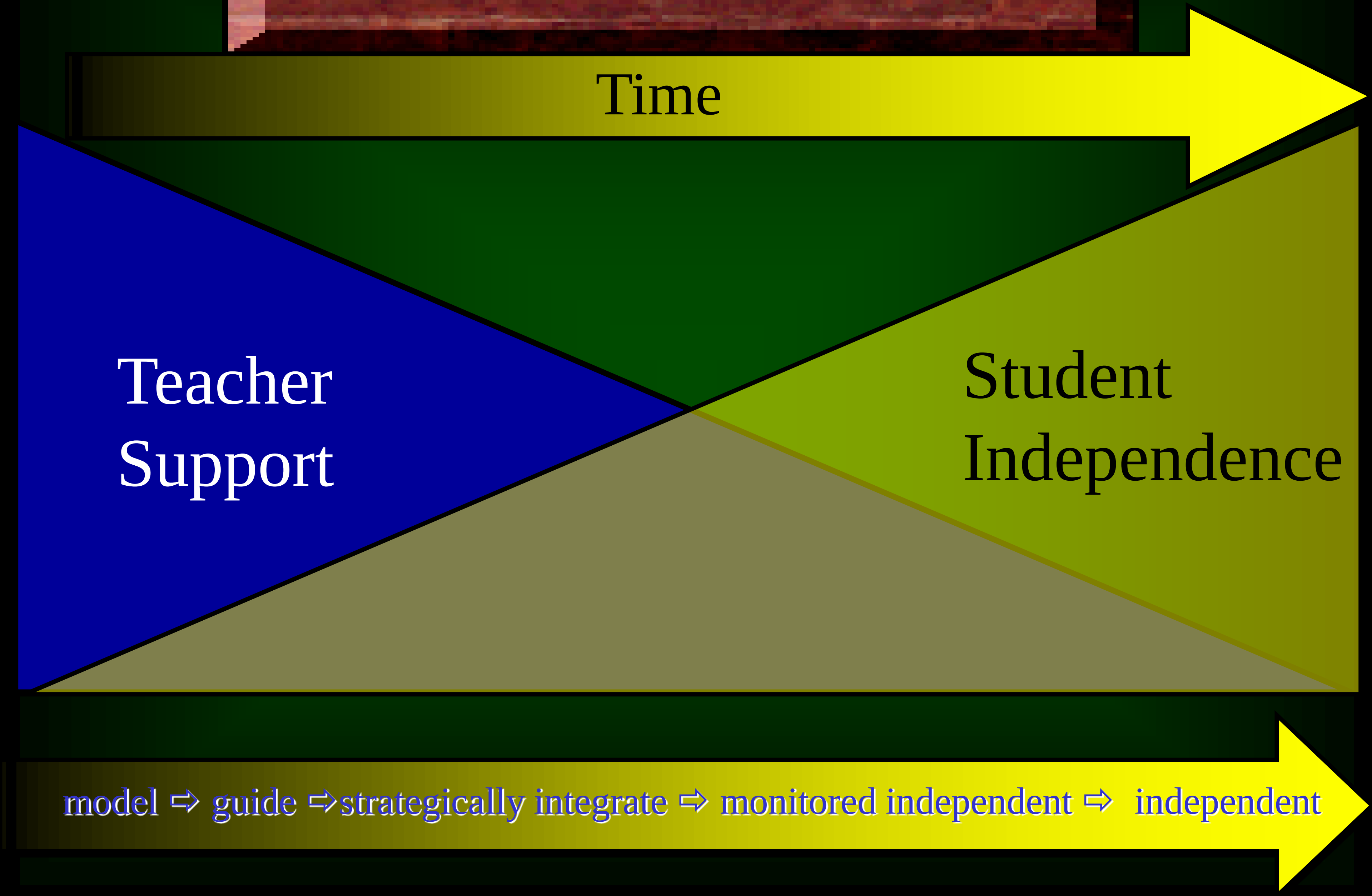
6 + 3 = ?

What are the prerequisite skills students need to learn in Pre-K and K before learning to add single digit numbers?

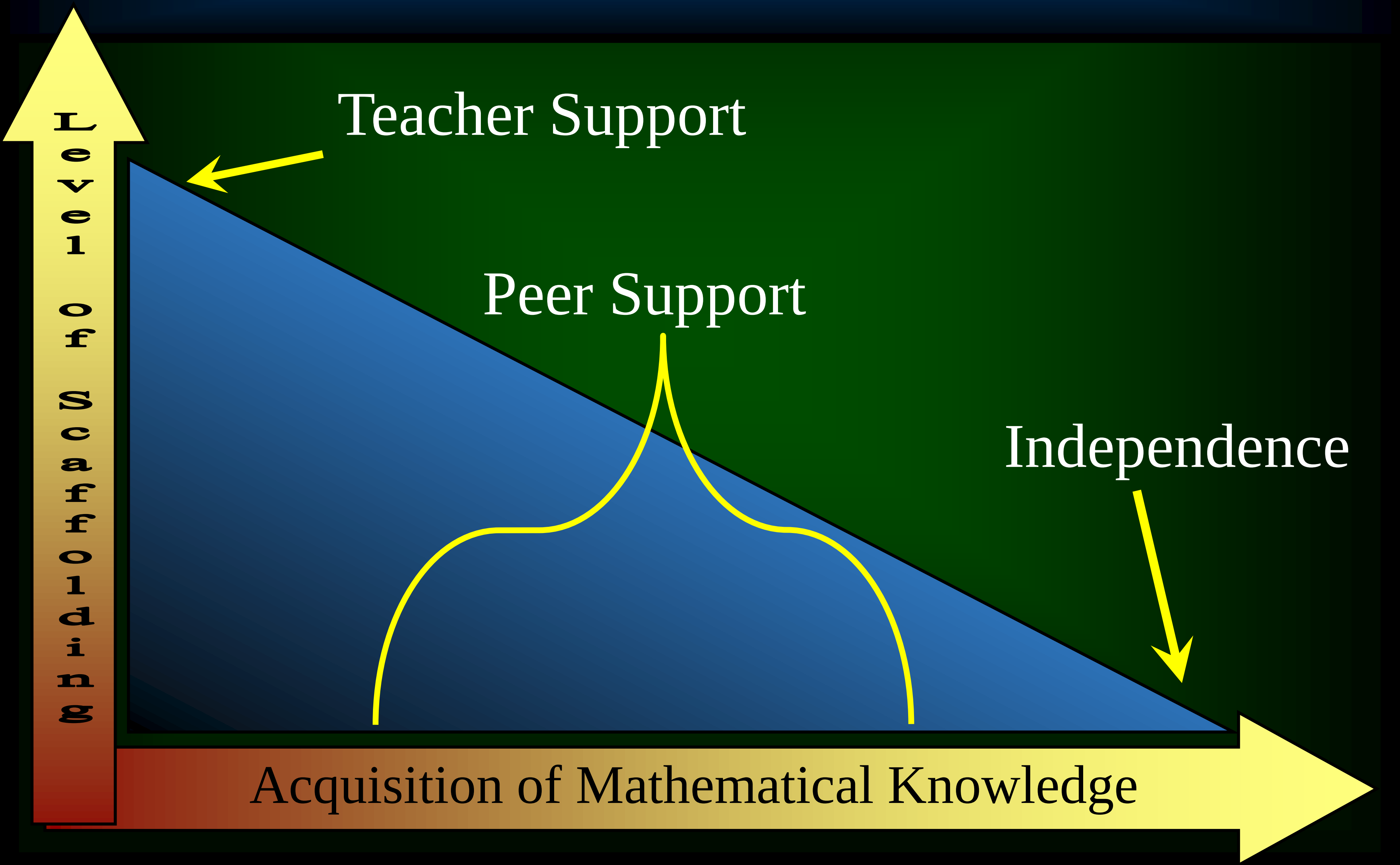
Oral counting; number vocabulary

| | | |
|---|---|---|
| Adding with concrete models | ⇨ | Subtracting with concrete models |
| Adding with semi-concrete objects<br>Or adding using a number line | ⇨ | Subtracting with semi-concrete<br>or number line |
| Min strategy with addition<br>(counting up from the larger addend) | ⇨ | Min strategy with subtraction<br>(counting up from the<br>subtrahend to the minuend) |
| Missing addend addition | | |
| Addition number families | ⇨ | Subtraction number families |
| Mental addition (+1; +2, +0)<br>Addition fact memorization | ⇨ | Mental subtraction<br>Subtraction fact memorization |

# Scaffold the Instruction



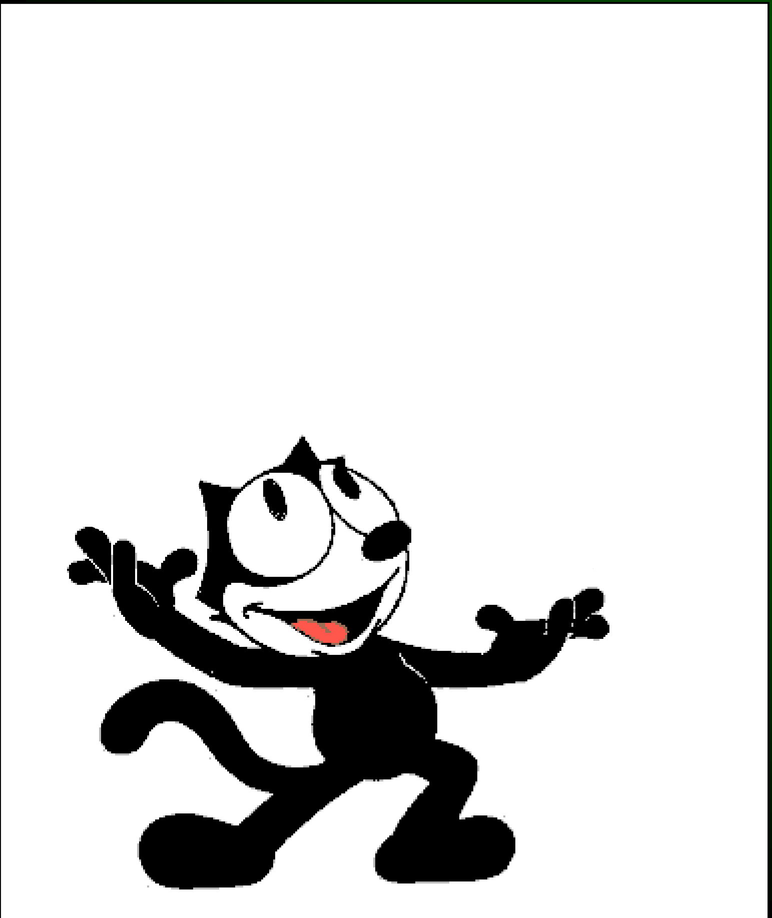
# Work Among Peers: Instructional Interactions



# Lesson Planning: Addition with Manipulatives

| | Scaffolding | | | | |
|---|---|---|---|---|---|
| | Model | Guide Strategy | Strategy Integration | Teacher Monitored Independent | Independent (no teacher monitoring) |
| Day 1 & 2 problems | | | | | |
| Day 3 & 4 problems | | | | | |
| Day 5 & 6 problems | | | | | |
| Day 7 & 8 problems | | | | | |
| Day 9 until accurate | | | | | |
| Until fluent | | | | | |

# Introduction to the Concept of Addition



# Addition of Semi-concrete Representational Models

$5 + 3 = ?$

||||| + ||| = ||||||||

$5 + 3 = 8$

||||| ||| = ||||||||

# The Min Strategy

5 + 3 = ?

5 + 3 = ? 8

|||

5 + 3 = 8

# Missing Addend Addition

4 + ? = 6

Early Algebraic Reasoning

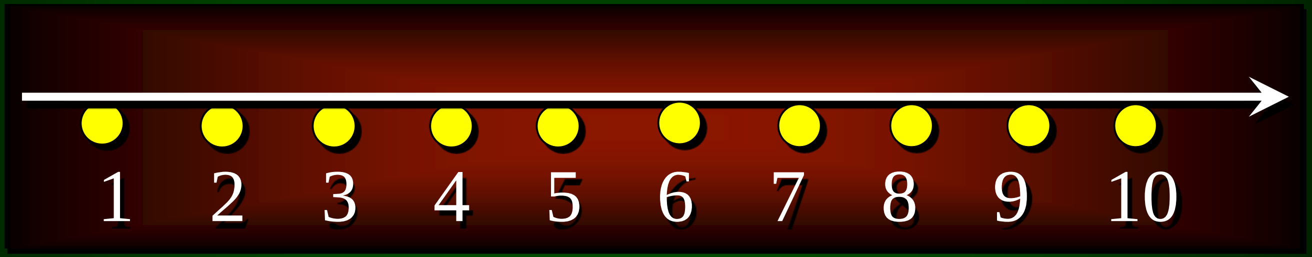
4 + ? = 6

|||| || = ||||||

4 + 2 = 6

|||| || = ||||||

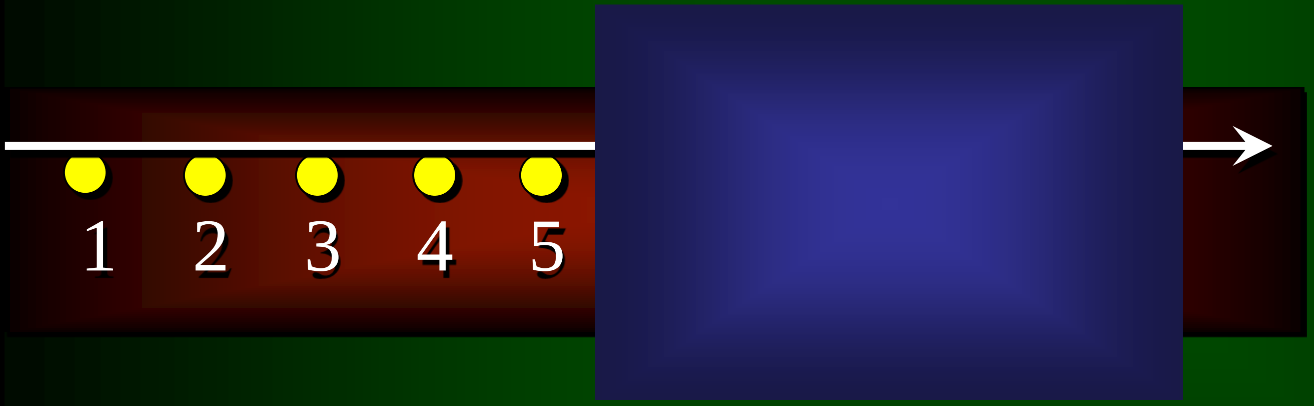
# Number Line Familiarity




“What is one *more than* 3?”

“What is 2 *less than* 8?”

# Mental Math



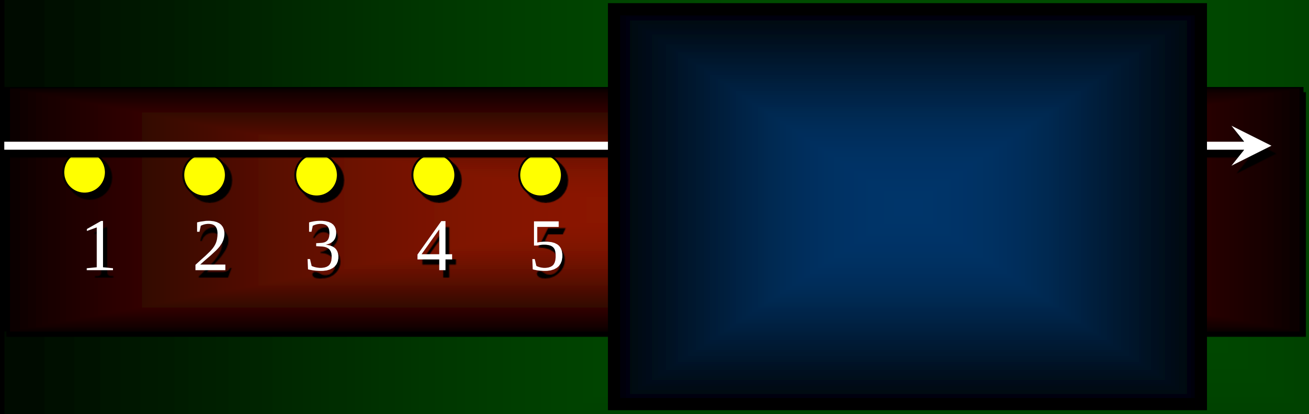

5 + 1 =

# Mental Math

1 2 3 4 5

5 + 1 =

# Mental Math




5 + 2 =

# Mental Math


"In the story of Aunt Flossie's Hats,
how many hats did Aunt Flossie have?"

# Number Families


4 + 3 = 7

7 - 4 = 3

3 + 4 = 7

7 - 3 = 4

# Fact Memorization

$$\begin{array}{r} 5 \\ +2 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ +6 \\ \hline \end{array}$$

1 + 8 =

4 + 3 =

$$\begin{array}{r} 4 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 2 \\ +7 \\ \hline \end{array}$$

5 + 2 =

6 + 0 =

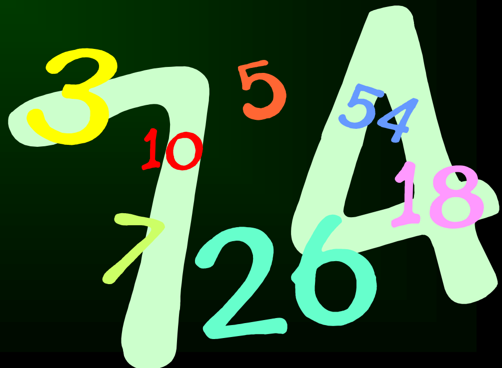
# Lesson Planning: Addition with Semi-Concrete Models

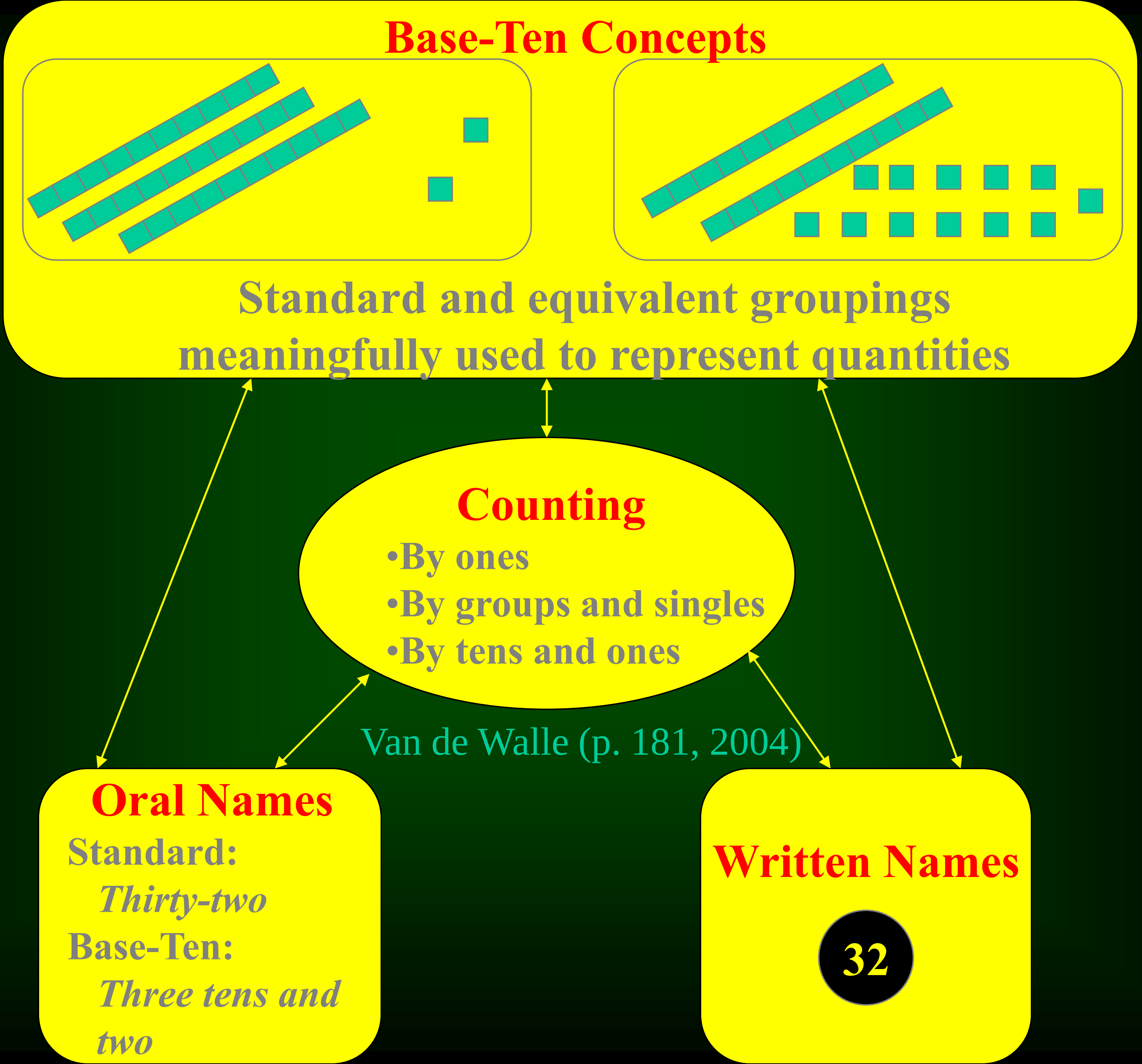
| | Scaffolding | | | | |
|---|---|---|---|---|---|
| | Model | Guide Strategy | Strategy Integration | Teacher Monitored Independent | Independent (no teacher monitoring) |
| Day 1 & 2 problems | 3 | 2 | | | |
| Day 3 & 4 problems | 2 | 4 | | | |
| Day 5 & 6 problems | 1 | 3 | 3 new, 2 conc | | |
| Day 7 & 8 problems | | 1 | 2 new, 2 conc | 4 new, 3 conc | |
| Day 9 until accurate | | | | 4 new, 2 conc | |
| Until fluent | | | | | 4 -5 daily |

# Place Value

Relational understanding of basic place value requires an integration of new and difficult-to-construct concepts of grouping by tens (the base-ten concept) with procedural knowledge of how groups are recorded in our place-value scheme, how numbers are written, and how they are spoken.

(Van de Walle, 2004)




Base-Ten Concepts
Standard and equivalent groupings meaningfully used to represent quantities
Counting
•By ones
•By groups and singles
•By tens and ones
Van de Walle (p. 181, 2004)
Oral Names
Standard:
Thirty-two
Base-Ten:
Three tens and two
Written Names
32

# Developing Whole Number Computation:Algorithms

*Algorithms* are steps used to solve a math problem.

When the number of digits or complexity of computation increases, the need for a traditional algorithm increases.

Traditional algorithms require an understanding of *regrouping*.

# Working With Models

1. **Display the problem** at the top of the place value mat:

$$\begin{array}{r} 27 \\ \underline{+54} \end{array}$$

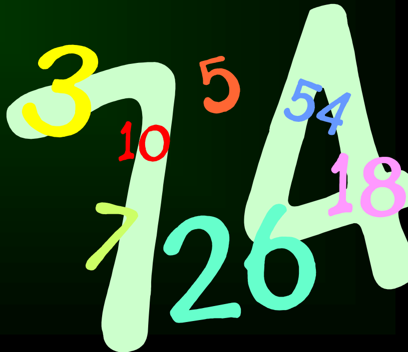
2. **Think aloud as you attempt to answer the question.**

**Teacher:** You are going to learn a method of adding that most “big people” learned when they were in school. Here’s one rule: You begin in the ones column. This is the way people came up with a long time ago, and it worked for them. Okay, I will start with the ones place. We have seven ones and 4 ones on our two ten-frames in the ones place. I am going to fill the first ten-frames by moving some ones in the second ten-frame. I filled up 10. There’s 11. That’s 10 and one. Can I make a trade?  Yes, because now I have a ten and an extra. So, I am going to trade the ten ones for a 10. I have one left in the ones column, which is not enough to trade tens. So the answer is 8 tens and a one – 81.

# Working With Models

3. Next, have students work through similar problems with you prompting them (ask leading questions) as needed.
4. Peer or partner group work or independent work with you monitoring it.
5. Finally, students should be able to do it independently and provide explanations.

Have students explain what they did and why. Let students use overhead models or magnetic pieces to help with their explanations.



Van de Walle (2004)

# In your group…

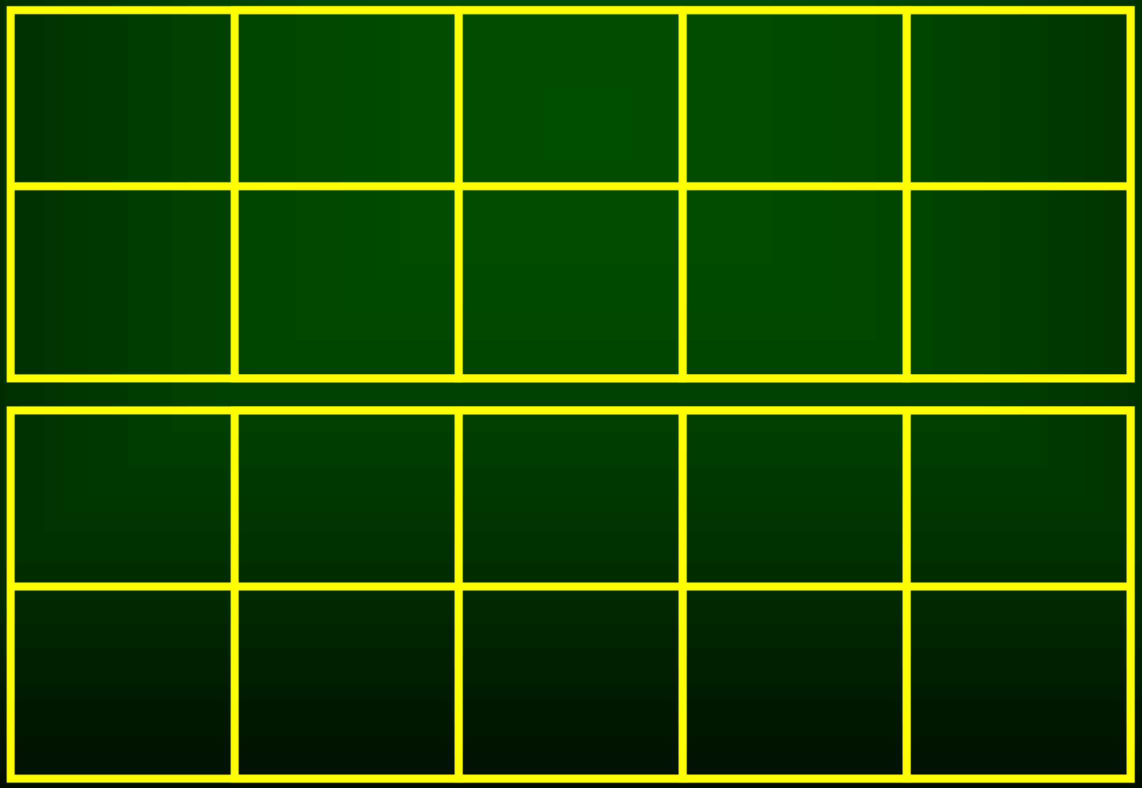
- Describe two visual (concrete or semi-concrete) models that would teach number and number operations (e.g., teach the concept of 4 and 9-4=5).

- What misconceptions could students develop based on your models?

13 - 5 = ☐

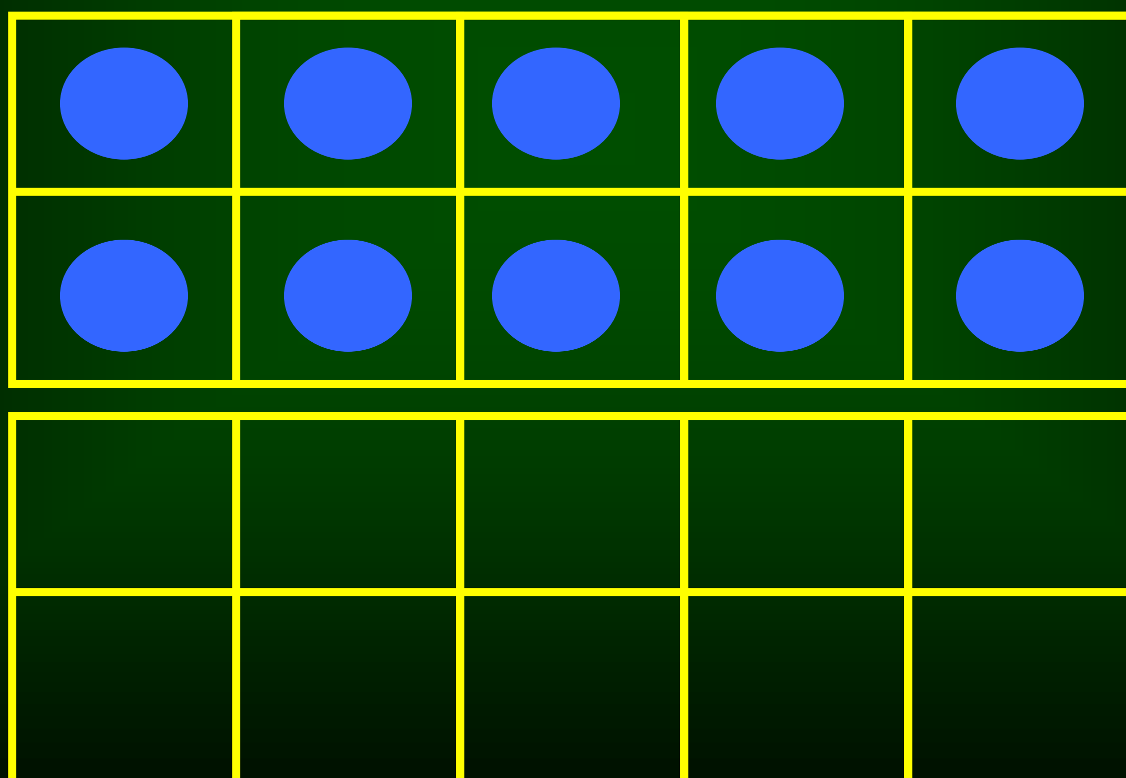
+10 +3 -3 -2

"Manipulative Mode"



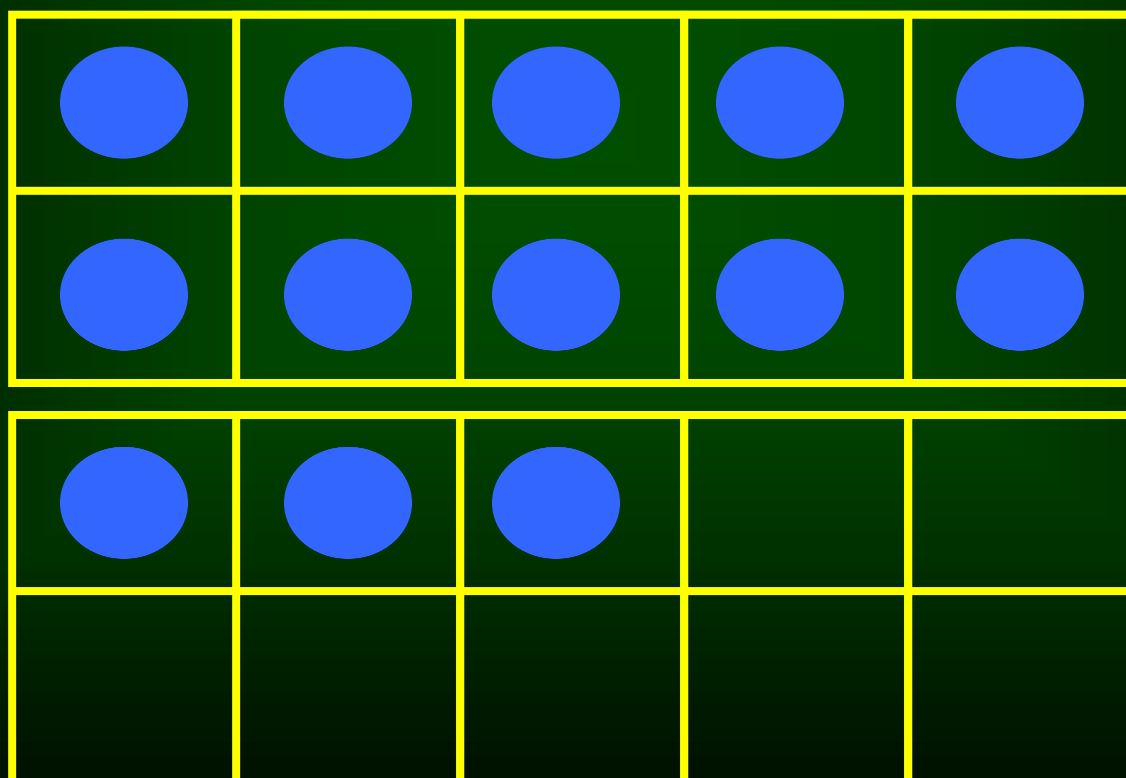
13 - 5 = ☐

+10 +3 -3 -2



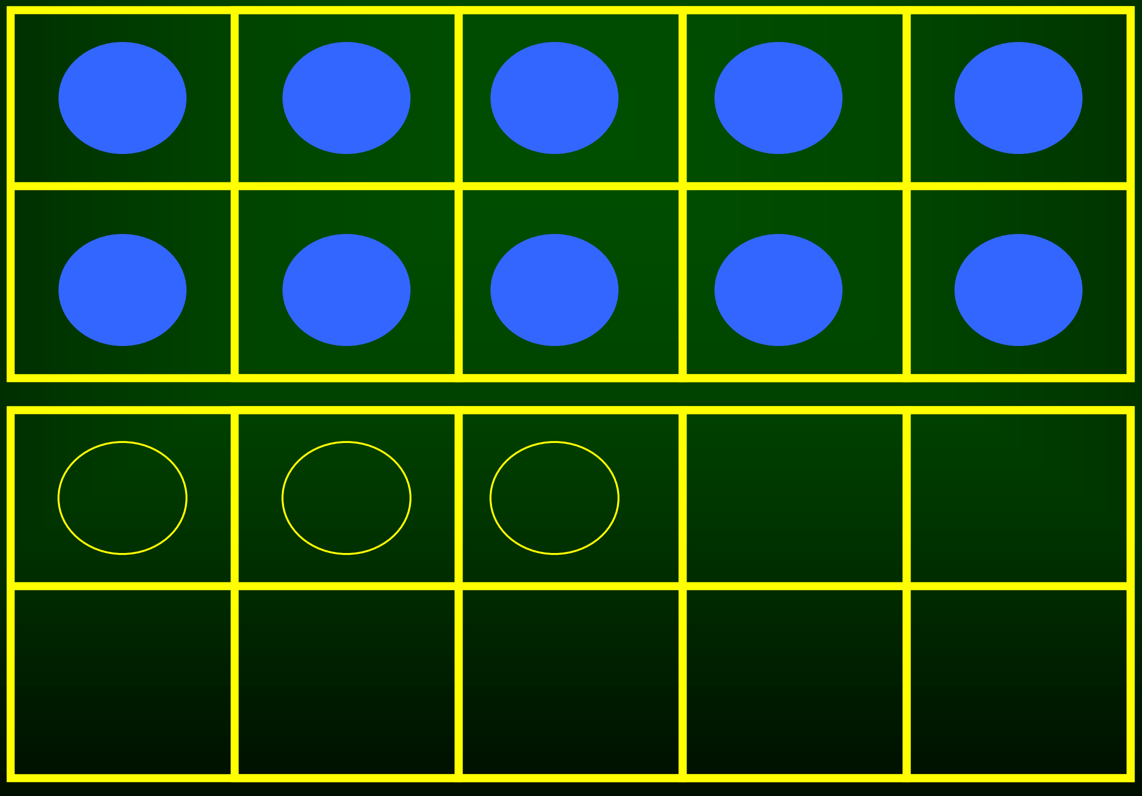
13 - 5 = ☐

+10 +3 -3 -2

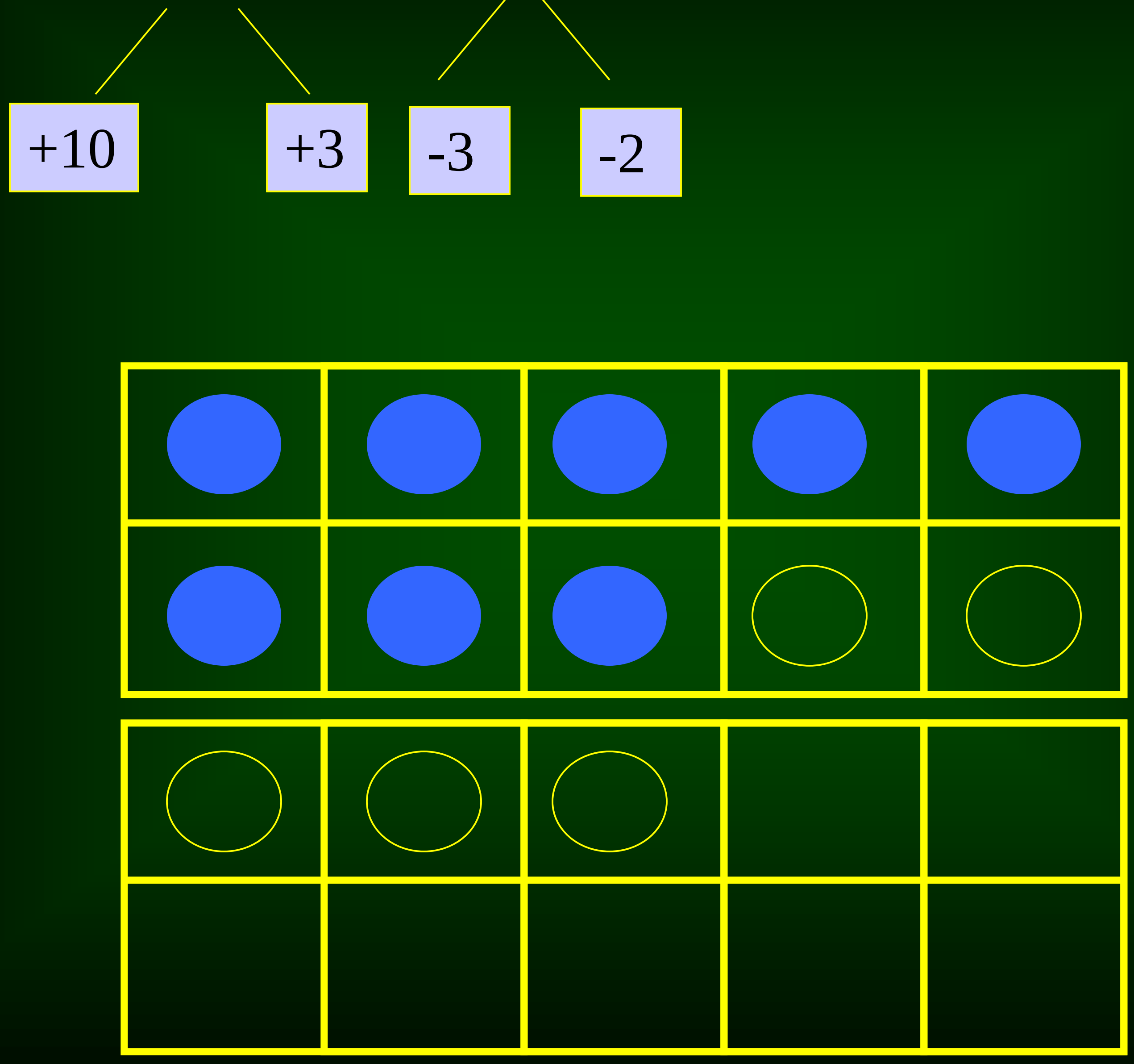


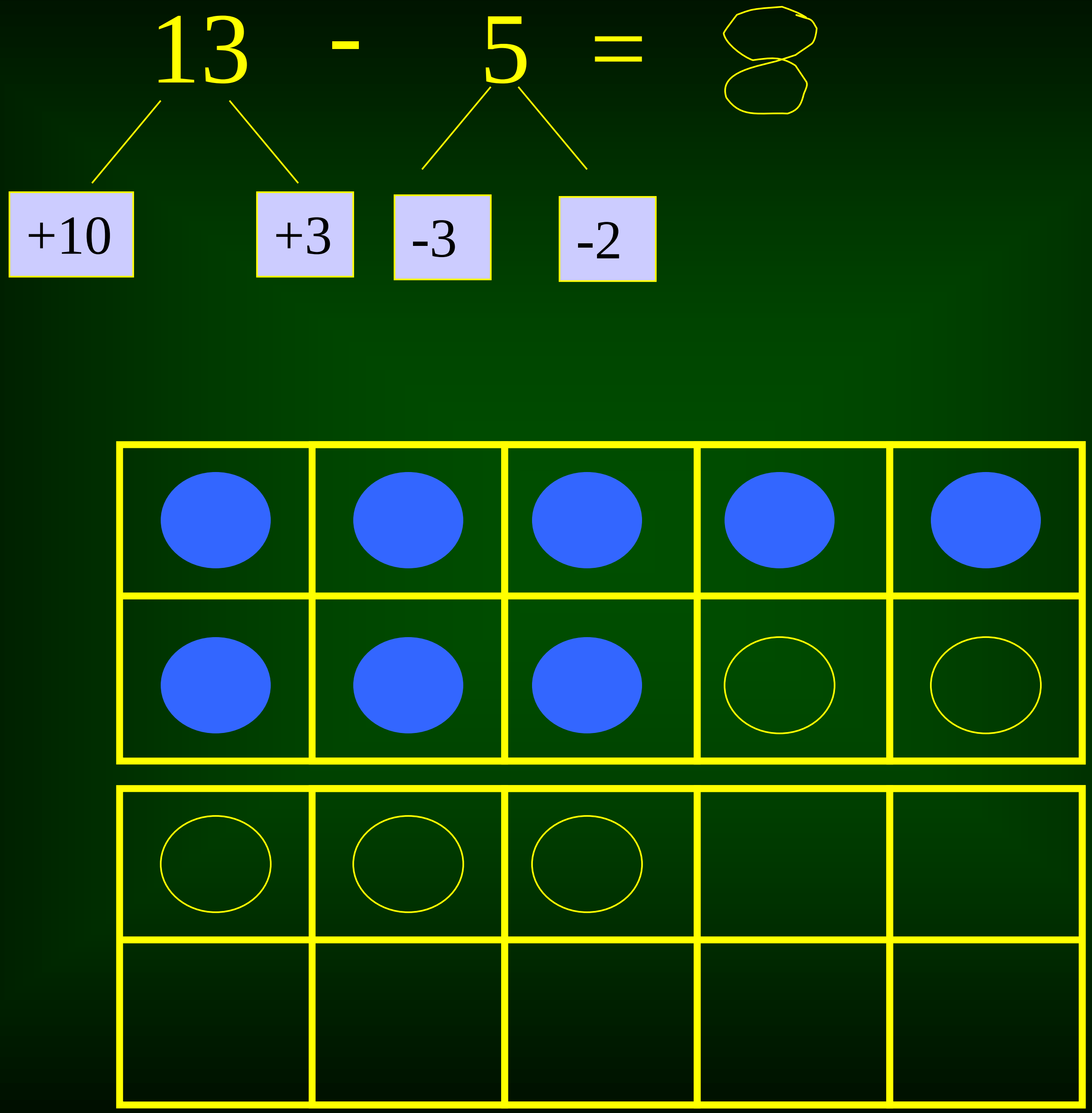
13 - 5 =

+10 +3 -3 -2



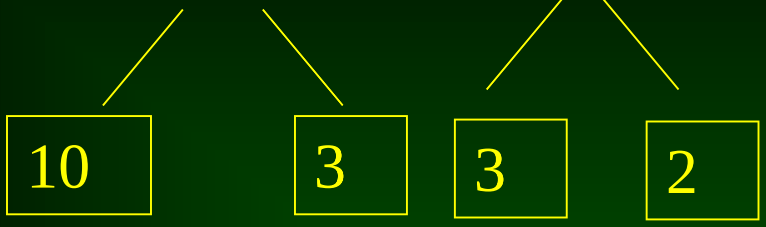
13 - 5 =

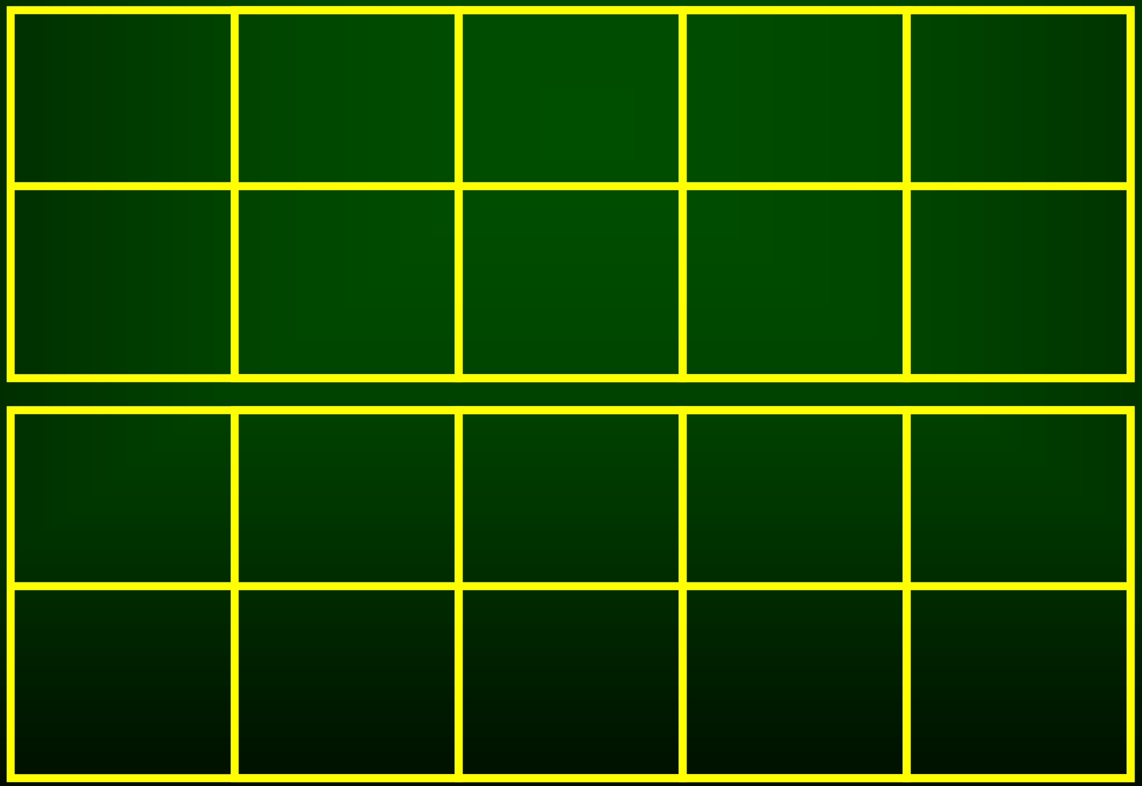




13 - 5 = 8
+10
+3
-3
-2

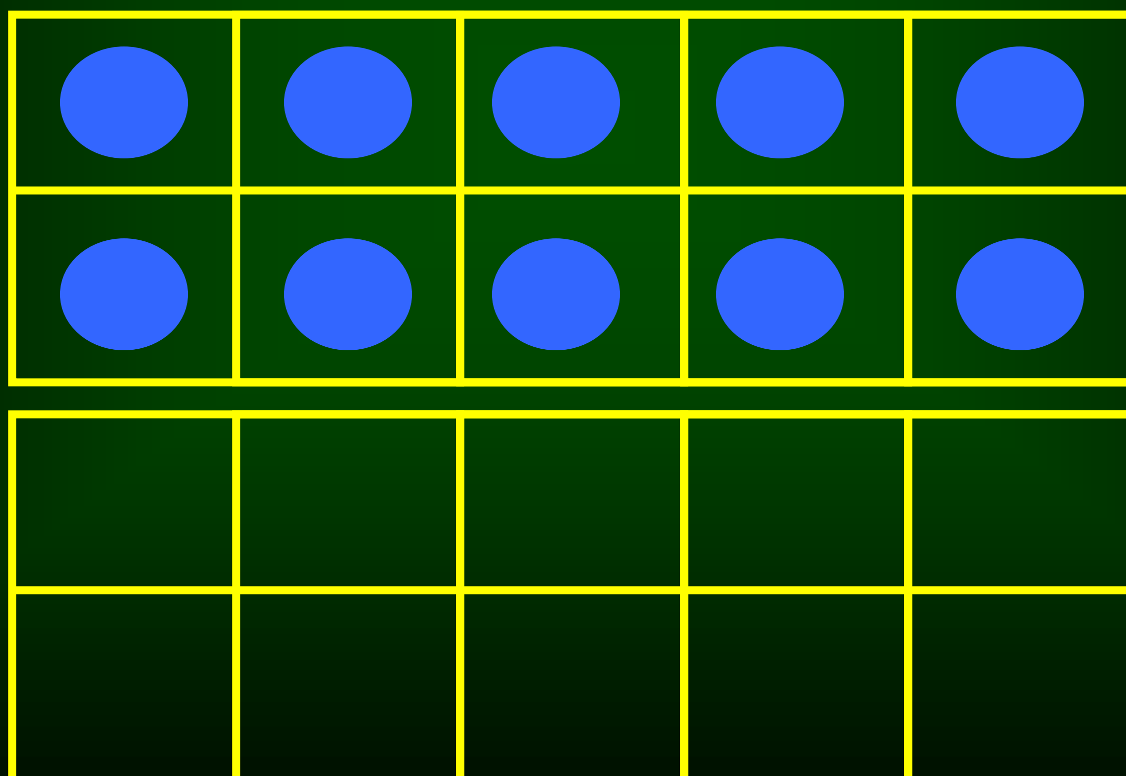
$$13 - 5 =$$



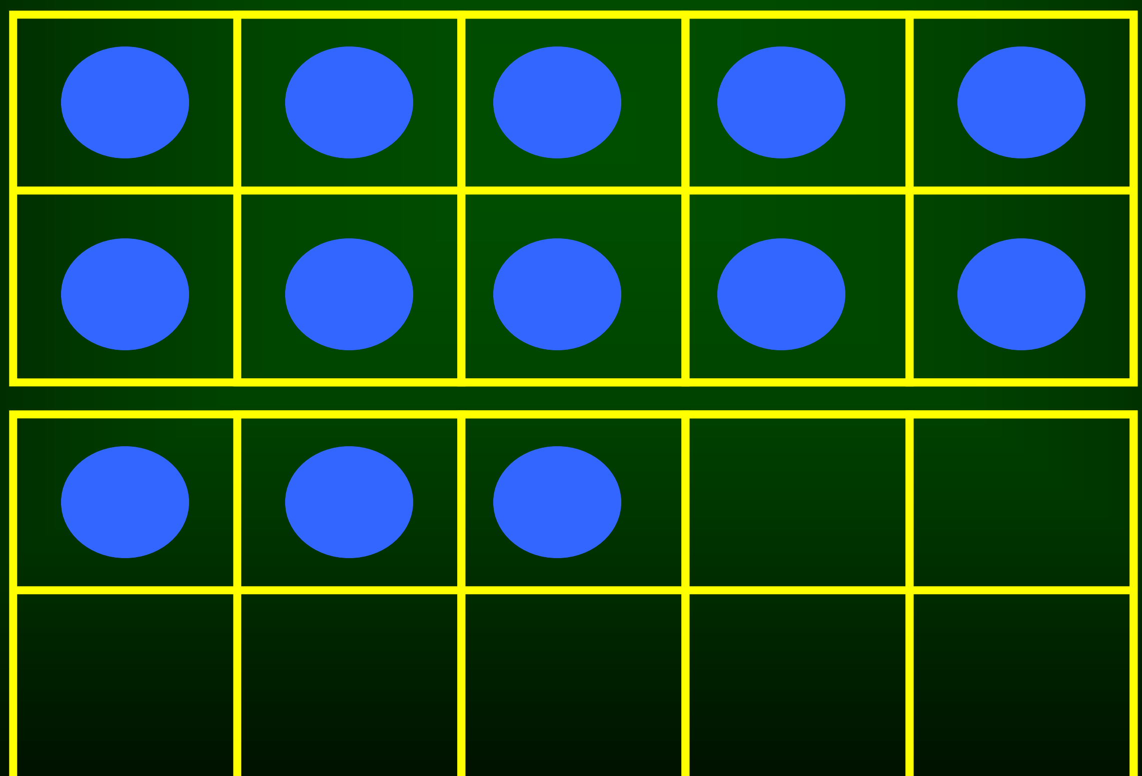
13 - 5 =

10 | 3 | 3 | 2



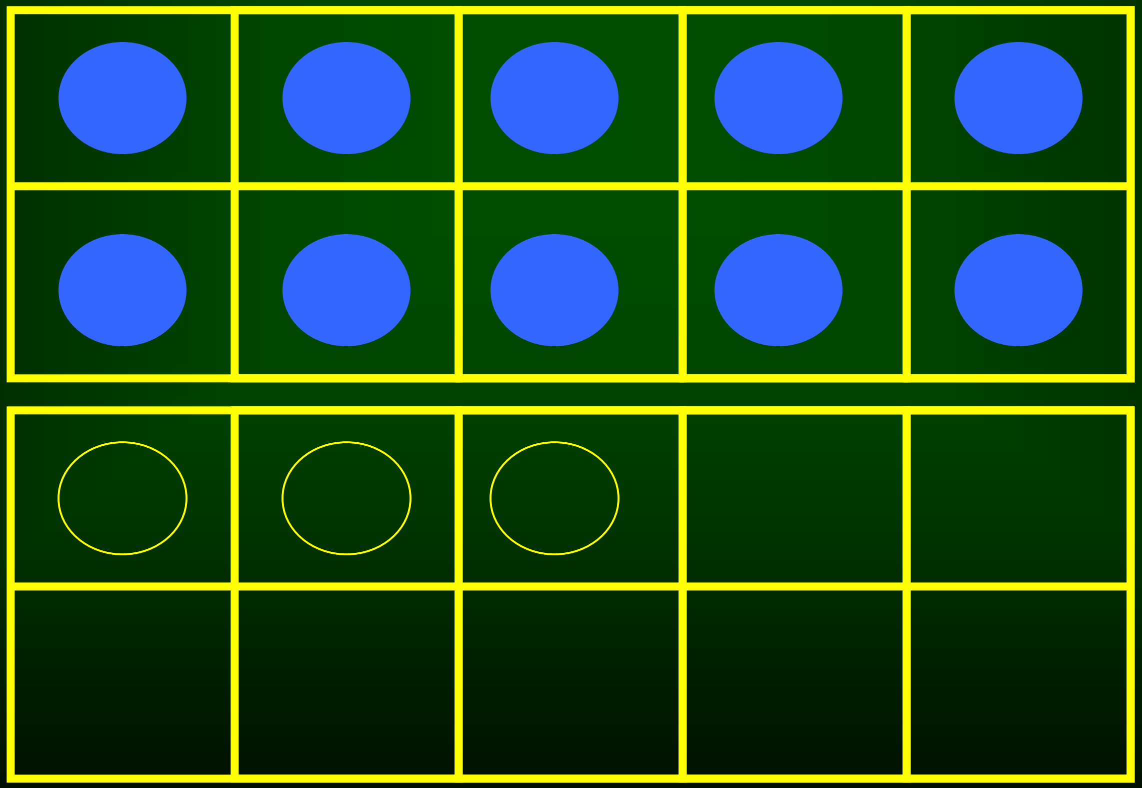
13 - 5 =

10 3 3 2



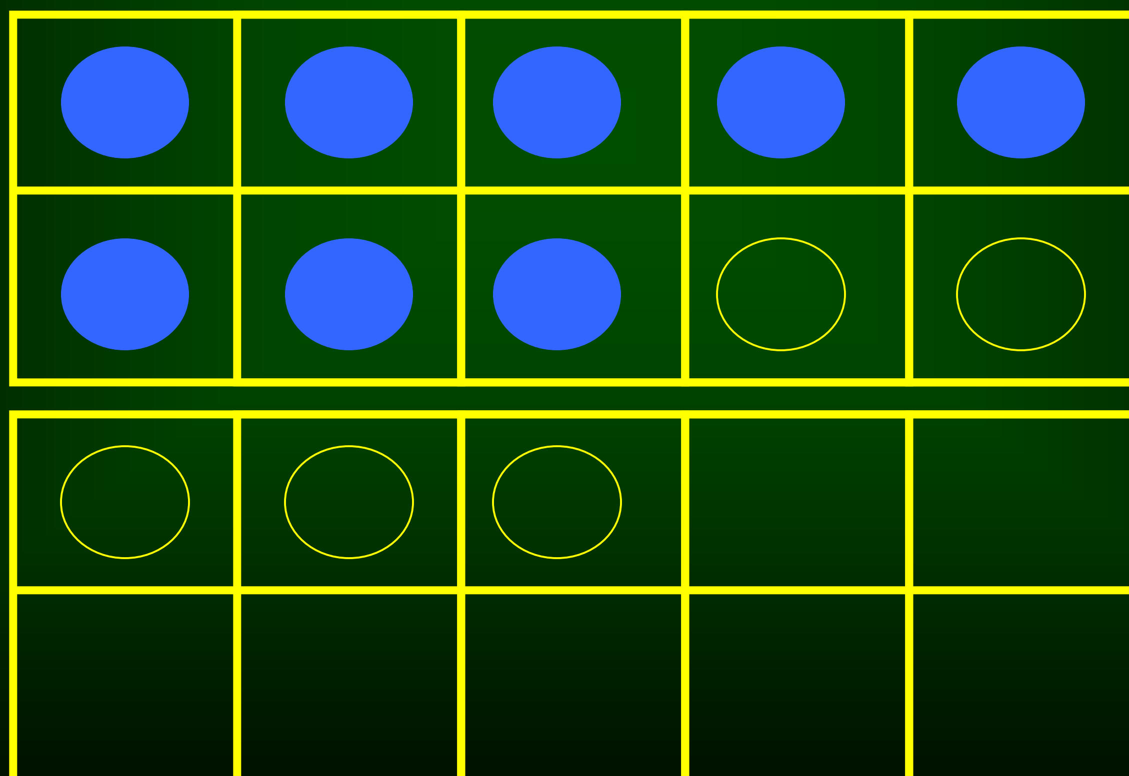
13 - 5 =

10 3 3 2



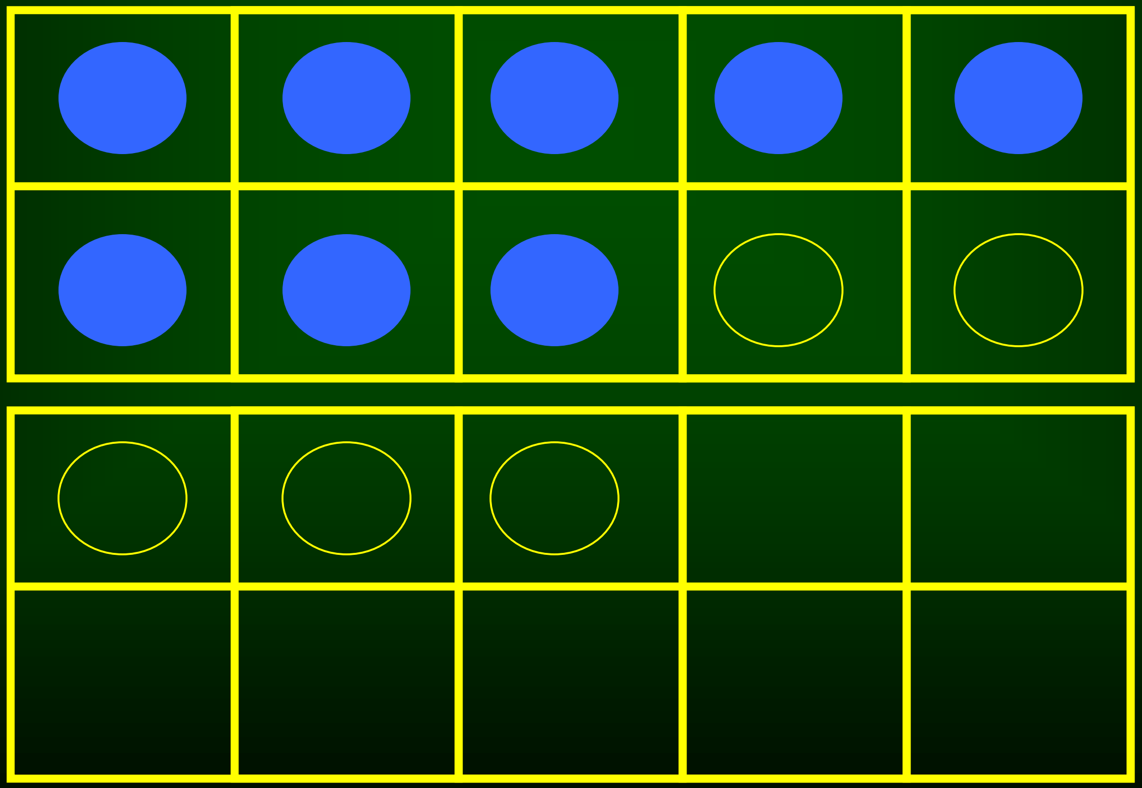
13 - 5 =

10 3 3 2



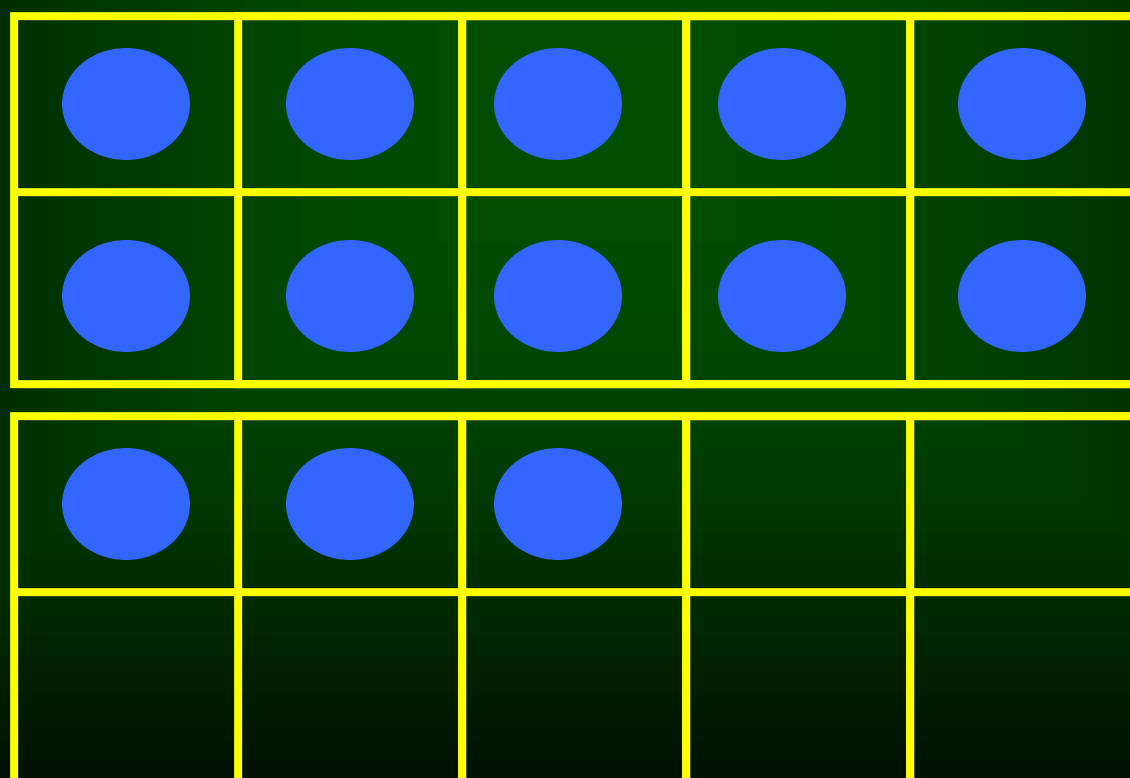
13 - 5 = 8

| 13 | | 5 | |
|---|---|---|---|
| 10 | 3 | 3 | 2 |

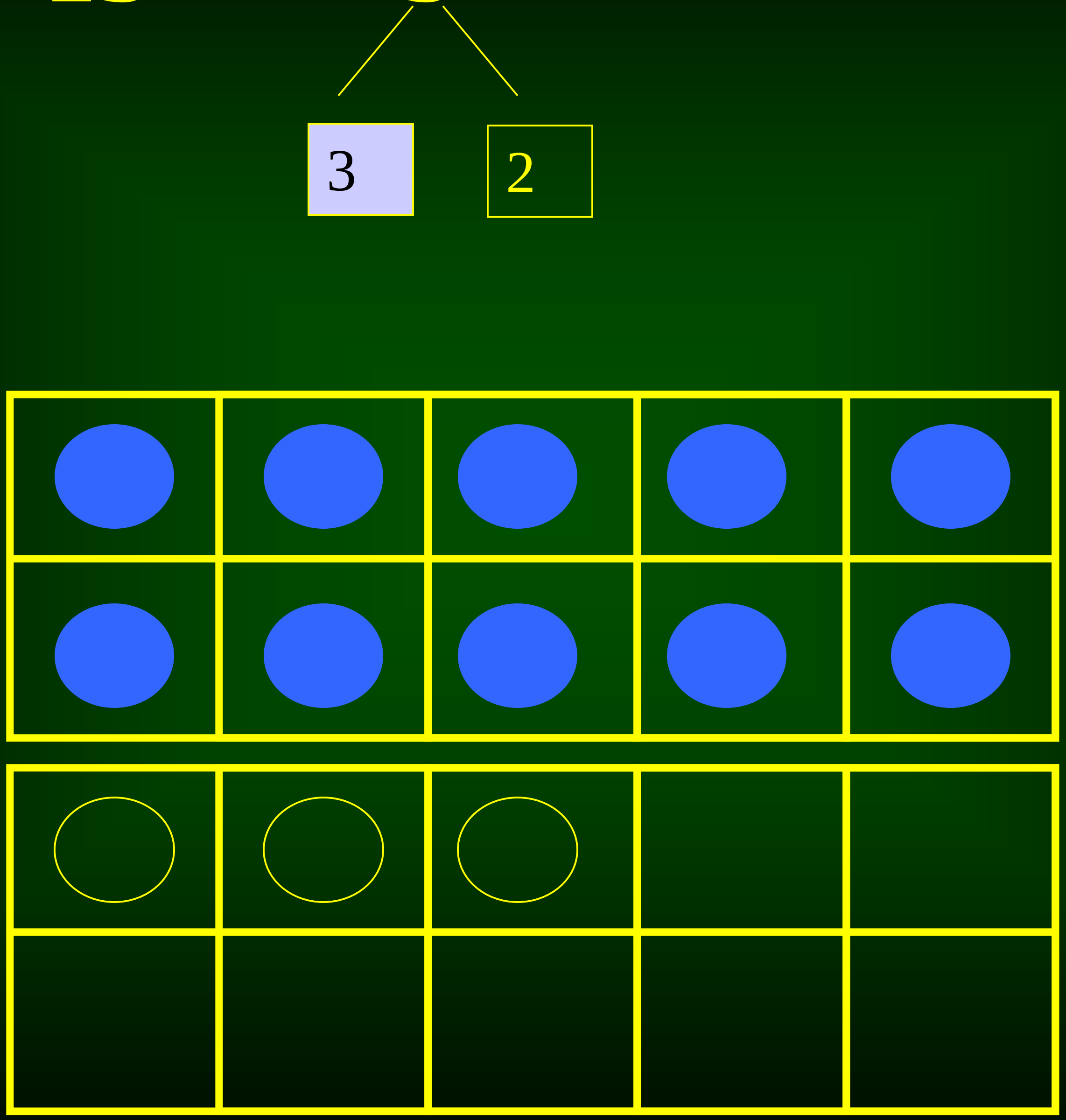


13 - 5 =

3 2

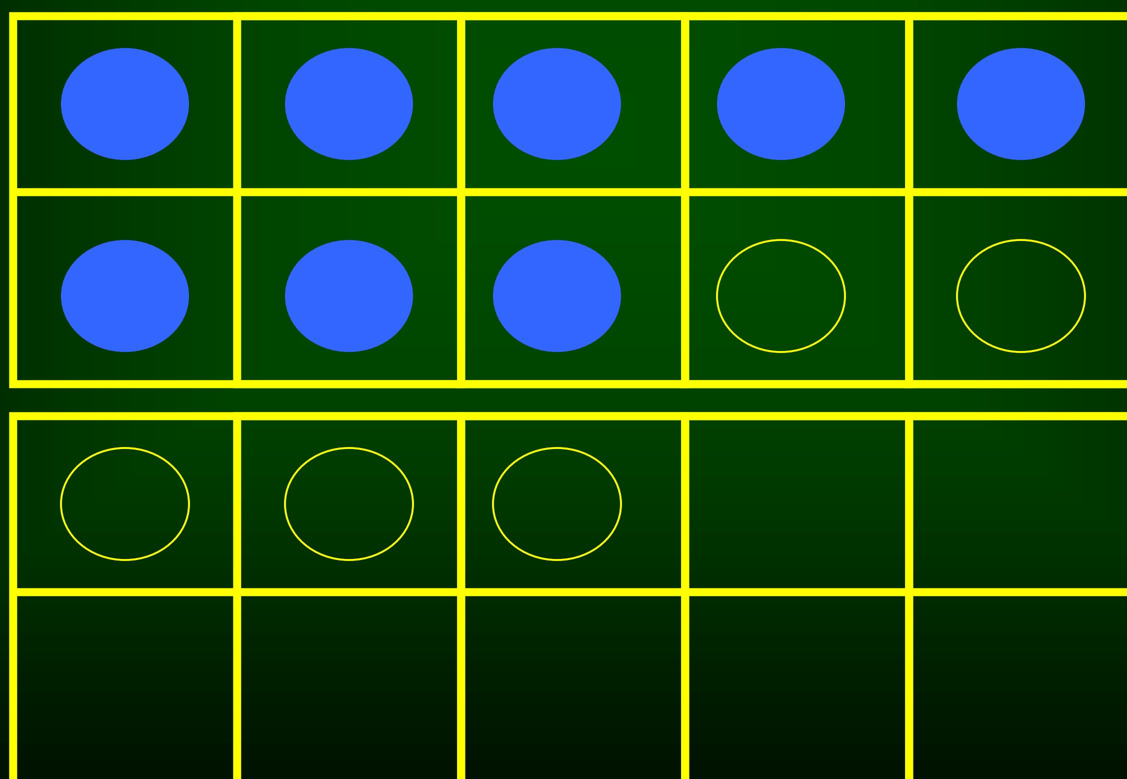


13 - 5 =



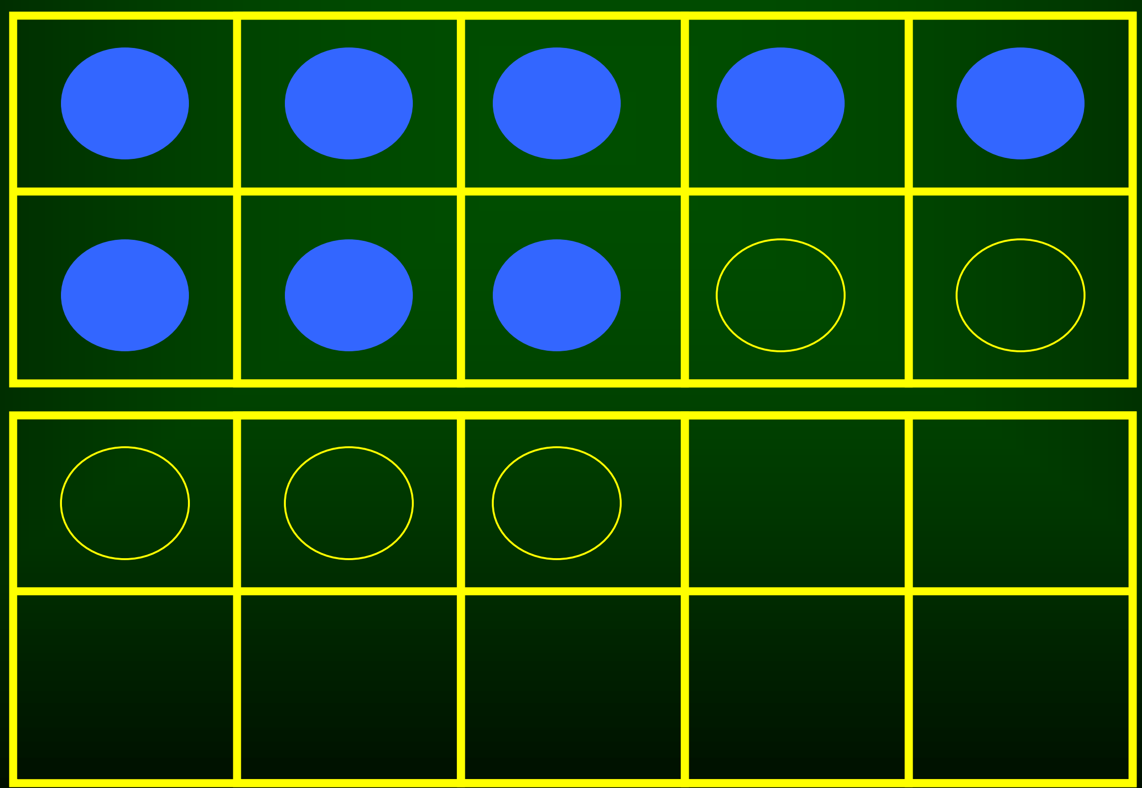
$$13 - 5 =$$

3 2

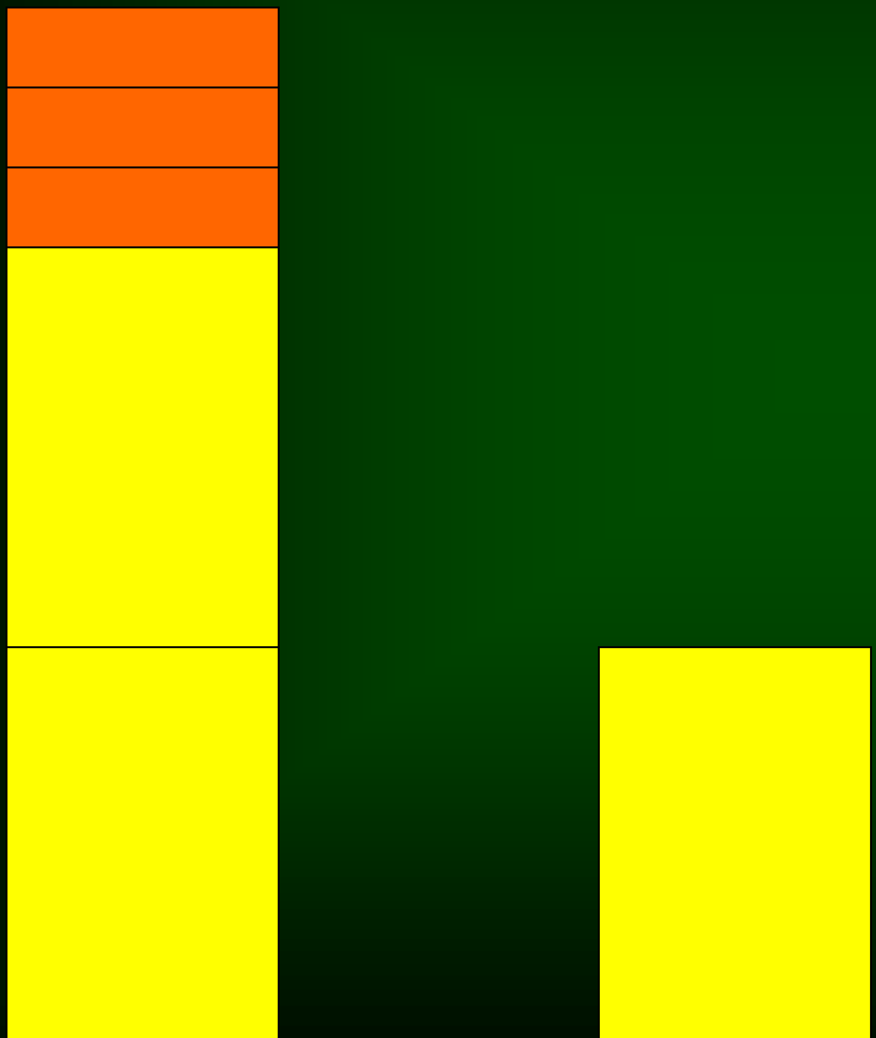


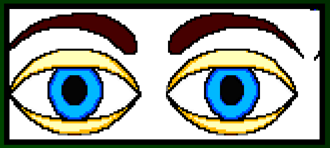
13 - 5 = 8

3 2

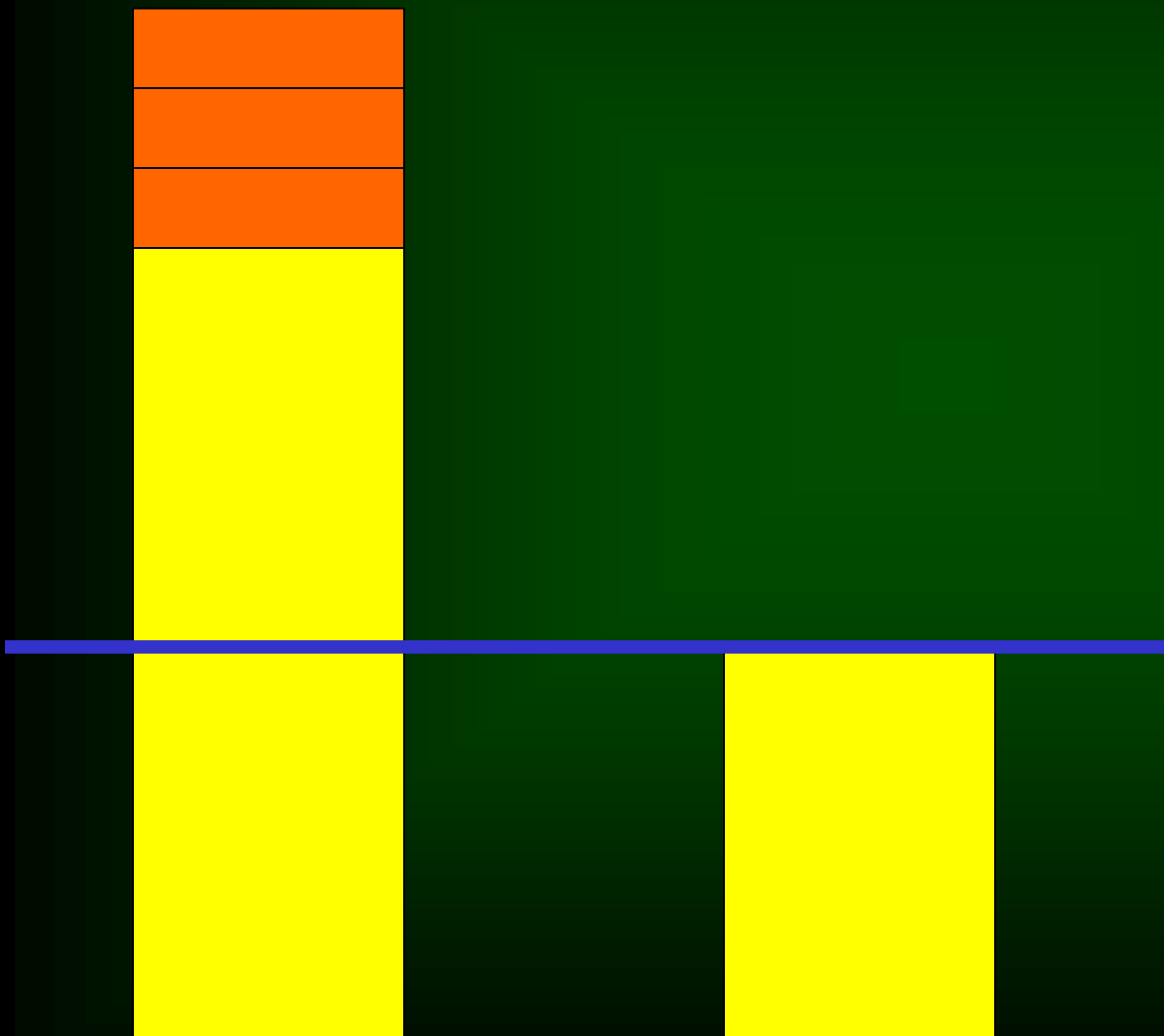


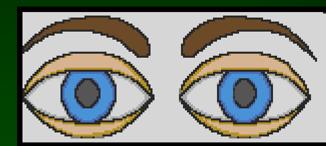
13 - 5 = ☐



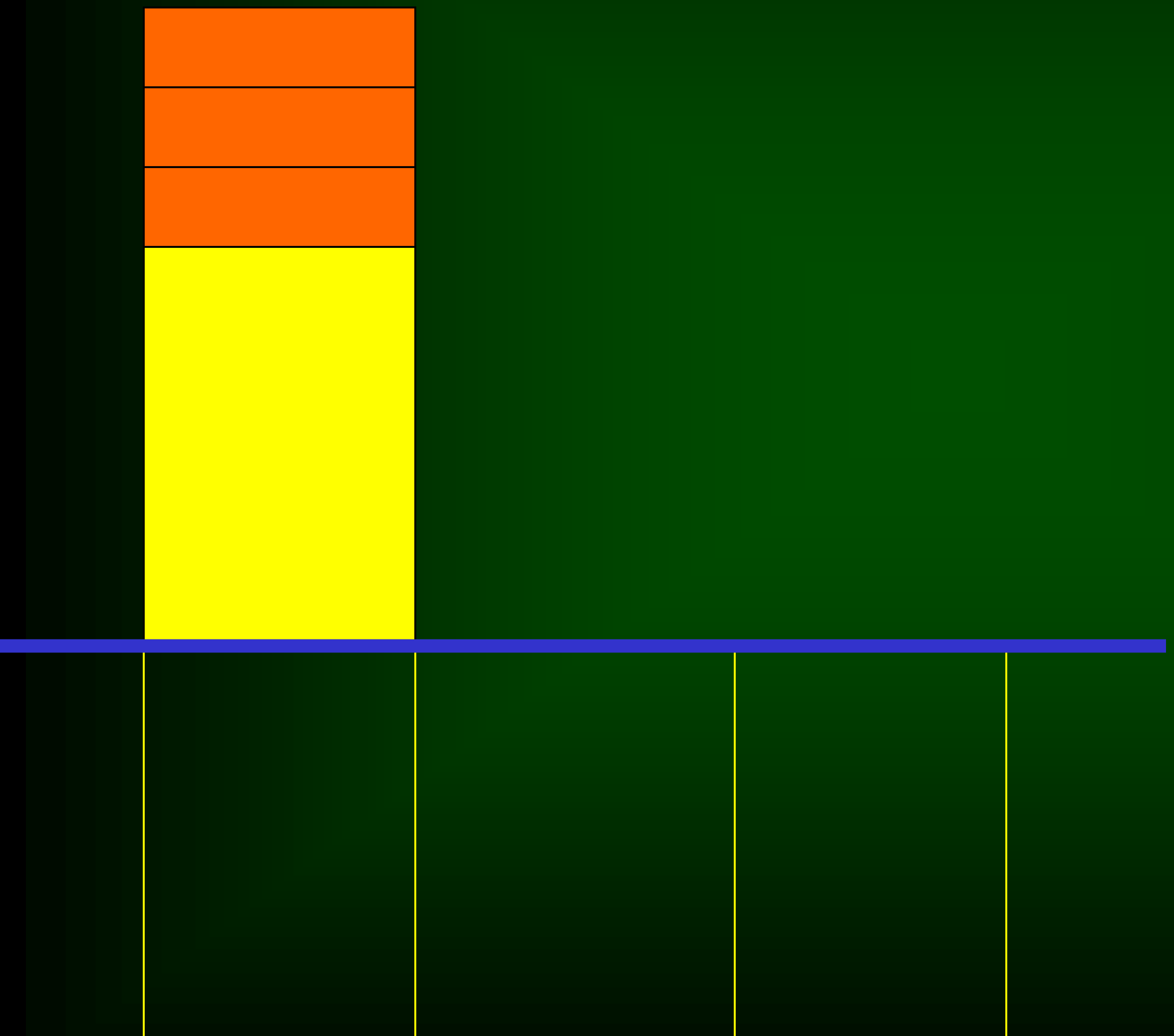


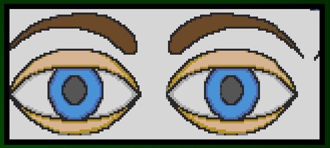
13 - 5 = ☐





$$13 - 5 = 8$$





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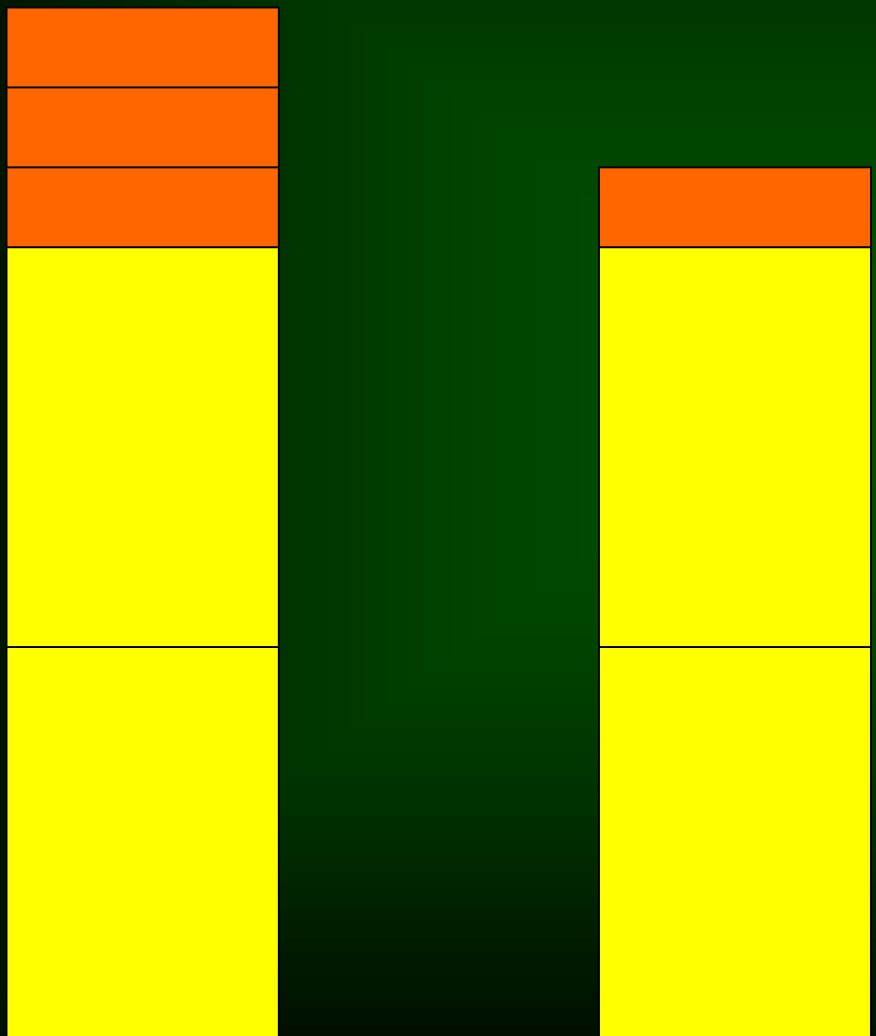

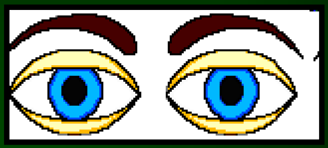

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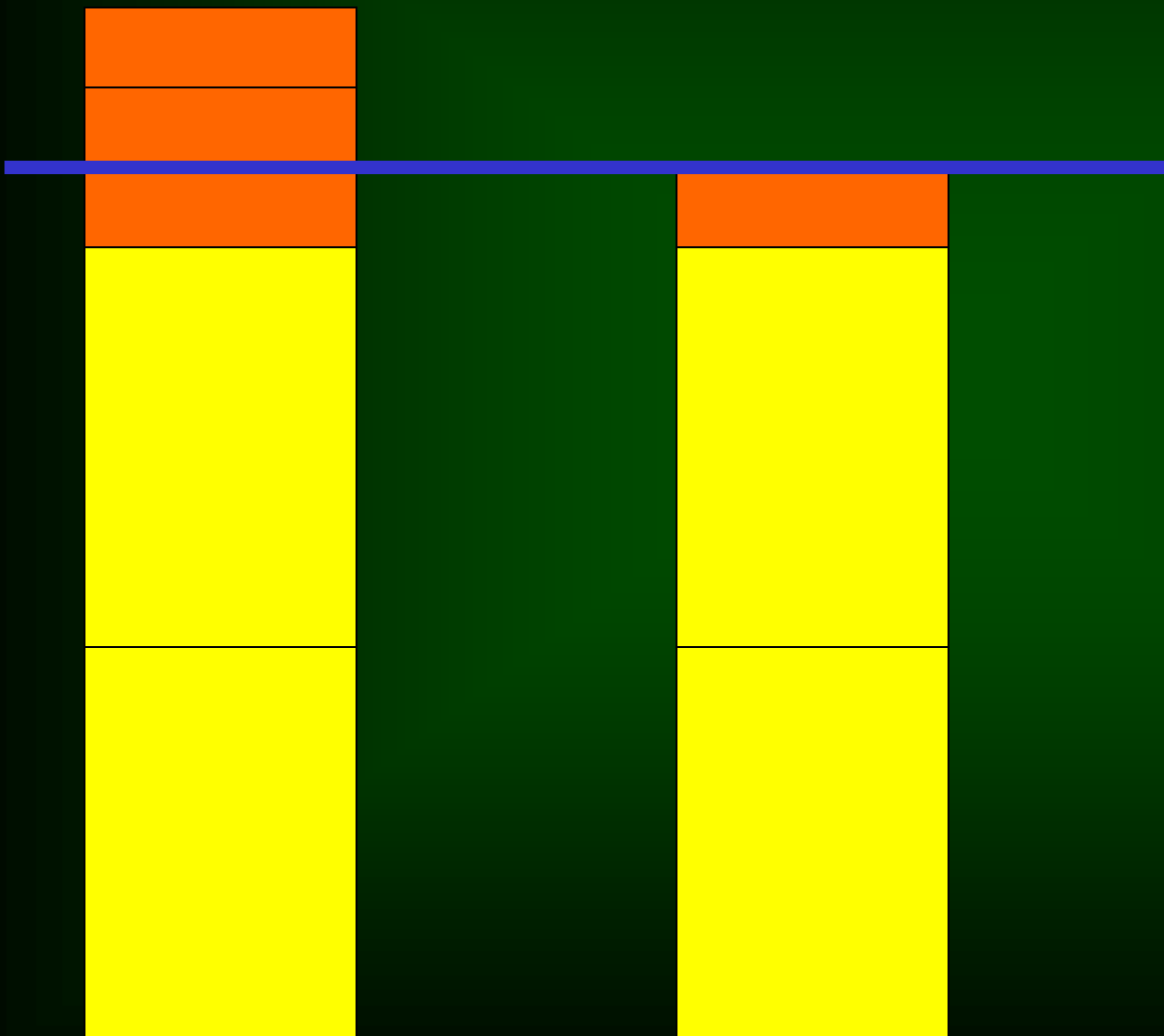
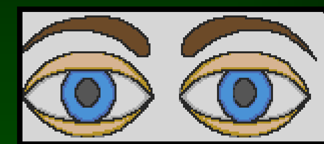
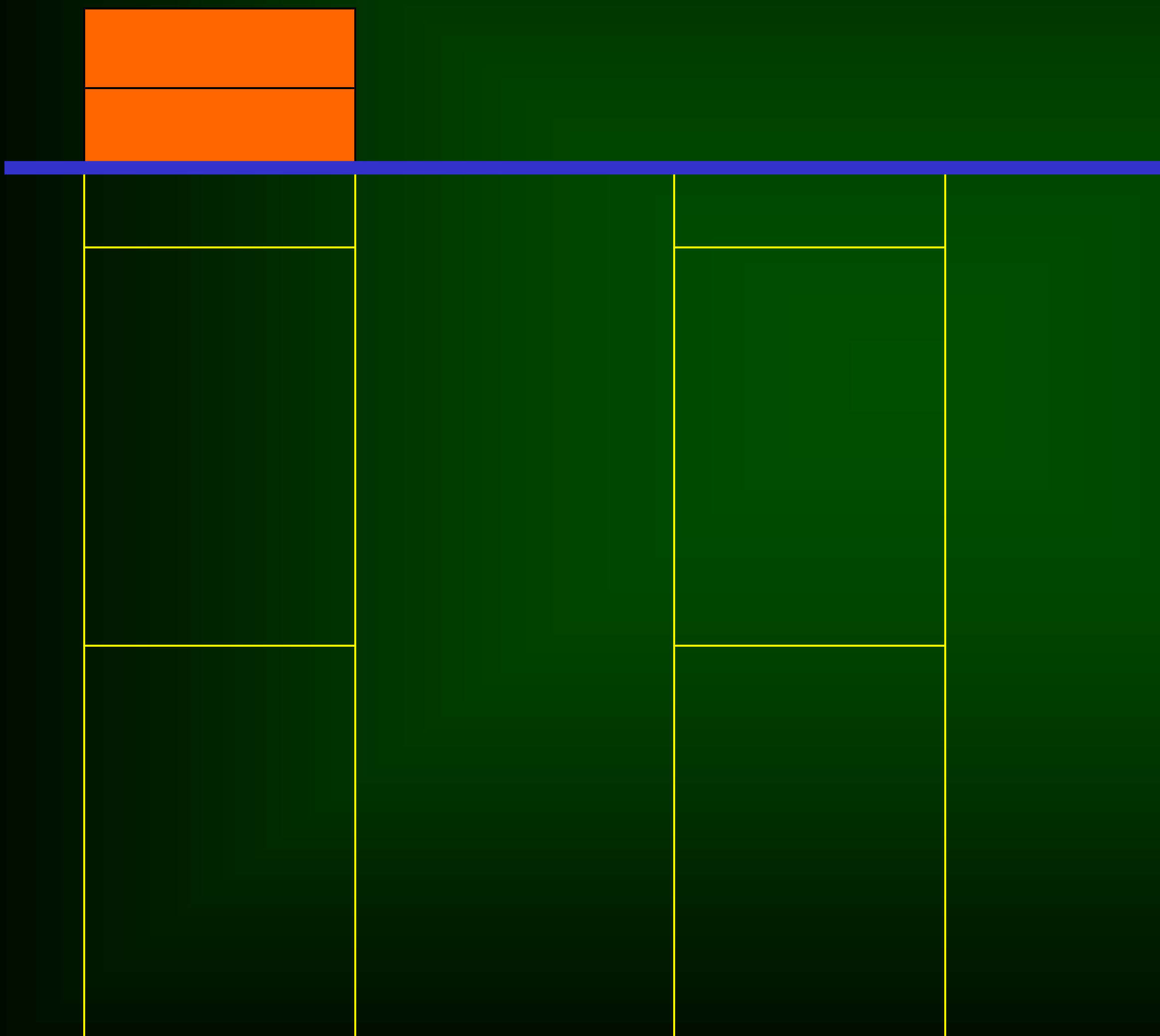
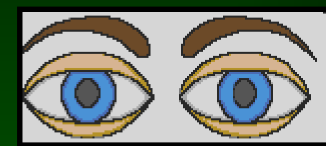




13 - 11 = 2





# Diagnosing and Remediating Errors

- Fact Errors
- Strategy Errors
- Components Errors
- Incorrect Operation or Sign Discrimination
- Random Errors

# Diagnosing & Remediating Errors

- Fact Errors:

```
      7
  3  8  2
-    1  6
---------
  3  6  5
```

☞ Extra Practice
or
Motivation

# Diagnosing & Remediating Errors

- Strategy Errors:

$$\begin{array}{r} 3\ 8\ 2 \\ -\quad 1\ 6 \\ \hline 3\ 7\ 4 \end{array}$$

☞ Reteach strategy beginning with teacher model

# Diagnosing & Remediating Errors

- Component Errors:

```
    9
 3  8  2
-   1  6
--------
 3  8  6
```

☞ Reteach that component, continue strategy

# Diagnosing & Remediating Errors

- Incorrect Operation:

```
   3 8 2
-    1 6
   -----
   3 9 8
```

☞ Precorrect or prompt correct operation.

For random errors - accuracy below 85%:

☞ Increase motivation

# Error Analysis

Fact? Component? Strategy? Sign?

Fact

```
  2 5
x 3 2
-----
  5 0
7 3
-----
7 8 0
```

```
  2 5
x 3 2
-----
  5 7
```

Sign

Component

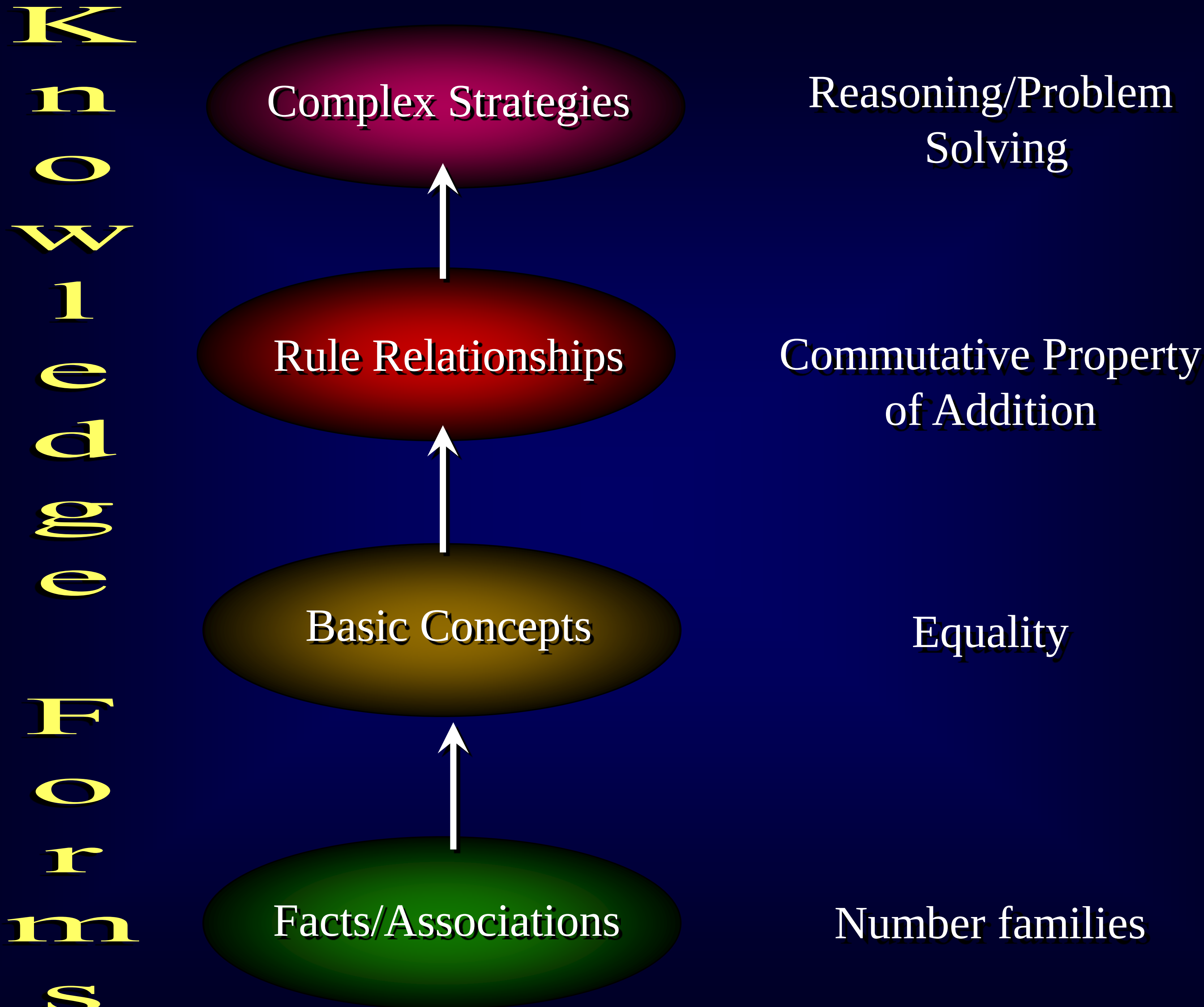
```
  2 5
x 3 2
-----
  5 0
  7 5
-----
1 2 5
```

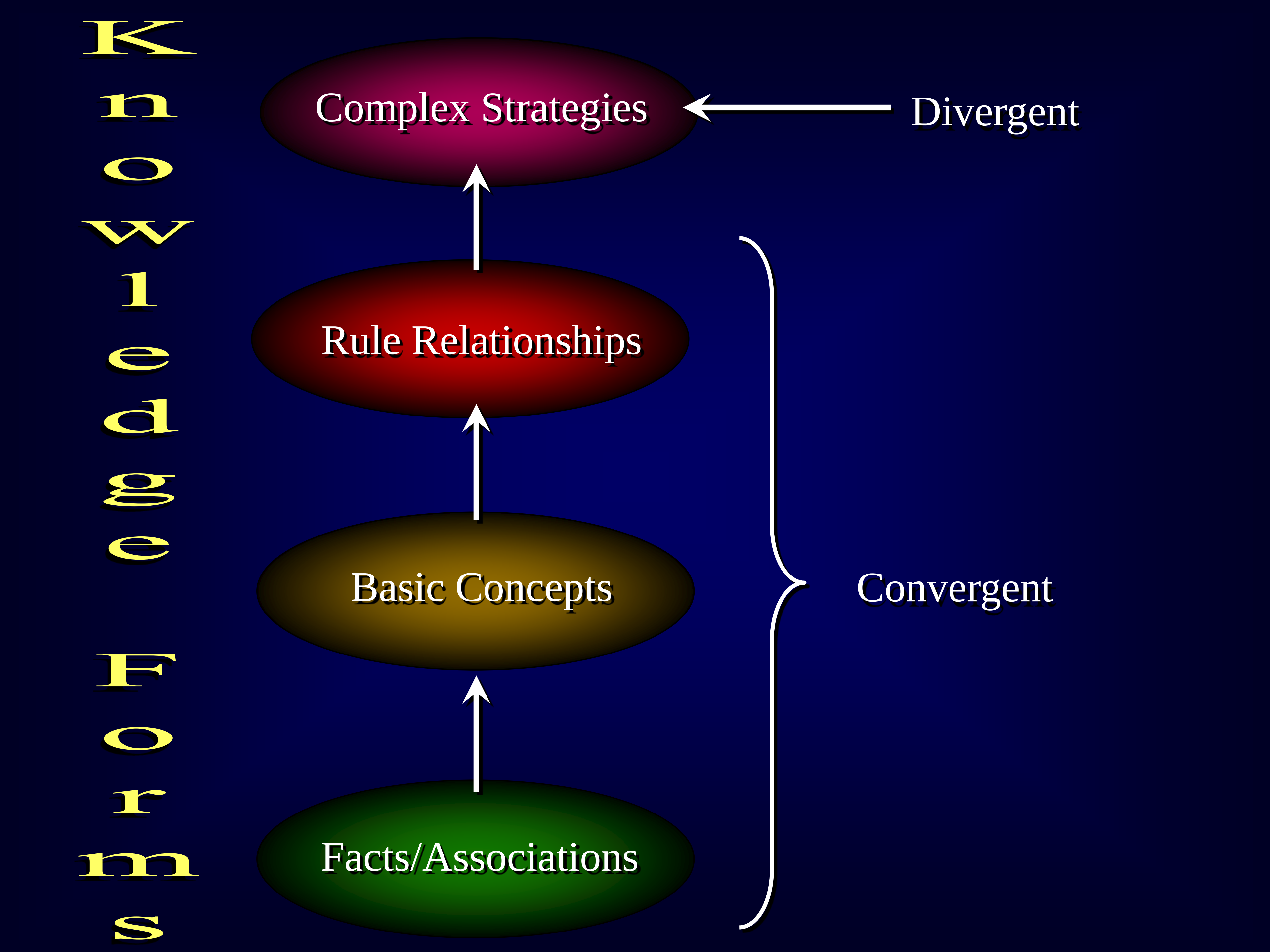
```
    2 5
  x 3 2
-------
7 5 5 0
```

Strategy

# Why all this error analysis?

- Errors or probable causes of errors imply considerations about
  - Preskills
  - Instruction
  - Remediation
- Identifying the type of error allows you to more efficiently address learner needs.


Knowledge Forms
Complex Strategies
Reasoning/Problem Solving
Rule Relationships
Commutative Property of Addition
Basic Concepts
Equality
Facts/Associations
Number families


Knowledge Forms
Complex Strategies
Divergent
Rule Relationships
Basic Concepts
Convergent
Facts/Associations

# Future Trends

- System of assessment to :

✓Detect students at risk for mathematics difficulties
✓Monitor student progress and fluency
✓Gauge instructional efficacy

- Increased emphasis on early algebraic reasoning

- More thorough teaching on narrower focus

- Professional knowledge, professional knowledge, professional knowledge