Massive Gas Jets Interaction with Magnetic Confined Plasma

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MS thesis under the supervision of
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Outline

a. Why do we use Massive Gas Injection?
   - Plasma major instabilities and Disruptions
   - Disruption Mitigation
b. Gas dynamics in vacuum
   - Jet flow expansion
   - Jet flow propagation into constant cross sectional duct
   - Some numerical results for H, He, Ne, Ar and Kr
c. Jet flow interaction with Plasma
d. Continuity equation
e. Electron heat balance equation
f. Ion heat balance equation

Disruptions Instabilities in tokamaks

- Plasma Disruptions (generally associated with an m=2 instability mode) are mainly caused by:

  (1) high electron density

  (2) high pressure values

  (3) lack of position control inside the tokamak.

- As a result:

  (1) complete loss of plasma confinement and magnetic field configuration (within ~10 ms),

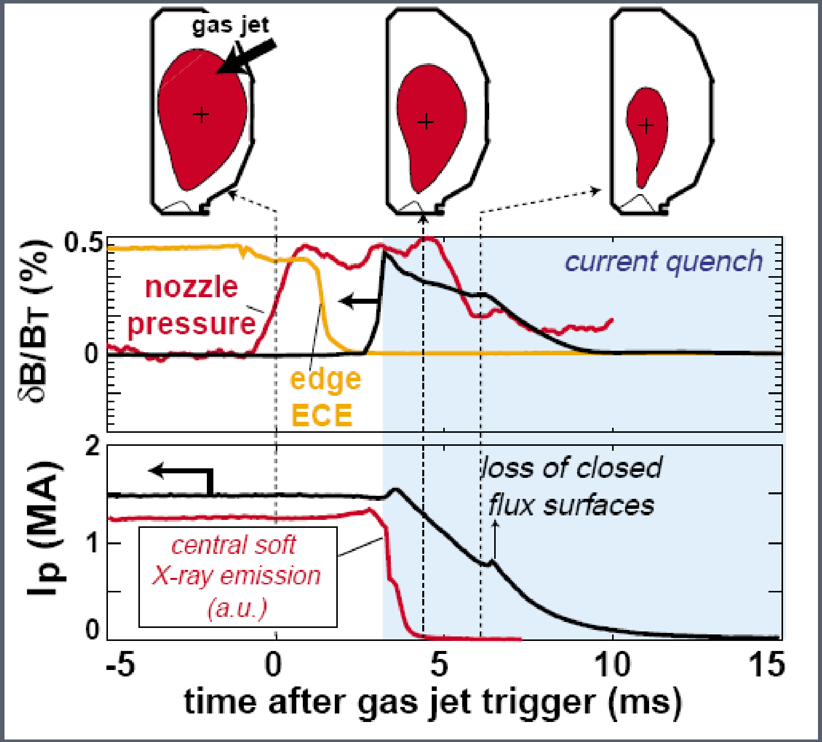
  (2) rapid temperature decrease,

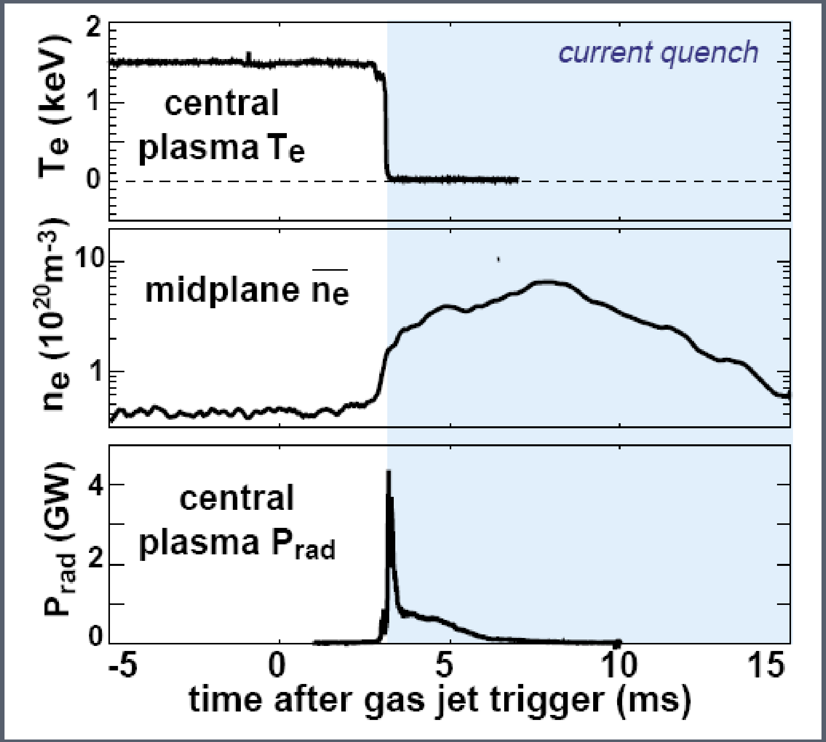
  (3) plasma current decay (100 MA/s)

  (4) possible development of runaway electrons.

Disruptive Instabilities in tokamaks

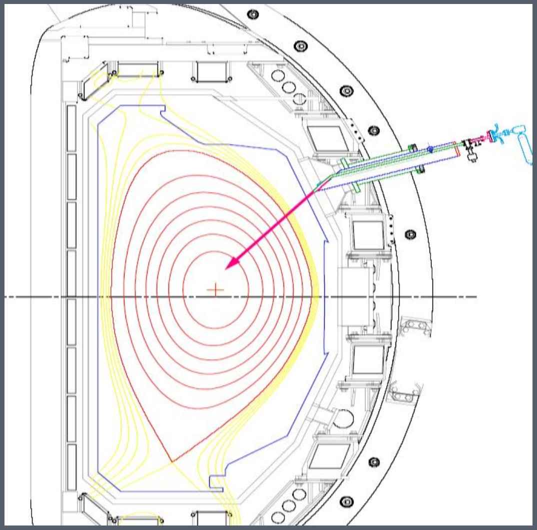
- All of these effects cause tremendous stress on the plasma vessels and serious damage to the tokamak

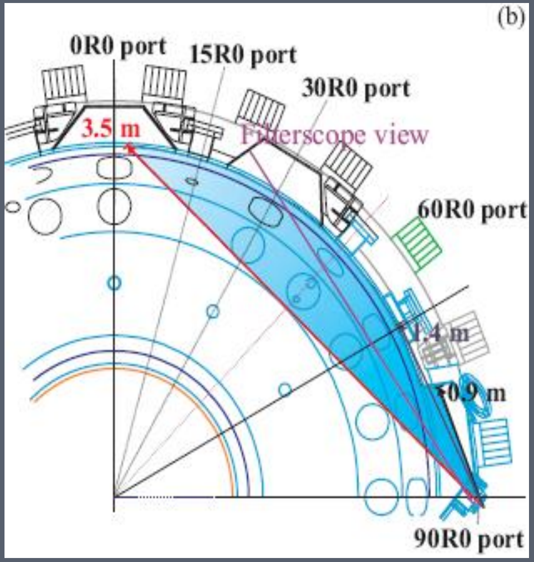




D.G. Whyte, IAEA Oct 2002

Massive gas jet disruption mitigation in a DIII-D tokamak viewed with a fast imaging camera with a D_α filter (1 μs exposure, 15 μs between frames)





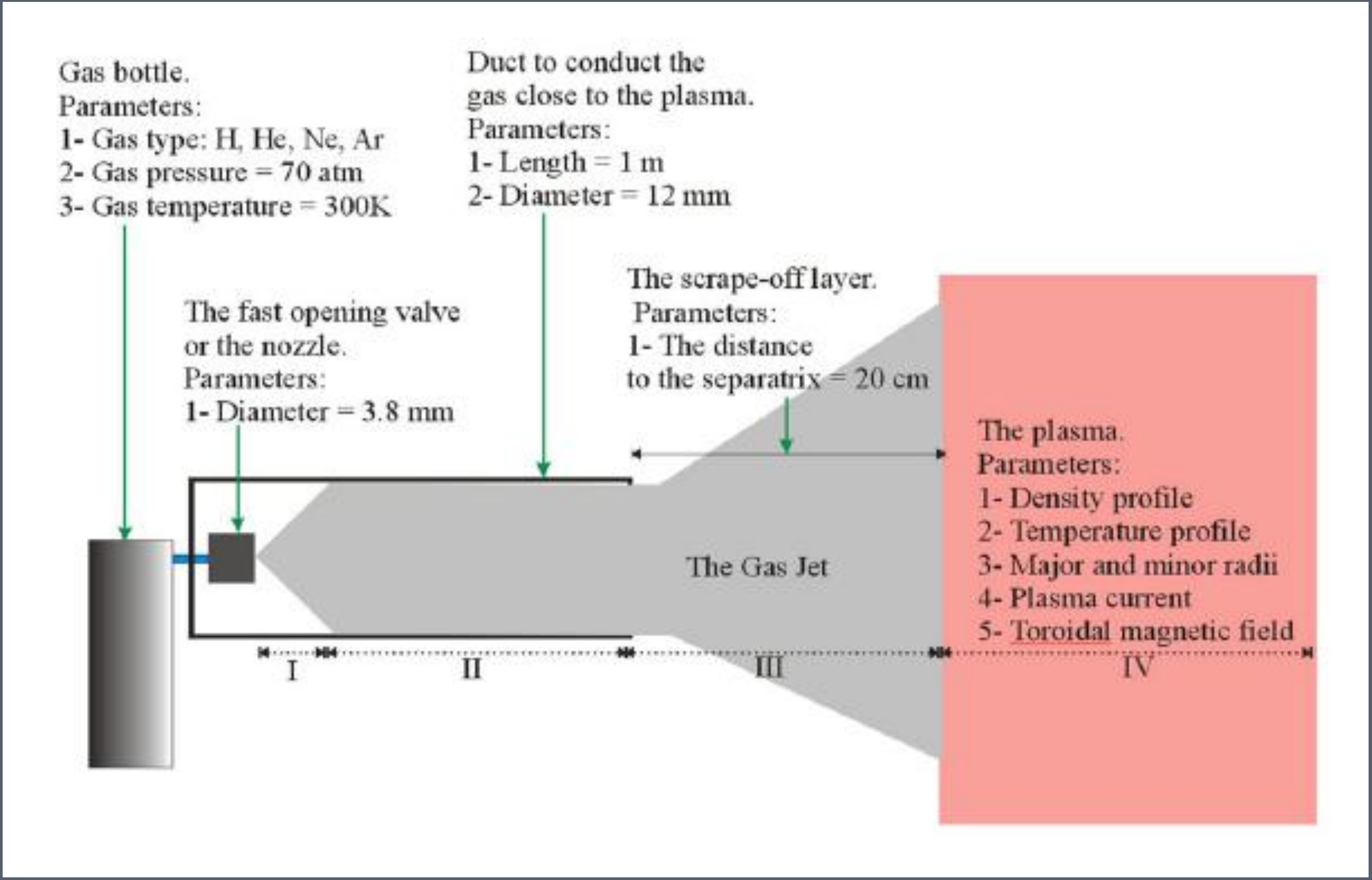
Disruption Mitigation

- Massive Gas puffing is one solution for disruption mitigation.
- The injected gas atoms collide with charged particles and get ionized.
- As a result, an increase in the electron density but a reduction in the temperature is achieved.
- This electron temperature decrease allows a controlled shut-down of the plasma without major heat load on the vacuum chamber.

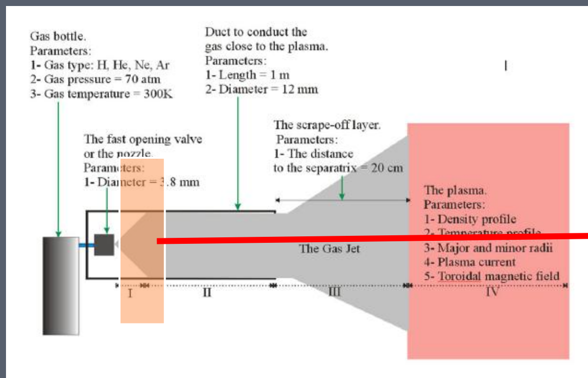
Our Goals

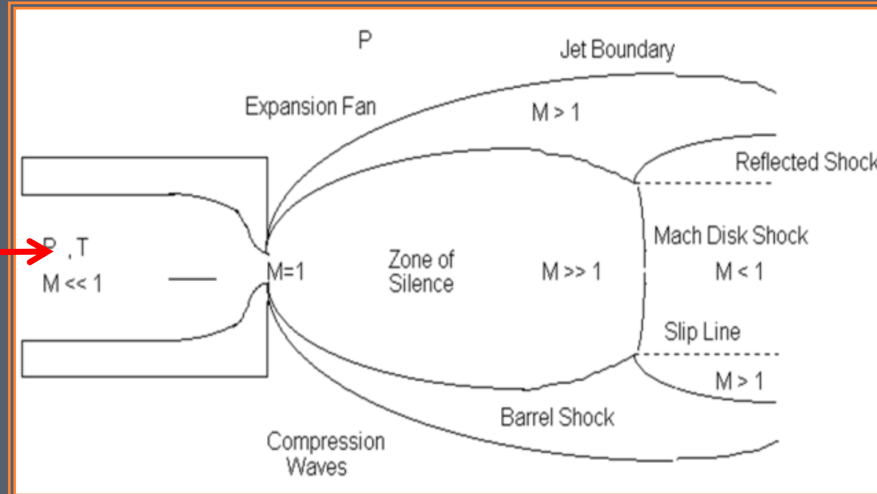
- Describe the gas jet dynamics in free expansion and in vacuum ducts.
- Describe the gas jet interaction with the plasma.
- Write a numerical code that solves the continuity and the heat equations while taking into account all atomic processes involved in plasma-neutrals interaction.
- Predict the feasibility of this method for present and future fusion devices (ITER-like devices).

Massive Gas Jet Injection (MGI) Setup



Gas Jet Free Expansion (Phase I)





$$\frac{P_0}{P^\star} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} ,$$

$$\frac{n_0}{n^\star} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{\gamma-1}} ,$$

$$\frac{T_0}{T^\star} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} ,$$

$$\frac{A_0}{A^\star} = \frac{1}{M}\left(\frac{(\gamma + 1)/2}{1 + (\gamma - 1)M^2/2}\right)^{\frac{\gamma+1}{\gamma-1}} .$$

- PV^γ = Constant (adiabatic process)
- $P = (R/W)nT$ (ideal gas)

Once the variation of M determined, the evolution of P_0, n_0, T_0 and A_0 are obtained.

M as a Function of Z for Free Expanding Jet

- The Mach number behavior is obtained by **numerical simulation of the full Navier-Stokes Equation of a compressible flow** with the adequate numerical scheme that takes into account the presence of shock waves.
- A **polynomial fit** of the Mach number dependence on distance to the nozzle is determined and leads to:

$$M = Z^{(\gamma-1)}\left(C_1 + \frac{C_2}{Z} + \frac{C_3}{Z^2} + \frac{C_4}{Z^3}\right) \quad \text{for } Z > 0.5,$$
$$M = 1 + AZ^2 + BZ^3 \quad \text{for } 0 < Z < 0.5.$$

	γ	C_1	C_2	C_3	C_4	A	B
H	1.4	3.606	−1.742	0.9226	−0.2069	3.19	−1.61
He, Ne, Ar and Kr	1.6	3.232	−0.7563	0.3937	−0.0729	3.337	−1.541

$$V_\infty = \sqrt{\frac{2R}{W}\left(\frac{\gamma}{\gamma-1}\right)T_0}$$

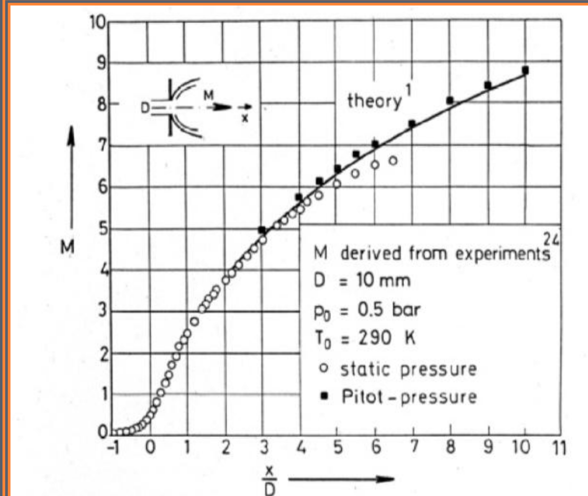
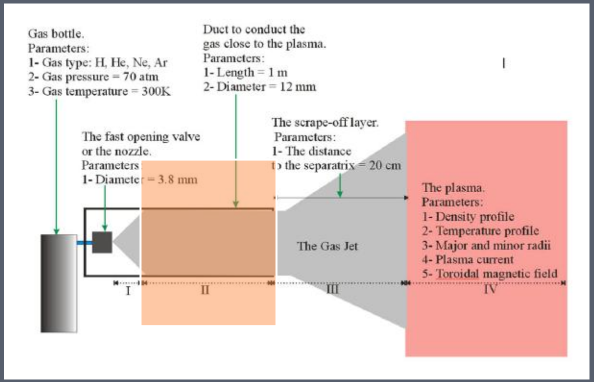


Figure 2. Sub- and transsonic free jet measurements: supersonic theory,[1] $\kappa = 1.40$.

G. Koppenwallner and C. Dankert, J. Phys. Chem., **91**, 2482 (1987)

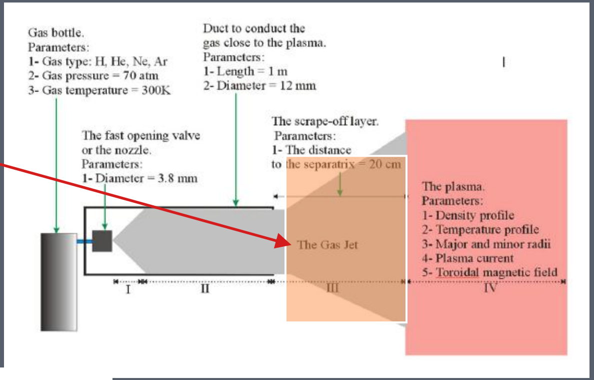
Jet propagation into constant cross-sectional duct (Phase II)

$$\frac{dM}{dx} = \frac{\gamma M^3}{2}\,\frac{1+\left(\frac{\gamma-1}{2}\right)M^2}{1-M^2}\,\frac{4f}{D}$$



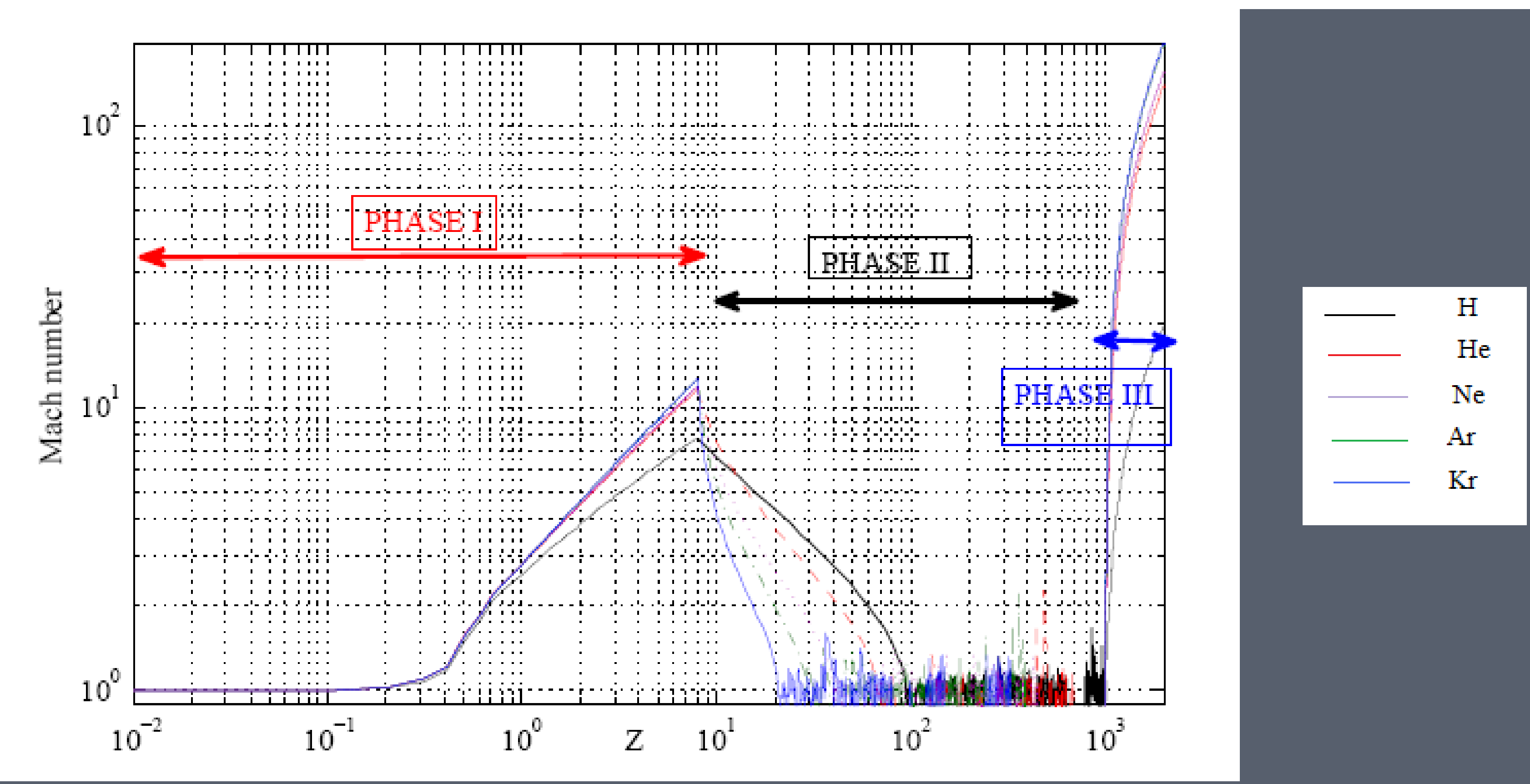
- Friction factor dependence on the flow judged to be turbulent or laminar according to Re number value (Re=ρ*VD*/μ).
- For Re > 12000 → turbulent flow $\frac{l}{\sqrt{f}} = -2log\left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$
- For Re < 1200→ laminar flow $f = \frac{64}{Re}$
- For 1200 < Re < 12000 → transitional regime flow $\frac{l}{\sqrt{f}} = 2log\frac{Re\sqrt{f}}{2.51}$

Numerical results for H, He, Ne, Ar and Kr

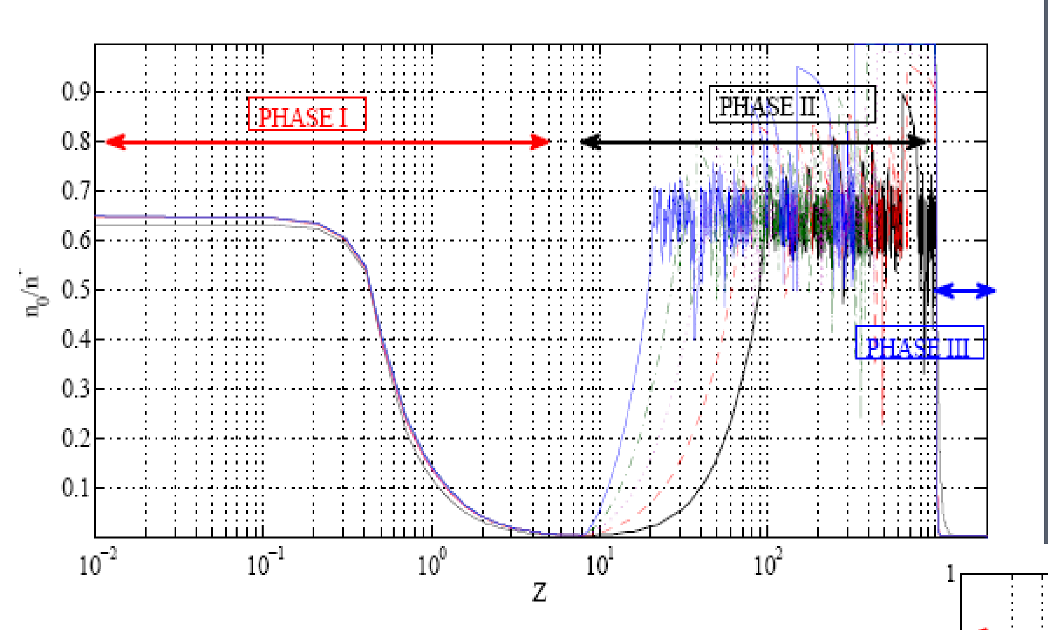


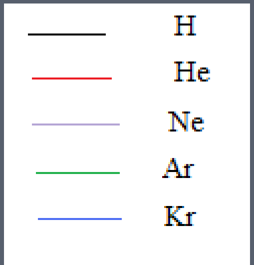
Phase III is a free expanding jet case similar to phase I

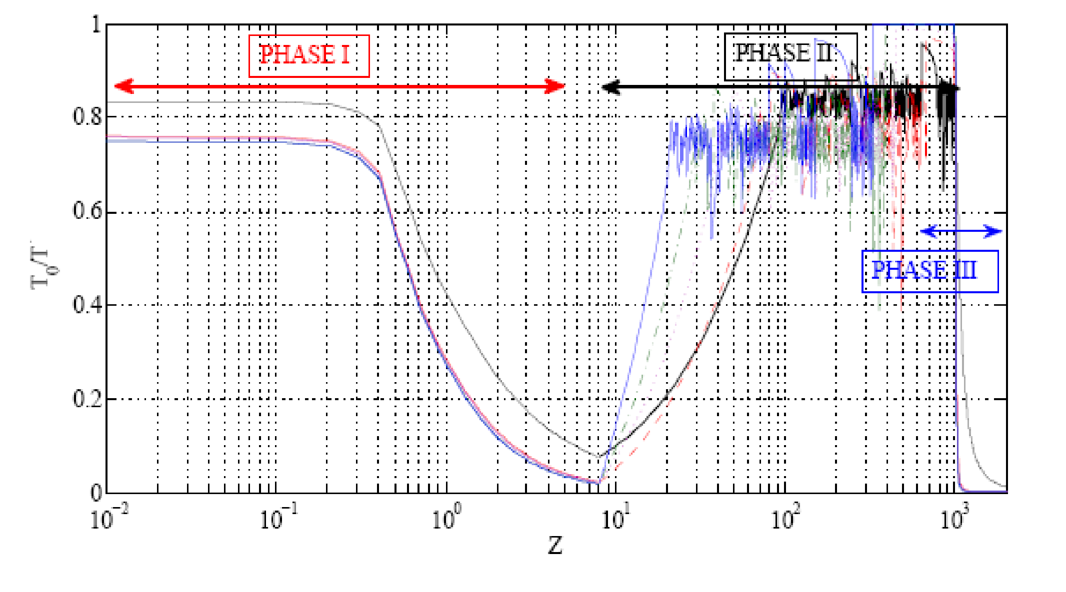
- Free gas jet expansion is characterized by the polynomial fit
- The propagation in ducts is characterized by *f* and Re



Numerical results for H, He, Ne, Ar and Kr gases







Conclusion I: Gas jet dynamics in vacuum and in ducts

1. We aim at using the polynomial fit to obtain the Mach number variations

2. Get all the jet parameters as a function of the distance to the nozzle.

3. The propagation in ducts is solved with the friction coefficient f obtained from the adequate formulas depending on the flow dynamics.

Equations to solve

1. Particle balance equation assuming quasi-neutrality ($n_e = n_i$)

2. Electron heat equation (T_e)

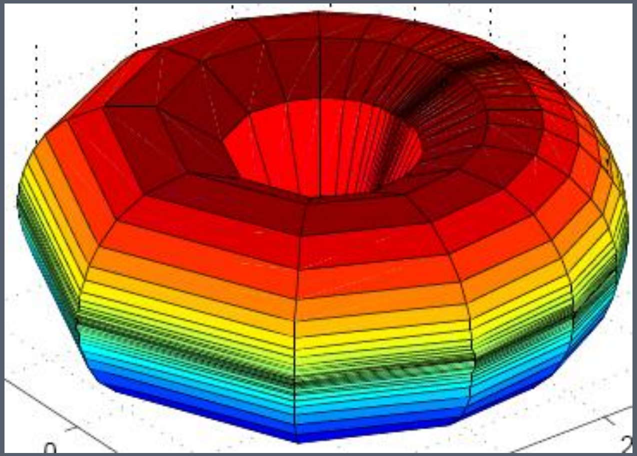
3. Ion heat equation (T_i)

4. Gas jet neutral density (n_0)

Dynamics in the toroidal direction and the poloidal plane

A. Toroidal direction:

1. Transport dominated by collisions
2. MGI OFF: no gradient therefore no transport of particles or heat.
3. MGI ON: ambipolar diffusion of particles.
4. No turbulence in the ϕ direction (with and without MGI)



B. Poloidal plane:

2. Transport is dominated by turbulence
3. Isotropy in the r and θ directions (with and without MGI)
4. MGI OFF: Diffusion and heat conductivity coefficients are r dependent only.
5. MGI ON: Diffusion and heat conductivity coefficients are (r,θ,ϕ) dependent.

Working Hypotheses

- **Major assumptions:**
  - Adiabatic electrons
  - Particle velocity dominated by **E**x**B**

- **Experimental data:**
  - the density fluctuations profile.

- **Atomic processes:**
  - ionization, recombination and charge exchange reactions.

- **Input parameters:**
  - Correlation coefficient between density and average radial velocity C
  - Average electron poloidal and toroidal velocities v_θ and v_ϕ

MGI OFF

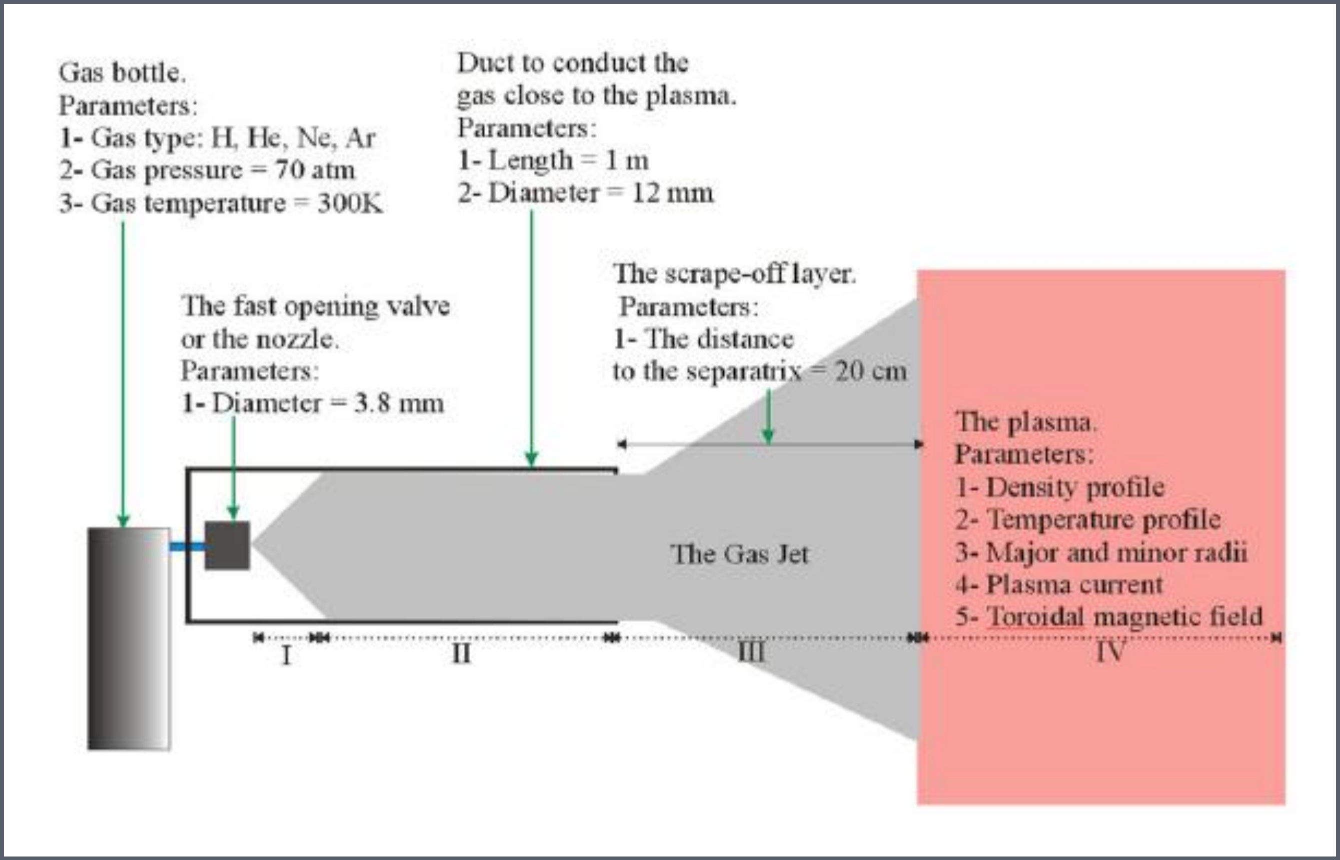
When MGI is OFF, we ought to have a steady state.

- For the continuity equation, the steady state leads to the determination of the ‘initial’ neutral sources which balance the radial transport.

- For the heat balance equation, it allows us to have the expression of the heat diffusion coefficient as a function of the plasma parameters.

- When the gas jet is ON, we insure a steady state far from where the MGI is injected

Gas interaction with Plasma: phase IV



Particle Balance Equation

$$\partial_t n + \nabla.(n\vec{v}) = S$$

The Particle Balance equation

$$\partial_t n + \nabla.(n\vec{v}) = S$$

MGI OFF (steady state)
Two main contributions:

- **Inherent plasma sources** → NBI; Wall-particle interaction; Continuous fuelling

- **Particle radial transport**

MGI ON

- Particle radial transport
- Inherent sources

+

- Ionization
  - (II) $e^- + A \rightarrow A^+ + 2e^-$
  - (RI) $h\nu + A \rightarrow A^+ + e^-$
- Recombination
  - (3BR) $2e^- + A^+ \rightarrow A + e^-$
  - (RR) $e^- + A^+ \rightarrow A + h\nu$
- Charge exchange → $A + B^- \rightarrow B + A^-$

Particle Conservation Equation: MGI OFF

- The gas particles dynamics obey the following continuity equation:

$$\boxed{\partial_t n + \nabla.(n\vec{v}) = S}$$

- Each quantity is set equal to its average and fluctuating value due to turbulence

$$n = n_0 + \tilde{n}$$

$$\vec{v} = \vec{v}_0 + \tilde{\vec{v}}$$

where $\langle \tilde{n} \rangle = \langle \tilde{\vec{v}} \rangle = 0$

- Gas jet off, the plasma is in steady state, $\partial_t n=0$ and let $S=S_{in}$
- The continuity equation becomes

$$\partial_t n_0 + \partial_t \tilde{n} + \nabla.[(n_0\vec{v}_0) + (\tilde{n}\tilde{\vec{v}}) + (\tilde{n}\vec{v}_0) + (n_0\tilde{\vec{v}})] = S_{in}$$

Time Averaging

$$\nabla.[(n_0\vec{v}_0) + \langle \tilde{n}\tilde{\vec{v}} \rangle] = S_{in}$$

1 2

Particle Conservation Equation: MGI OFF

- No radial expansion of the plasma and $\partial_\theta X = \partial_\phi X = 0$ $\rightarrow$ 1 $\boxed{\nabla.(n_0 \vec{v}_0) = 0}$

- $$S_{in} = \nabla.\langle \tilde{n}\tilde{\vec{v}} \rangle = \frac{1}{r}\partial_r(r\langle \tilde{n}\tilde{v}_r \rangle) + \frac{1}{r}\partial_\theta \langle \tilde{n}\tilde{v}_\theta \rangle) + \partial_\phi \langle \tilde{n}\tilde{v}_\phi \rangle$$

Isotropic <u>turbulence</u> across B

$$\langle \tilde{n}\tilde{v}_r \rangle = \langle \tilde{n}\tilde{v}_\theta \rangle$$

<u>Collisions</u> along B

$$\partial_\phi \overline{X} = 0$$

No gradient of the flux in the toroidal direction

- Let C be the correlation coefficient between density and velocity.
- It is assumed to be constant

$$\langle \tilde{n}\tilde{v}_r \rangle = C(\langle \tilde{n}^2 \rangle . \langle \tilde{v}_r^2 \rangle)^{\frac{1}{2}}$$

Particle transport due to turbulence when MGI OFF

$$\langle \tilde{n}\tilde{v}_r \rangle = \langle \tilde{n}\tilde{v}_\theta \rangle = C(\langle \tilde{n}^2 \rangle . \langle \tilde{v}_r^2 \rangle)^{\frac{1}{2}} = -D\nabla n_0$$

a b

Turbulent density fluctuating expression obtained experimentally

$$\frac{(\langle \tilde{n}^2 \rangle)^{\frac{1}{2}}}{n_0} = I_c + I_e (r/a)^{\xi}$$ a

Assuming adiabatic electrons

$$\frac{\tilde{n}}{n_0} = \frac{\tilde{\Phi}}{\Phi_0}.$$

Assuming particles velocity dominated by an **ExB** drift

$$v_r = E_\theta B_\phi = -\frac{B}{r}\partial_\theta \Phi$$

$$\langle \tilde{v}_r^2 \rangle = \frac{2\pi B^2 \Phi_0^2}{r^2 l^2} \langle \frac{\tilde{n}^2}{n_0^2} \rangle$$ b

So to obtain finally the expression of D and that of the turbulence contribution to particle transport

2 $$\langle \tilde{n}\tilde{v}_r \rangle = -D\partial_r n_0 = \frac{(2\pi)^{1/2} C B \Phi_0}{rl} (\frac{\delta n}{n_0})^2 n_0$$

Particle Conservation Equation: MGI ON

$$\partial_t n_0 + \nabla(n_0 \vec{v}_0) + \nabla(\widetilde{\tilde{n}\vec{v}}) = S_{in} + S_{ext}$$

1 2 3 4 5

- 1 $\partial n/\partial t$ is not equal to 0 since MGI is on.

- 2 n is no longer r dependent only but (r,θ,φ)-dependent .

$$\nabla(n_0\vec{v}_0) = n_0(\partial_r v_{0r} + \frac{1}{r}\partial_\theta v_{0\theta} + \partial_\phi v_{0\phi}) + v_{0r}\partial_r n_0 + \frac{v_{0\theta}}{r}\partial_\theta n_0 + v_{0\phi}\partial_\phi n_0$$

4 Was previously obtained for MGI OFF

5 plasma particle loss and gain via ionization and recombination expressions

Particle Conservation Equation: Turbulence contribution

$$\partial_t n_0 + \nabla(n_0\vec{v}_0) + \nabla(\tilde{n}\tilde{\vec{v}}) = S_{in} + S_{ext}$$

1 2 3 4 5

3

$$\langle\nabla.(\tilde{n}\tilde{\vec{v}})\rangle = \frac{1}{r}\partial_r\langle r\tilde{n}\tilde{v}_r\rangle + \frac{1}{r}\partial_\theta\langle\tilde{n}\tilde{v}_\theta\rangle + \partial_\phi\langle\tilde{n}\tilde{v}_\phi\rangle$$

$$\nabla.(\langle\tilde{n}\tilde{\vec{v}}\rangle) = (\frac{1}{r} + \partial_r + \frac{\partial_\theta}{r})\langle\tilde{n}\tilde{v}_r\rangle + \partial_\phi\langle\tilde{n}\tilde{v}_\phi\rangle$$

$$\langle\tilde{n}\tilde{v}_\phi\rangle = -D_{||}\partial_\phi n_0$$

$$\underbrace{D_{||} = \frac{(T_e + T_i)D_i D_e}{T_i D_e + T_e D_i}.}_{\text{Ambipolar diffusion}}$$

$$\langle\tilde{n}\tilde{v}_r\rangle = -D\partial_r n_0 = \frac{(2\pi)^{(1/2)}CB\Phi_0}{rl}(\frac{\delta n}{n_0})^2 n_0$$

Electron Heat Equation

$$\frac{3}{2}n_e\partial_t T_e + \frac{3}{2}n_e\vec{v}_e.\nabla T_e = -\nabla.\vec{q}_e + Q_e$$

Electron Heat equation: MGI OFF

$$\frac{3}{2}n_e\partial_t T_e + \frac{3}{2}n_e\vec{v}_e.\nabla T_e = -\nabla.\vec{q}_e + Q_e$$

1 2 3 4

1 For the gas jet off, steady state => $\partial_t T_e = 0$

2 $\vec{v}_e.\nabla T_e = 0$

4

$$Q_e = Q_{OH} + Q_{e,i} + Q_{e,I} - Q_{e,n} - Q_{rad}.$$

$Q_{e,n}$=-n($E_i\sigma_i n_0$ + 3/2$T_e\sigma_r n$)

$$Q_\Omega = 2.8\times 10^{-9}\frac{I^2}{a^4 T_e^{3/2}}$$

$$Q_{e,i} = -Q_{i,e} = \frac{3m_e n_e}{m_i\tau_e}(T_e - T_i)$$

$$Q_{rad} = Q_{Br} + Q_{rec} + Q_{line}$$

$$Q_{RR} = 4.1\times 10^{-46}n_e n_Z T_e^{3/2}Z^6$$

$$Q_{line} = 1.8\times 10^{-44}n_e n_Z T_e^{1/2}Z^4$$

$$Q_{Br} = 1.69\times 10^{-26}n_e T_e^{1/2}\Sigma_i[Z^2 n_i]$$

Electron Heat equation: Heat Flux Vector

3 $$\nabla.\vec{q}_e = \nabla.\vec{q}_e^{\,0} + \nabla.\vec{q}_e^{\,turb}$$

3a 3b

Frictional heat flux

$$\vec{q}_e^{\,u} = n_e T_e (0.71\vec{u}_{\parallel} + \frac{3}{2w_{ce}\tau_e}\hat{\phi} \times \vec{u})$$

$$\nabla.\vec{q}_e^{\,u} = \partial_\phi q_e^\phi = \partial_\phi (0.71 n_e(r) T_e(r) u_\phi) = 0$$

Thermal heat flux

$$\vec{q}_e^{\,T} = \frac{n_e T_e \tau_e}{m_e}(-3.16\nabla_{\parallel} T_e - \frac{4.66}{w_{ce}^2\tau_e^2}\nabla_{\perp} T_e - \frac{5}{2w_{ce}\tau_e}\hat{\phi} \times \nabla T_e)$$

$$\nabla.\vec{q}_e^{\,T} = \frac{-4.66}{m_e w_{ce}^2 \tau_e r}\partial_r (r n_e T_e \partial_r T_e)$$

3a

Electron Heat equation: turbulence contribution and heat conductivity expression

$$\langle \vec{q}^{turb} \rangle = \vec{v}_0 \langle \tilde{n}\tilde{T} \rangle + T_0 \langle \tilde{n}\tilde{\vec{v}} \rangle + n_0 \langle \tilde{T}\tilde{\vec{v}} \rangle$$

3b $$\vec{q}^{turb} = \vec{q}_{\perp}^{turb} = q_r^{turb}\hat{r} + q_\theta^{turb}\hat{\theta}$$

No turbulence in the toroidal direction

$$q_\theta^{turb} = q_r^{turb} = \underbrace{\Gamma T_0}_{\text{Turbulent particle flux}} + \underbrace{\kappa \partial_r T_0}_{\langle \tilde{T}\tilde{v}_r \rangle = \frac{\kappa}{n_0}\partial_r T_0}$$

- The gas jet OFF case, yields the expression of the electron heat conductivity κ according to the heat equation (which is *r* dependent only):

$$\frac{1}{r}\partial_r (r\kappa\partial_r T_0 + rT_0\Gamma) = -\nabla . q_0 + Q_e \longrightarrow \boxed{\kappa = \frac{\frac{r}{2}(Q_e - \nabla . \vec{q}_e^{\,0}) - T_0\Gamma}{\partial_r T_0}}$$

Electron Heat equation: MGI ON

$$\frac{3}{2}n_e\partial_t T_e + \frac{3}{2}n_e\vec{v}_e.\nabla T_e = -\nabla.\vec{q}_e + Q_e$$

1 2 3 4

1 Time-dependent problems $\partial_t T_e \neq 0$

2 $$\frac{3}{2}n_e\vec{v}_e.\nabla T_e = \frac{3}{2}n_e\left(\frac{v_{e\theta}}{r}\partial_\theta T_e + v_{e\phi}\partial_\phi T_e\right)$$

3 $$\nabla.\vec{q}_e = \nabla.\vec{q}_e^{\,0} + \nabla.\vec{q}_e^{\,turb}$$

$$\nabla.q_e^0 = \nabla.q_e^T + \nabla.q_e^u$$ → **Collision contribution due to thermal and frictional heat fluxes**

$$\nabla.\vec{q}_e^{\,turb} = \frac{1}{r}\partial_r(r q_r^{turb}) + \frac{1}{r}\partial_\theta q_\theta^{turb}$$ → **Turbulence contribution**

$$q_r^{turb} = q_\theta^{turb} = \frac{r}{2}(Q_e - \nabla.\vec{q}_e^{\,0})$$

4 Q_e is determined from the expressions of plasma heat losses and gains by ionization, recombination charge exchange and radiation

Ion Heat Equation

$$\frac{3}{2}n_i\partial_t T_i + \frac{3}{2}n_i\vec{v}_i.\nabla T_i = -\nabla.\vec{q}_i + Q_i$$

Ion Heat equation: heat conductivity expression for MGI OFF (steady state)

$$\boxed{\underbrace{\frac{3}{2}n_i\partial_t T_i}_{1} + \underbrace{\frac{3}{2}n_i\vec{v}_i.\nabla T_i}_{2} = \underbrace{-\nabla.\vec{q_i}}_{3} + \underbrace{Q_i}_{4}}$$

1 $\partial_t T_i = 0$

2 $\vec{v}_i.\nabla T_i = 0$

4
$$\begin{cases} Q_i = Q_{i,e} + Q_{i,n} \\ Q_{i,e} = \dfrac{3m_e n_e}{m_i \tau_e}(T_e - T_i) \\ Q_{i,n} = \dfrac{-3n_i n_e T_i \sigma_r}{2} \end{cases}$$

Ion Heat equation: heat conductivity expression for MGI OFF (steady state)

3 $$\nabla.\vec{q}_i = \nabla.\vec{q}_i^{\,0} + \nabla.\vec{q}_i^{\,turb}$$

$$\nabla.\vec{q}_i^{\,0} = \frac{-2}{m_i w_{ci}^2 r}\partial_r\left(\frac{r n_i T_i \partial_r T_i}{\tau_i}\right)$$ → **Collision contribution due to ion heat fluxes**

$$q_\theta^{turb} = q_r^{turb} = \Gamma T_0 + \kappa\partial_r T_0,$$ → **Turbulence contribution**

- The gas jet OFF case, yields the expression of κ according to the ion heat equation:

$$\frac{1}{r}\partial_r(r\Gamma T_0 + r\kappa\partial_r T_0) = Q_i - \nabla.\vec{q_0}$$ → $$\kappa = \frac{\frac{r}{2}(Q_i - \nabla.\vec{q}_i^{\,0}) - T_0\Gamma}{\partial_r T_0}$$

Ion Heat equation: MGI ON

$$\frac{3}{2}n_i\partial_t T_i + \frac{3}{2}n_i\vec{v}_i.\nabla T_i = -\nabla.\vec{q}_i + Q_i$$

1 2 3 4

1 $\partial_t T_i \neq 0$

2 $\vec{v}_i.\nabla T_i \neq 0$

3 $$\nabla.\vec{q} = \nabla.\vec{q}_0 + \frac{1}{r}\partial_r(\frac{r^2}{2}(Q_i - \nabla.\vec{q}_i^0)) + \frac{1}{r}\partial_\theta(\frac{r}{2}(Q_i - \nabla.\vec{q}_i^0))$$

4 Q_i is determined from the expressions of plasma heat losses and gains by:

- **ionization,**
- **recombination,**
- **charge exchange,**
- **and radiation**.

MGI ON & OFF

1- For the gas jet OFF case: we used the fact of a stationary density profile to find the so-called inherent sources and push the discussion further to find the expression of the particle diffusion coefficient and the heat thermal conductivity as a function of the plasma parameters, namely n, T and Γ_r.

2- For the gas jet ON: we have all the terms that allow us to simulate the behavior of the density profile and the electron and ion temperature profile in space time coordinates. In addition we can investigate the new expressions of the diffusion coefficient and heat conductivity that are (r,θ,ϕ)-dependent.

The Simulation Code

Write a numerical code that solves the continuity and the heat equations for both electrons and **singly** ions while adopting the following strategies:

- Finite difference method.
- Explicit dependence of plasma parameters on time
- Spatial three dimensional (r,θ,ϕ) dependence of the plasma parameters.
- Non-uniform grid spacing.

Conclusion