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Horse Race of Utility-Based Asset Pricing Models: Ranking through Specification Errors

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Horse Race of Utility-Based Asset Pricing Models: Ranking through Specification Errors

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Abstract

In this paper, we examine a specific class of asset pricing models typically referred to as consumption-capital asset pricing models (CCAPM). Our contribution is to provide a ranking of these intertemporal utility-based models, based on the size of their pricing errors as analyzed relative to aggregate information arriving in the US stock market. This will be the one criterion upon which they will be assessed- a criterion commonly known as the Hansen and Jagannathan specification error or distance-measure test. In brief, our findings suggest little supportive evidence in favor of one model vastly outperforming the rest. However, we are able to document a few patterns where there are clear benefits to using one model over another. We also find that some models perform better at quarterly than annual samples, and vice versa.

Introduction

In this paper, we examine a specific class of asset pricing models typically referred to as consumption-capital asset pricing models (CCAPM). Our contribution is to provide, in an explicit quantitative example, a ranking of the intertemporal utility-based asset-pricing models that heretofore have been extensively investigated, each in isolation. Waiting in the wings today are several alternatives to the standard expected utility case. How the relative performance of these models compares in US data clearly qualifies as a legitimate question. Our answer to such a question integrates the models into one unified framework, as analyzed relative to aggregate information arriving in the stock market. Particularly, in this paper, the models will be ranked according to the size of their pricing errors. This will be the one criterion upon which they will be assessed - a criterion commonly known in the literature as the Hansen and Jagannathan specification error or distance-measure test.

In the recent past, the theories and surrounding applications of each model specification[1] have been largely tested by the available econometric techniques of GMM and the Hansen and Jagannathan volatility bounds test (1991). Unfortunately, the stochastic discount factor (SDF) paradigm has proved only partially successful. For example, among the most damaging pieces of evidence against the standard expected utility case is that aggregate consumption is too smooth to justify the volatility in stock returns (Mehra and Prescott, 1985). Also, while state and time non-separabilities in consumption have shown incremental power over the standard case, they have all been weighed down by some deficiency challenging their preeminence. As a result, the literature finds *all* asset-pricing models mere approximations, at best. Many, unfortunately, are also non-admissible.[2]

Admissibility, however, is not a necessary requirement for a research question to be meaningful to researchers. Even when it is understood that a particular

1. Every model is essentially a specification of the stochastic discount factor or SDF.

2. Hansen's J-test is observed to have rejected scores of models, and many SDFs even fail to fall inside the Hansen and Jagannathan bounds.

model does not price correctly the vector of securities in an empirical analysis, it still pays to see which of the approximations is most useful. By way of ranking the models according to the size of their specification errors (given a common data set), the paper demonstrates how good these models are as *approximations* to the truth.

The project assumed in this paper is compatible with the course embraced by the field in the last decade or so. To a great extent, researchers in this area, rather than fully devising a model prior to testing it, have been working their way backwards making certain whatever SDF they propose satisfies a now well-known checklist of properties required for explaining asset-market data. Note that, in this project, all the SDFs are drawn from the pool of utility-based asset pricing models. There are no portfolio-based models simply because comparisons between consumption or utility-based models on the one hand *and* return or portfolio-based models on the other are not very informative. The reason is that returns are far better measured than consumption data, so pricing errors for return-based models that use the mimicking portfolio for marginal utility will be smaller than the underlying consumption-based model.

The remainder of the paper is organized as follows: The next section reviews the Hansen and Jagannathan (HJ, hereafter) procedure for calculating the pricing error of a candidate model. In addition to unconditional models, close consideration is given to conditional asset pricing, which introduces lagged variables that serve as instruments for publicly available information. Section three presents a summary of the HJ methodology in the literature. Section four introduces the parameterized preference models and identifies their respective SDF proxies. Section five briefly discusses the data and related summary statistics, and a separate appendix presents the variable sources in more extensive detail. The empirical results of ranking through specification errors are reported in section six. The section also interprets the values of our estimates of the various preference parameters. Finally, section seven overtly concludes the findings.

The Hansen-Jagannathan Distance Procedure

The HJ-distance or specification error test methodology used herein involves a number of theoretical merits over and above the conventional methods for testing asset-pricing models. Following is a brief listing of the key properties underlying the testing methodology.[3]

- The HJ-specification error test analyzes the general case where an asset pricing model may be misspecified. In the present context, misspecification translates into incorrect pricing of the vector of assets in the economy. This is important since formal statistical tests in the literature find that the hypothesis which claims that pricing errors are zero is a low probability event under most popular models. The HJ distance measure is intended to make comparable measures of model misspecification. In brief, a model is deemed superior among a set of models if it has the smallest HJ distance.
- The HJ method is particularly suitable for models whose SDFs are explicitly specified like consumption-based models. Its advantage in this setting is that the weighting matrix in the HJ distance measure is the same for different asset-pricing models, which makes it comparable across various models (nested or not). Another benefit for having a fixed weighting matrix is the absence of penalty/advantage to having a volatile SDF.[4]
- In addition to assessing misspecified models, the test can be used in a number of ways. For example, it can be used to examine the information content of different sets of asset market data. It can also be used even when a candidate discount factor or SDF depends on parameter values unavailable to the econometrician. Luckily as well, it is directly applicable when there are market frictions in general, such as transaction costs or short-sale constraints.

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3. The discussion is only sufficient for the purpose of this project, so the reader is additionally referred to the original papers by Hansen and Jagannathan (1997) and Jagannathan and Wang (1996).

4. This is not true of GMM where Hansen's J-test (a x^2under the null) is inversely related to the variance of pricing errors- in evident favor of models with pricing errors that are more variable across different assets. As in Jagannathan and Wang (1996), optimal GMM cannot determine which model among a set of competing models generates the smallest pricing error simply because the weighting matrix used in the J-statistic varies with changes in the SDF function.

In what follows, we consider first the case of no conditioning information. Initial attention is thus limited to the case where the lagged instrument is a fixed constant. In the later part of this section, the analysis will generalize to allow for a non-constant set of predetermined variables.

The Distance Measure without Conditioning Information

Assume that in a complete and frictionless economy is an *N*-vector of test assets, with gross return $R^{i}_{t,t+1}$ to holding the *i*-th asset between *t* and *t*+1. Gross returns are adjusted for risk when multiplied by a stochastic discount factor so that the expected present-value per dollar invested is equal to one (zero if the returns are excess) when a model is error-free. The stochastic discount factor "*m*" (or pricing kernel) is always an intertemporal marginal rate of substitution (IMRS) in this paper. Assuming this random variable obtains from_a true/admissible model, the fundamental pricing equation with no conditioning information is such that:

$$(1) \qquad E_t\left(m^{*}_{t,t+1} R_{t,t+1}\right) = 1$$

The model comparison tool in this paper has been developed by Hansen and Jagannathan (1997). For the set of asset payoffs with norm equal to one, these authors suggest a measure of the *maximum* pricing error (δ) associated with a given asset pricing model. Every candidate asset-pricing model is identified by its implied stochastic discount factor proxy as denoted by the letter *m*. As proxies can be mis-specified, the underlying model yields a specification error that is possibly positive in magnitude. The HJ specification error test, then, provides estimates of the size of the pricing errors of mis-specified pricing models. The maximal error measure has the alternative interpretation of the least squares distance between the SDF of an approximate model and the family of SDFs that correctly prices the vector of securities. Provided no arbitrage opportunities exist, the set of true SDFs is non-empty and typically large. It is signified by *M*, of which m^{*} is a member satisfying the first equation. For the purpose of estimation and successive drawing of inference, the error measure is given by:

$$(2) \quad \delta^{2}_{HJ} = \min_{m^{*} \in M} E\left[\left(m - m^{*}\right)^{2}\right]$$

Defined as a quadratic function of the pricing errors as weighted by the inverse of the second moments of gross returns, δ also takes on the alternate form:

$$(3) \qquad \delta = \sqrt{\left\{E\left(R_{t,t+1}m_{t,t+1} - 1_N\right)^{'}\left[E\left(R_{t,t+1}R_{t,t+1}^{'}\right)\right]^{-1}E\left(R_{t,t+1}m_{t,t+1} - 1_N\right)\right\}}$$

where $E\left(R_{t,t+1}R_{t,t+1}^{'}\right)$ is positive definite. Following Jagannathan and Wang (1996), we refer to the above measure as the HJ-distance. The SDF with the least distance measure performs best in the sample. At the extreme, if the model is correct, the HJ-distance is zero and there are no pricing errors.

Hansen and Jagannathan (1997) show that their HJ-distance measure can be estimated via GMM using for their weighting matrix (in place of an estimate of the variance matrix of pricing errors) the inverse of the sample second moment matrix of returns. However, SDFs are a function of data and a number of unknown preference parameters. To this end, we follow HJ in fixing these free parameters by minimizing the maximum pricing error. The overall procedure is henceforth two-step:

(a) A sample estimate of the unknown preference parameters (like the risk aversion coefficient) is GMM-obtained, based on the moment condition $E\left(R_{t,t+1}m_{t,t+1} - 1_N\right) = 0_N$ and the weighting matrix $\left[E\left(R_{t,t+1}R_{t,t+1}^{'}\right)\right]^{-1}$

(b) The HJ-distance is then computed given the estimate of the SDF parameters in part (a).

The Distance Measure with Conditioning Information

Following Hansen and Jagannathan (1997), the test assets allow for conditioning information, as well. This consideration is of practical importance since asset returns contain predictable components. If the fundamental pricing equation with conditioning information is $E\left(m_{t,t+1}^{*}R_{t,t+1}\middle|\Omega_t\right) = 1$ (where Ω_t is the information set at *t*), then by the law of iterated expectations, the set can be replaced by its subset of instrumental variables Z_t as in $E\left(m_{t,t+1}^{*}R_{t,t+1}\middle|Z_t\right) = 1$.

For the purposes of our empirical investigation, we incorporate conditioning information into our set of test assets by following the suggestion in Breen, Glosten, and Jagannathan (1989). These authors construct "synthetic portfolios" or dynamic trading strategies that, by design, are still returns with unit price. For example, if there are two original returns $R_1$and R_2, then a managed portfolio as such can be constructed in which $(1-z)$ units are invested in the first asset and z units in the second where z is a variable from the previous time period so that it is in the conditioning information set of investors at the time of investment.

Asymptotic Distribution of the HJ Distance Measure

With sample moments replacing their population counterparts, pricing errors of inaccurate models are evaluated via equation (3). Hansen, Heaton, and Luttmer (1995) develop an econometric method to provide consistent estimators of the specification error measure. They derive the asymptotic distribution of d (= δ) and prove that the resulting theory extends to the present case where there are unknown preference parameters that must be estimated. In this paper, the standard errors of d will be calculated according to proposition 2.2 in Hansen et.al.(1995) using 12 lags for the correction procedure in Newey and West (1987a).

Literature Review of the Hansen-Jagannathan Specification Error Test

Jagannathan and Wang (1996) use pricing errors to justify the addition of macroeconomic factors to the traditional CAPM model. For his version of the unconditional multi-factor CAPM, Cochrane (1996) reports pricing errors about half those of the static model on size portfolios. In a similar vein, Jagannathan, Kuota, and Takehara (1998) and Hodrick and Zhang (2001) evaluate several *linear* asset pricing models[5] by computing their HJ-distance measure. Farnsworth, Ferson, Jackson, and Todd (2002) also study the use of four non-utility-based stochastic discount factor models in evaluating the investment performance of portfolio managers and find that measures of performance are not highly sensitive to their set of SDF models.

5. Cumulatively, these models are the CAPM, a linearized consumption-CAPM, the Jagannathan and Wang (1996) conditional-CAPM, the Campbell (1996) dynamic asset pricing model, a linearized-version of the production-based model in Cochrane(1996), and the Fama and French (1993) three-factor and five-factor models.

On the econometric front, Hansen, Heaton, and Luttmer (1995) develop asymptotic distribution theory for specification errors on SDFs, and Ferson and Siegel (2002) examine these standard errors by simulation. In 1999, Ahn and Gadrarowski study, in a Monte Carlo setup, the finite-sample properties of the HJ-distance methodology as applied to linear factor models. Following, Kan and Zhou (2002) derive the exact distribution for linear settings, only. As just-implied, linear portfolio-based models have been the workhouses for almost all applications (empirical or non-empirical) on the HJ specification error test. Sensibly, given these model *are specializations* of the canonical consumption-based model and given the temporal and state non-separabilities in utility-based asset pricing only generalize and extend on the canonical case, it is natural to conceive, in general, the applications of the HJ methodology to utility-based models.. For a start, Bakshi and Naka (1997) use specification errors in examining the empirical performance of four consumption-based models in a 1971–1990 sample of Japanese aggregate consumption and security markets data. Chen and Ludvigson (2003) also use HJ-distances to compare SDFs under flexibly-estimated habit formation models with proxies from a variety of linear factor models. They find that, relative to the latter set, their habit models possess the ability to explain portfolios of equity returns double-sorted on size and book-to-market characteristics but face greater difficulty justifying the behavior of a Treasury bill rate. In just another rare attempt to apply the Hansen and Jagannathan methodology to nonlinear models, Campbell and Cochrane (2000) explore the poor performance of the standard C-CAPM model in a fully specified economy simulated on the basis of their external habit-formation paper (Campbell and Cochrane,1995). As recognized by these authors, such an exercise is of limited value. While they test how good several models are as approximations to the assumed one, it is of greater relevance to researchers to quantify how far these models are from the truth. This, again, is the focus of the present paper.

In what follows is a brief presentation of the discrete-time versions of the candidate utility functions.

SDF Proxies of Candidate Utility-Based Models

Nearly all candidate models in this project concern themselves with the further gains from relaxing the time/state-separable assumptions in the constant relative risk aversion (CRRA) expected utility function, so they all collapse to the standard case. As will be made apparent in Table 1, each model captures some well-documented feature of human behavior. Hence, in each case, the corresponding Euler equation implies that consumption risk is not the only risk that should be compensated for in equilibrium. For each specification of utility, we specify the form, the utility function, and the corresponding IMRS.

Table 1 Model Description

1. Standard CRRA	Displays state and time separability.
2. Recursive Utility (Epstein-Zin-Weil)	Disentangles the coefficient of relative risk aversion from the elasticity of intertemporal substitution. Allows for state-inseparability.
3. Capitalist Spirit Model	Includes wealth not only for its implied consumption reward but also for its resulting social status.
4. Gul's Disappointment Aversion Model	Weighs bad outcomes more heavily than good ones. Bad and good are defined endogeneously, with respect to the certainty equivalent measure of a gamble.
5. Abel's Consumption Externality Model	Models 'catching-up-with-the-Joneses'. Defined over one's own consumption relative to lagged aggregate consumption per capita. Habit is external and in ratio form. Allows for differing degrees of time-nonseparability as governed by the parameter kappa.
6. Ferson and Constantinides's Habit Persistence and Durability Model	Models internal habit persistence/durability formation in difference perspective.
7. Campbell and Cochrane's Habit Formation Model	Models external habit formation in a slow-moving non-linear manner.

All the models have been amenable to empirical analysis using market data and standard asset-pricing methodologies. More specifically, they have been tested for their usefulness in addressing the equity-premium and the risk-free rate puzzles. None afford an all-encompassing explanation, but all of them find some empirical support. Whether in text or footnotes, we include a word on each specification in the empirical literature.

Finally, note that throughout the paper, the time discount factor will take on a fixed value. This shall permit easier comparison of the remaining preference parameters. We assume the subjective discount factor β takes on a fixed value equal to 0.9975. This corresponds to a reasonable 3% annual rate of time preference.

Standard Power Utility Model

The time-and-state separable iso-elastic power utility function is given by:

$$U_t^{TSS} = E_t\left[\sum_{j=0}^{\infty}\beta^j \frac{C_{t+j}^{1-\gamma}-1}{1-\gamma}\right]$$

where γ is the RRA coefficient and β is the time-discount factor. This most widely studied (and widely rejected) preference specification implies a stochastic-discount factor of the form:

$$m_{t,t+1}^{TSS} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$

where the elasticity and relative risk aversion coefficients are reciprocal of one another. This standard formulation serves as the benchmark case for subsequent model developments. It has not performed well in explaining the behavior of consumption and asset returns over time as in Hansen and Singleton (1982, 1983), and Wheatley (1988): particularly, the SDF plots outside the sample HJ bound and Hansen's J-test rejects the model outright.

Epstein and Zin (1990) and Weil (1989) relax the state-separability assumption, and then Constantinides (1990) relaxes time-separability, arguing that this may be the key restriction generating the equity-premium puzzle of Mehra and Prescott (1985).

Recursive Utility Model

Independently, Epstein and Zin (1990, 1991a) and Weil (1989) investigate a specific class of recursive preferences (a generalization of the Kreps and Porteus (1978) framework) over intertemporal consumption lotteries. An important feature of these preferences is that they permit attitudes towards risk, as captured by the coefficient of relative risk aversion, to be separate from the degree of intertemporal substitution.[6] Current consumption and a certainty equivalent measure of random future utility are combined via an aggregator function to determine current utility. The objective function can be expressed recursively as:

$$U_t^{EZW} = \left[(1-\beta) C_t^{(1-\gamma)/\nu} + \beta \left(E_t U_{t+1}^{1-\gamma} \right)^{1/\nu} \right]^{\frac{\nu}{1-\gamma}}$$

where $\gamma > 0$ is the coefficient of relative risk aversion and $\sigma = \dfrac{\nu}{\nu + \gamma - 1}$ is the elasticity of intertemporal substitution. Hence, the recursive formulation allows the two effects to be separated. The parameter $\boldsymbol{v}$ is set to govern the degree of state non-separability. For $v = 1$, the non-expected utility model is identical to C-CAPM in which case the individual is indifferent to the resolution of uncertainty. Given no restrictions on the value of v, the systematic risk of every asset is determined by the covariance with both the market portfolio and consumption growth. The resulting formula for the SDF is given by:

$$m_{t,t+1}^{EZW} = \beta^{\nu} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma-\nu} R_{m.t,t+1}^{\nu-1}$$

where $R_{m.t,t+1}$ is the simple gross return on the portfolio of all invested wealth[7]. It is often called the market return and its subscript m should not be confused with the m of a model SDF proxy.

Intertemporal asset pricing models with Kreps and Porteus (1978) preferences have fared poorly in explaining the first two unconditional moments of the risky and risk-less asset returns (Epstein and Zin ,1990 and Weil,1989). The volatility bounds test in Bakshi and Naka (1997) indicates that early resolution of uncertainty is more consistent with a more volatile IMRS and that the ability of the model to generate the latter originates from the volatile wealth index rather than from consumption growth. On a separate note, Melino and Yang (2003) demonstrate that the recursive model can perfectly match US historical data if it is generalized to allow for a state dependent elasticity of intertemporal substitution (EIS) and a state-dependent relative risk-aversion (RRA) coefficient.

6. To highlight the advantage of this separation, recall that risk aversion (well-defined in atemporal contexts) is not meaningful in deterministic settings, whereas the opposite is true of the elasticity of intertemporal substitution.

7. This often includes human capital in addition to financial assets.

Capitalist Spirit Model

Unlike the recursive model, capitalist-spirit investors acquire wealth not only for its implied consumption reward but also for its resulting social status. Further support for this hypothesis and for the unremitting acquisition of wealth by already rich individuals (with or without children) can be found in Bakshi and Chen (1996) and the references therein.

The capitalist-spirit period utility is provided by:

$$U_t^{CS} = E_t \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\gamma}}{1-\gamma} W_{t+j}^{-\lambda}$$

where $\lambda_{cs} > 0$ when $\gamma \geq 1$ and $\lambda_{cs} < 0$ when $\gamma = 0$ The variable W_t denotes the investor's time t absolute wealth, and the λ_{cs} parameter measures the extent to which the investor cares about status. The Arrow-Pratt relative risk-aversion in wealth[8] is given by

$\gamma + \lambda_{cs}$. Absolute wealth is simply status in this model so that higher wealth means higher status (independent of the wealth distribution for the group of people with whom the investor has social or professional contacts).[9] The corresponding stochastic discount factor is decreasing in both wealth growth and consumption growth with the former serving as a proxy for social status. This can be seen in:

$$m_{t+1}^{CS} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{W_{t+1}}{W_t} \right)^{-\lambda_{cs}} \left(1 + \frac{\lambda_{cs}}{\gamma - 1} \frac{C_{t+1}}{W_{t+1}} \right)$$

8. The more the investor cares about status, the more she is averse to wealth risk. As in the standard model, nonetheless, the elasticity of intertemporal substitution is the reciprocal of γ.

9. Other formulations of the capitalist spirit, which involve the wealth distribution of other groups in the economy, appear in Bakshi and Chen (1996). In some sense, the omitted formulations express a `catching-up-with-the-Joneses' spirit, as in the next model. However, the difference is that the reference level is group-specific and not necessarily aggregate wealth, and the very specifications do not allow for aggregation under the identical preferences/distinct endowments assumption. Indeed, they can only be subjected to individual consumer data, which leaves them outside the scope of this paper.

Note that the capitalist-spirit functional and parameter restrictions are fundamentally different from the particular class of Epstein-Zin-Weil recursive preferences[10]. Wealth in the recursive model is only a stand-in for tomorrow's utility index, whereas Bakshi and Chen's investors desire the wealth-induced status whose risk, as a result, is compensated for in equilibrium. Overall, Bakshi and Chen (1996) find that, for a battery of tests[11], the estimated values and signs of the preference parameters are supportive of the spirit-of-capitalism hypothesis.[12]

Gul's Disappointment Aversion Model

Unless some precautionary motive is brought into play (as in the Campbell and Cochrane (1999) proposition), habit models, as will become clear, unfortunately generate too much variation in the risk-free rate in order to obtain a sufficiently variable stochastic discount factor. This is not the case with disappointment-averse individuals as they weigh bad outcomes more heavily than good ones (where bad and good are defined with reference to a certainty equivalent measure of a gamble). In fact, it is precisely the existence of a bad state that lowers the risk-free rate and increases the average stock return, thereby allowing the model to better match both real and excess returns.

Disappointment-averse preferences have been axiomatized by Gul (1991) to offer a solution to a number of decision paradoxes including the Allais paradox.[13]

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10. Under the latter notation, the IMRS is $m_{t+1}^{ezw} = \beta^{1-\lambda_{ezw}} \left(C_{t+1} / C_t \right)^{-\gamma} \left(W_{t+1} / W_t \right)^{-\lambda_{ezw}}$ where $\gamma > 0$ if $\lambda_{ezw} < 1$, $\gamma < 0$ if $\lambda_{ezw} > 1$ and $\gamma = 0$ if $\lambda_{ezw} = 1$ Also, observe that the Epstein-Zin formula as stated in Bakshi and Chen (1996) mistakenly passes a unit power for the impatience parameter. However, this does not change any of the paper's results since the authors assume $\beta = 1$ all along.

11. These include GMM, the Hansen and Jagannathan (1991) minimum volatility bounds test and the Hansen and Jagannathan (1997) maximum specification error test.

12. The authors also find that, as in the Epstein-Zin-Weil utility, the ability of their model to generate a volatile SDF comes mostly from the spirit of capitalism. This is only because their wealth growth series (with a standard deviation of 4.4 percent) is ten fold more volatile than its consumption counterpart.

13. Just as in experiments, the individual exhibits preference for a much smaller gain for sure to a small risk of getting nothing, the hope is that (in analogous terms of security returns) the agent may want to settle for the less return from the risk-less asset to avoid getting disappointed with a stock.

The resulting functional form is a single parameter extension of expected utility for which A=1. Epstein and Zin (1989) propose the following constant elasticity of substitution (CES) function as the aggregator function combining current consumption C_t with μ_t, the certainty equivalent of random future utility given time t public information, to obtain the current-period lifetime utility:

$$U_t^{DA} = \left[C^{\alpha} + \beta\mu_t^{\alpha}\right]^{\frac{1}{\alpha}}$$

where μ_t is based on risk preferences that are disappointment averse as in Gul (1991).[14]

For a disappointment-averse consumer ($A\neq 1$), the SDF for excess returns[15] depends on consumption growth (and only implicitly on the return on the market portfolio) as in:

$$m_{t+1}^{DA} = \beta/\left(C_{t+1}/C_t\right)^{-\gamma} I_A\left\{\beta^{1/(1-\gamma)}\left(C_{t+1}/C_t\right)^{\gamma/(\gamma-1)} R_{m.t,t+1}^{1/(1-\gamma)}\right\}$$

where γ is positive, $0<A\leq 1$, the indicator function

$$I_A(x) = \begin{cases} 1 & x \leq 1 \\ A & x \geq 1 \end{cases}$$

and $R_{m.t,t+1}$ is the return on the market portfolio. The restriction $A<1$ provides a straightforward disappointment-aversion interpretation. It also implies first-order risk-aversion.[16]

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14. The certainty equivalent function is more formally defined by: $\int \phi_{DA}\left(x/\mu(p)\right)dp(x) = 0$ where

$$\phi_{DA}(x) = \begin{cases} v(x) - v(1) & x \leq 1 \\ A\left(v(x) - v(1)\right) & x \geq 1 \end{cases} \text{ and } v(x) = \begin{cases} \left(x^{\alpha} - 1\right)/\alpha & \alpha \neq 0 \\ \log(x) & \alpha = 0 \end{cases}$$

15. We conveniently follow the extant literature in using the excess (not individual) returns SDF. For practical purposes, the familiar approach of using the individual asset SDF encounters serious difficulty in this model since it involves a conditional expectation that is difficult to compute.

16. First-order risk-aversion refers to the fact that the risk-premium on a small gamble about certainty is proportional to the standard deviation of the gamble (and not to its variance as is the case with second order risk-aversion). In this model, the parameter A measures first-order risk-aversion so that a unit A implies first-order risk-neutrality, and risk-aversion is said to increase as either γ (second-order risk aversion) increases or A falls.

Epstein and Zin (1989,1991b) integrate these preferences in an intertemporal asset pricing model under a recursive utility framework and show that their resulting IMRS satisfies the Hansen and Jogannathan volatility bounds (1991) for a large set of values of γ and β when $A < 1$. Bonomo and Garcia (1994) generate both the first and second unconditional moments of the equity premium and the risk-free rate by both (1) endowing their agents with disappointment averse preferences and (2) making the joint process of consumption and dividends follow a bivariate three-state Markov switching model.[17] Finally, Ang , Beart , and Liu (2002) find realistic portfolios with DA utility functions exhibiting low curvature. For moderate variations in their parameters, they are also able to generate optimal *non-participation* in the stock market.

Abel's Consumption Externality Model

The consumption externality or 'catching-up-with-the-Joneses' case is obtained for a utility function of the ratio of one's own consumption to the lagged level of aggregate consumption per capita:

$$U_t^{CE} = E_t\left[\sum_{j=0}^{\infty}\frac{\left(C_{t+j}/X_{t+j}\right)^{1-\gamma}-1}{1-\gamma}\right]$$

where the habit level X_{t+j} is equal to $\left[C_{t+j-1}^{k}\overline{C}_{t+j-1}^{1-k}\right]^{\gamma}$,and C_{t+j-1} (the consumer's own consumption in period *t+j-1*) is also aggregate consumption per capita $(\overline{C}_{t+j-1})$ in the same equilibrium period for a representative agent setting as such. The parameter κ governs the degree of time non-separablity while γ governs the degree of risk-aversion. The SDF corresponding to this specification is:

$$m_{t+1}^{CE} = \beta\left(C_t/C_{t-1}\right)^{\kappa(\gamma-1)}\left(C_{t+1}/C_t\right)^{-\gamma}$$

This is the general formulation. For κ between 0 and 1, the formulation is the relative consumption model or "catching up with the Joneses" first studied by Gali in April 1989. In the present paper, both the general and κ =1-restricted cases are considered.

17. Unfortunately, the researchers demonstrate that *both* conditions are necessary to fitting the two moments and that this can only be achieved via a high implicit risk-aversion.

Overall, accounting for consumption externality does not result in a much better performing SDF. For example, while Abel (1990) finds that the unconditional expected returns on stocks, bills, and consols generated by this model are much closer to their US historical averages than their counterparts in the standard time-separable case for either of a lognormal or a 2-point *i.i.d.* consumption growth distribution, his standard deviations are unrealistic.

Ferson and Constantinides's Habit Persistence and Durability Model

A second habit-forming model relaxes the time separability of preferences of the von Neumann-Morgenstern type, as in Constantinides (1990) and Ferson and Constantinides (1991). In contrast to Abel's ratio model of external habit formation, the habit level here is internal.[18]

It is modeled in the difference perspective $(C_t - X_t)$. The formalization of this statement assumes that a habit-forming individual has a preference representation of the form:

$$U_t^{HP/D} = E_t\left[\sum_{j=0}^{\infty}\beta^j \frac{\left(C_{t+j} - X_{t+j}\right)^{1-\gamma} - 1}{1-\gamma}\right]$$

where the stochastic subsistence level X_t is proportional to the one-period lagged consumption as in θC_{t+j-1}. The factor of proportionality, which basically governs the degree of time-nonseparability, is θ. A positive θ suggests habit persistence in the consumption good. Alternatively, if θ <0, consumption is said to be durable. This form of habit formation drives a wedge between the coefficient of relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS) in consumption. To maintain a positive RRA coefficient,[19] γ (though only

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18. Habit here depends on the agent's own consumption rather than on the aggregate consumption level. Ferson and Constantinides (1991) utilize a more general lag structure for Xt as in an exponentially weighted sum of the past flows of consumption services (Ryder and Heal, 1973). I do not employ this more general utility specification for X not only because it is more econometrically challenging to assume a longer lag structure but also because, as in Ferson and Constantinides, the coefficients in specifications with lags as short as two periods cannot be estimated reliably.

19. As in the capitalist spirit model of Bakshi and Chen where wealth enters the utility function, it would be improper to compute the value for the RRA coefficient using an atemporal gamble that changes consumption (either current or at some specified date) by the outcome of the gamble. Rather, risk-aversion must be defined in terms of an atemporal gamble that changes wealth. RRA in the Constantinides model is a function of both wealth and the subsistence level.

approximately equal to the former) must necessarily take on a positive value. The model-implied SDF is:

$$m_{t+1}^{HP/D} = \beta\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \frac{(C_{t+1}/C_t - \theta)^{-\gamma} - \beta\theta(C_{t+1}/C_t)^{-\gamma} E_{t+1}(C_{t+2}/C_{t+1} - \theta)^{-\gamma}}{(C_t/C_{t-1} - \theta)^{-\gamma} - \beta\theta(C_1/C_{t-1})^{-\gamma} E_t(C_{t+1}/C_1 - \theta)^{-\gamma}}$$

where the conditional expectations are replaced by their ex-poste values as in Ni (1997) and Palacios-Huerta (2002), among others.

Constantinides (1990), and Ferson and Constantinides (1991) argue that the model fares empirically better than the time-separable model, which it nests. Heaton (1995) also proposes the model as a resolution of both the equity-premium and risk-free rate puzzles, although he finds that it generates a volatile short rate. Conversely, Daniel and Marshall (1997) do not find a sizeable out-performance (over the standard power utility function) for horizons of one, four, and eight quarters. Interestingly, though, they demonstrate amazing out-performance at two-year horizons (i.e. using two-year returns).

Campbell and Cochrane's Habit Formation Model

Except for the different perspective to modeling Xt, the habit-forming utility function developed by Campbell and Cochrane (1999) is the same as in Constantinides (1990). Habit is external in this model, and it varies in a slow-moving non-linear manner. How habit reacts to consumption is summed up in a recession indicator defined as the surplus consumption ratio $S_t = \frac{C_t - X_t}{C_t}$.

According to Campbell and Cochrane (1999), consumption growth is *i.i.d.* normal and the log surplus consumption ratio s_t ($=lnS_t$) follows the conditionally normal and heteroskedastic $AR(1)$ process $s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda^{cc}(s_t)\varepsilon^c_{t+1}$, where ε^c_{t+1} is the innovation to the consumption growth process distributed *i.i.d.* N(0,σ^2_ε). The steady state value of s is $\bar{s} = \ln(\bar{S}) = \ln\left[\sigma_t\sqrt{\gamma/(1-\phi)}\right]$ with ϕ measuring the level of habit persistence or the speed at which s_t reverts to $\bar{s}$. The sensitivity function $\lambda^{cc}(s_t)$ is negatively related to s_t according to the (non-linear) square-root process:

$$\lambda^{cc}(s_t) = \begin{cases} \frac{1}{S}\sqrt{\left[1-2(s_t - \bar{s})\right]-1} & for \quad s_t \le s_{\max} \\ 0 & otherwise \end{cases}$$

where $s_{\max}$ is the value of s_t at which the first expression in the above equation runs into zero. The model SDF is given by:

$$m_{t,t+1}^{cc} = \beta\left(\frac{S_t}{S_{t-1}}\frac{C_t}{C_{t-1}}\right)^{-\gamma}$$

so that covariances with both the S and C shocks drive average returns.[20]

This preference ordering makes the individual extremely averse to consumption risk even when the risk-aversion is small.[21] Here, the local curvature is time varying. In particular, it depends on both S_t and the power γ as in $\eta_t = -Cu_{cc}/u_c = \gamma/S_t$.[22] As consumption falls towards habit, additional reductions in consumption become less intolerable which explains the increase in the risk-aversion of the consumer.

We follow Li (2002) in calculating the surplus consumption ratio from actual data. Particularly, in calculating $\bar{s}$, the curvature parameter is set to 2 and σ^2 is estimated via the method of moments estimation.[23] Then for ϕ on a grid[24] between 0 and 1 and with s_t starting at its steady state value, the actual consumption series is used to compute both s_t and λ_{t-1}^{cc}

Campbell and Cochrane (1999) demonstrate, through a calibration argument, that their model can generate a large number of observables, including the equity-premium, the risk-free rate, the price dividend ratio, a time-varying and counter-

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20. Similar to the recursive and capitalist-spirit models, the variation in C_{t+1}/C_t hardly accounts for any risk-premia. In this model, this role is virtually occupied by the volatility in S_{t+1}/S_t.

21. Indeed, all the habit formation and subsistence consumption models surveyed herein have been able to produce a large enough equity premium only by resigning themselves to using an implausibly high effective risk-aversion.

22. The terms u_{cc} and u_c denote (respectively) the first and second derivatives of the utility function with respect to consumption.

23. This is tantamount to using the two just-identified moment conditions (in addition to those implied by our pricing equation):

$$E\left(q_t - \mu_q\right) = 0 \text{ and } E\left(q_t q_{t-1} - \mu_q^2\right) = 0$$

where $q_t = \ln\left(C_t/C_{t-1}\right)$ denotes the consumption growth rate with mean equal to μ_q.

24. Like every other preference parameter left to our discretion in this project, the value of ϕ is chosen over [0,1] to minimize the maximum pricing error associated with this model. Additionally, to avoid the unit root problem and insure that $\overline{S} < 1$, the restriction $0 < \phi < 1 - \gamma\sigma^2$ is imposed.

cyclical Sharpe ratio, the volatility of both the excess return and the P/D ratio and the long-horizon forecastability of stock returns. In the process, however, they invoke implausibly high counter-cyclical variations in the effective risk-aversion. This latter also takes on implausibly high values in times of recessions.

Before we present our main empirical findings, a number of data-related issues deserve a few clarifications.

Data

We assume that consumer's decisions happen at the same fixed interval over which asset returns and consumption are measured. Empirical results are first obtained for an interval of every quarter. Then, annual intervals are tried as a robustness check against the sampling frequency. Notice that the analysis based on annual data is not susceptible to seasonality in the consumption and returns series within the year. Thus, comparing the results from the two sampling frequencies also doubles as a check against seasonality of the adjustment procedure.

The size of our sample is dictated by our available measure of consumption to wealth ratio, as required by the capitalist-spirit model. We use the data taken from Lettau and Ludvigson (2001a) through which they develop a consumption-aggregate wealth measure, made available from their website. For the quarterly models, the sample period is 1951:04 to 2002:04, for a total of 205 observations. The available annual data covers a shorter time series and a total of 54 observations for a sample period covering the years 1948 to 2001. A detailed description of every item in our data set can be found in the appendix.

Table 2 Univariate Summary Statistics

Panel A: Quarterly Sample (1952Q1-2002Q4)		
Variable	**Mean (%quarter)**	**Std Dev.**
Real Cons Growth*	0.501817548	0.4710657
Log Cons to Wealth Ratio** : LCAY	-7.22549E-05	0.0123067
Real Return on 3 Mnth Tbill: RR3Tb	0.84257	0.65121
Real Return on Decile 1: RRDEC1	2.94537	10.32072
Real Return on Decile 5: RRDEC5	1.9481	7.62552
Real Return on Decile 10: RRDEC10	1.5935	5.55906
Real Return on CRSP-VW Index: RRVW	1.1966	5.62449
Real Return on CRSP-EW Index: RREW	1.71741	7.39831
Real Return on Market Portfolio : RRMkt	0.50112	0.82998
Diff(-1L)*100*RR3Tb + [1-Diff(-1L)*100]*RRDec5: RRZ1	2.236345916	9.6688572
Slope(-1L)*100*RR3Tb + [1-Slope(-1L)*100]*RRDec5: RRZ2	1.540233144	6.5983873
TB1MO(-1L)*100*RR3Tb + [1-TB1MO(-1L)*100]*RRDec5:RRZ3	0.731503116	6.0166571
LCAY(-1L)*100*RR3Tb + [1-LCAY(-1L)*100]*RRDec5: RRZ4	0.231837884	14.196915
Panel B: Annual Sample (1949-2001)		
Variable	**Mean (%/year)**	**Std Dev.**
Real Cons Growth*	2.00911	1.13887
Log Cons to Wealth Ratio** : LCAY	9.230111111	0.368938
Real Return on 1 Mnth Tbill: RR1Tb	2.9895	2.45278
Real Return on 3 Mnth Tbill: RR3Tb	3.30986	2.71437
Real Return on CRSP-VW Index: RRVW	5.26729	10.20283
Real Return on CRSP-EW Index: RREW	7.08607	12.91717
Real Return on Market Portfolio : RRMkt	2.08125	1.74861
Slope(-1L)*100*RR1Tb + [1-Slope(-1L)*100]*RREW: RRZ	4.940362179	8.1595486

*Notes**: The mean and standard deviation of real consumption growth are employed in the Campbell-Cochrane model which assumes an i.i.d. normal specification for this series.**: For this variable, the rel;event statistics are not in percentage terms.*

In table 2, we report the univariate summary statistics for consumption and asset returns data. Many of the stylized facts resemble those shown in other studies. For example, the average real-consumption growth is 0.502% per quarter (2.01% per year) with a standard deviation of 0.471% (1.139%), which is quite smooth relative to the volatility of stock returns. Decile 1 (the smallest firms) has the highest average return and the highest standard deviation among the set of original asset returns.

In our choice of asset returns, we tag on the greater part of the existing literature by only focusing on security market data. Everywhere in this paper, real asset returns are the CRSP-provided nominal returns deflated by the appropriate price deflator. For *quarterly unconditional* models, the first set of test assets[25] includes a 90-day treasury bill in addition to a subset of the size deciles of value-weighted portfolios of common stocks traded on the New York Stock Exchange (NYSE). To capture most of the stock return behavior while keeping the number of test assets at a proper minimum, only deciles 1, 5 and 10 are used (as in Ferson and Constantinides,1991). In an attempt to check the robustness of our results to seasonality, we also experiment with an alternate (second) vector of quarterly asset returns. This includes the same choice of assets in our *annual unconditional* models: a 90-day treasury bill and the CRSP value and equally-weighted indices. We use the CRSP Index (NYSE, AMEX, and Nasdaq), and not the S&P index, because it is a much broader measure which provides a better proxy for nonhuman components of total asset wealth.

For conditional models, the set of instrumental variables is embedded in the set of test assets, as previously described. Here, we experiment with one set of asset returns for each of the annual and quarterly models. For *quarterly conditional* models, the returns vector includes six assets: the real return on the 90-day t-bill, the real return on decile 5 of the CRSP value-weighted index, and real return on four “managed” portfolios in which z units are invested in the three-month Treasury bill and $(1-z)$ units in the fifth decile portfolio. All $\boldsymbol{z}$s are denominated in percentage terms and selected from the previous time period so they constitute legitimate instruments. They have all been proposed in the literature studying predictability. For the third return, the portfolio weight $\boldsymbol{z}$ is the dividend yield on the CRSP equally-weighted stock index minus the dividend yield on the value-weighted index (Ferson and Constantinides, 1991); for the fourth return, $\boldsymbol{z}$ is the three-month Treasury bill rate less the one-month Treasury bill rate (Fama and Schwert, 1977); for the fifth return, $\boldsymbol{z}$ is the nominal one-month Treasury bill rate ((Fama and French,1989); and for the last return, $\boldsymbol{z}$ is the log consumption-

25. This is case (a) in the results tables and figures.

aggregate wealth ratio (Lettau and Ludvigson, 2001b). In a similar fashion, the *annual conditional* models include in their assets vector: the real return on the 30-day Treasury bill, the real return on the equally weighted portfolio, and the real return on one "managed" portfolio. This is constructed as $(1-z)$ units invested in the 30-day t-bill and $\boldsymbol{z}$ units in the equally weighted stock index, where $\boldsymbol{z}$ is the three month Treasury bill rate minus the one-month Treasury bill rate, all lagged once.[26]

Empirical Results

HJ-Distance Measure

The empirical results emerge from tables 3a to 3e. In each race, we set β =0.9975 and report the distance measure obtained by choosing the unknown preference parameters in the last four columns to minimize the Hansen and Jagannathan specification error. The standard errors underneath each distance measure are calculated under the null hypothesis that the true distance is not equal to zero**.** A lag length of $T^{1/3}$ (where T is the sample size) is used in the computation of the Newey and West (1987a) covariance matrix. Absenting statistical consideration to the HJ-distance standard errors, we can summarize our main findings in few points:

(a) The worst performing model is always the standard model with a pricing error larger than any of the other competing models. This should not come as a surprise since the time and state-separable case was originally thought of as too rigid a specification, giving rise to the other seven utility functions in the literature.

(b) No single parametrized model consistently dominates the rest. However, there is a frontrunner in each time frequency, irrespective of whether the models are conditional or not. Among the competing models with *quarterly data* (tables 3a to 3c), sampling evidence overly favors the Abel (1990) specification with

26. Non-singularity is the only restriction we need to impose on our positive definite sub-optimal weighting matrix. We verify that for every one of our five asset return vectors, $W = E\left(R'R\right)^{-1}$ is non-singular.

unrestricted κ. Alternatively, the Epstein-Zin-Weil model always provides the most accurate pricing in the annual sample (tables 3d and 3e).[27]

(c) Some broad patterns also appear to hold over the different rankings:

(i) When concerns about status are reflected in preferences as in the capitalist-spirit hypothesis, the distance measure is reliably modest.

(ii) Despite an additional free parameter and a complex evolution of the habit stock in the Campbell and Cochrane (1999) model, there is little guidance on its relative performance with respect to Abel's model without κ. Both hover somewhere in the middle between rows 3 to 6.

(iii) The performance of Gul's (1991) disappointment aversion model is generally unimpressive, possibly because we employ the excess (not individual) returns SDF. With quarterly data, it offers little improvement over the standard power utility function.Also, the improvement in performance in the annual sample is not steady.

(iv) In quarterly samples, the Ferson and Constantinides (1991) model with our one-lag specification is a steady third. This is row 3 in tables3a to 3c. Nevertheless, with annual data the model performance significantly deteriorates.

(d) There seems to be no advantage to having a larger number of preference parameters to be estimated. For example, the Abel (1990) model without κ (and hence only one parameter to be estimated) fares second in table 3b, outperforming five other models with two preference parameters each.

(e) Tables 3b and 3d can be interpreted as evidence against robustness to varying the sampling frequency or to seasonality in adjusting the consumption and returns procedures. The proof is in the dramatic reshuffling of positions across the two tables where models are required to price the same exact set of assets. It is also evident that because of the smaller number of observations in the annual sample, the performance of every model (as reflected in the magnitude of its HJ-distance) worsens as we move from quarterly (table 3b) to annual data (table 3d).

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27. Where they are not winners, the Abel with κ unrestricted and the Epstein-Zin-Weil models do not fare a good deal better than the middle.

Table 3a Quarterly Unconditional Models (1)

1. Abel with Kappa	**HJD** Std.Err.	**0.1601** 0.1306	γ Std.Err.	224.3351 56.31621	κ Std.Err.	0.4882 0.10874
2. Capitalist Spirit	**HJD** Std.Err.	**0.1842** 0.0592	γ Std.Err.	75.4428 45.9381	λ Std.Err.	32.8415 35.2209
3. Ferson-Constantinides	**HJD** Std.Err.	**0.2022** 0.084	γ Std.Err.	34.6821 14.4384	θ Std.Err.	0.41564 0.07379
4. Epstein-Zin-Weil	**HJD** Std.Err.	**0.2052** 0.0627	γ Std.Err.	22.6258 16.8028	υ Std.Err.	-28.5012 21.48384
5. Campbell-Cochrane	**HJD** Std.Err.	**0.2095** 0.0768	γ Std.Err.	0.65195 0.16293	ø	0.665
6. Abel w/o Kappa	**HJD** Std.Err.	**0.2265** 0.0749	γ Std.Err.	53.5871 19.1074		
7. Disappointment Aversion	**HJD** Std.Err.	**0.2364** 0.0682	γ Std.Err.	0.53889 45.3841	A Std.Err.	0.86583 0.44929
8. Standard CRRA	**HJD** Std.Err.	**0.2451** 0.0719	γ Std.Err.	4.73179 1.35356		

Notes: The asset vector includes 4 assets: a 90-day Treasury bill plus Deciles 1, 5, and 10 of the CRSP value-weighted portfolio. For each estimation, we set the subjective rate of discount = 0.9975. The reported HJD measure is obtained by choosing the unknown preference parameters in columns 4 and 6 to minimize the Hansen-Jagannathan specification error. A lag length of 6 is used in the computation of the Newey-West (1987a) covariance matrix. *The asymptotic standard errors are based on a non-zero null and are computed according to proposition 3.2 in Hansen et al. (1995).*

Table 3b Quarterly Unconditional Models (2)

1. Abel with Kappa	**HJD** Std.Err.	**0.0224** 0.0834	γ Std.Err.	188.8249 179.8058	κ Std.Err.	0.577592 0.405307
2. Abel w/o Kappa	**HJD** Std.Err.	**0.0879** 0.0619	γ Std.Err.	54.35711 15.66152		
3. Ferson-Constantinides	**HJD** Std.Err.	**0.0998** 0.0578	γ Std.Err.	14.32003 80.16932	θ Std.Err.	0.542681 0.635039
4. Campbell-Cochrane	**HJD** Std.Err.	**0.1046** 0.0583	γ Std.Err.	0.466876 0.211398	ø	0.6975
5. Capitalist Spirit	**HJD** Std.Err.	**0.1125** 0.0777	γ Std.Err.	40.65423 36.08729	λ Std.Err.	9.455126 18.87757
6. Epstein-Zin-Weil	**HJD** Std.Err.	**0.1205** 0.074	γ Std.Err.	8.601189 11.18571	υ Std.Err.	-9.733472 19.23452
7. Disappointment Aversion	**HJD** Std.Err.	**0.1263** 0.063	γ Std.Err.	0.412299 47.04733	A Std.Err.	0.948601 0.436807
8. Standard CRRA	**HJD** Std.Err.	**0.1288** 0.062	γ Std.Err.	3.05429 1.055481		

Notes: The asset vector includes 4 assets: a 90-day Treasury bill plus the CRSP value and equally-weighted indices. For each estimation, we set the subjective rate of discount = 0.9975. The reported HJdistance measure is obtained by choosing the unknown preference parameters in columns 4 and 6 to minimize the Hansen-Jagannathan specification error. A lag length of 6 is used in the computation of the Newey-West (1987a) covariance matrix. The asymptotic standard errors are based on a nonzero null and are computed according to proposition 3.2 in Hansen et al. (1995).

Table 3c Quarterly Conditional Models

1. Abel with Kappa	**HJD** Std.Err.	**0.121** 0.111	γ Std.Err.	209.0118 65.06607	κ Std.Err.	0.5517 0.1522
2. Capitalist Spirit	**HJD** Std.Err.	**0.1703** 0.1186	γ Std.Err.	142.835 49.82935	λ Std.Err.	70.394 22.9770
3. Ferson-Constantinides	**HJD** Std.Err.	**0.2212** 0.0634	γ Std.Err.	19.3689 13.43931	θ Std.Err.	0.5232 0.1023
4. Abel w/o Kappa	**HJD** Std.Err.	**0.2383** 0.0542	γ Std.Err.	75.75842 14.19688		
5. Campbell-Cochrane	**HJD** Std.Err.	**0.2605** 0.0544	γ Std.Err.	0.562262 0.230452	ø	0.627
6. Epstein-Zin-Weil	**HJD** Std.Err.	**0.272** 0.0537	γ Std.Err.	20.94219 13.37204	υ Std.Err.	-26.564 16.6816
7. Disappointment Aversion	**HJD** Std.Err.	**0.2852** 0.0504	γ Std.Err.	0.73909 27.52826	A Std.Err.	0.8771 0.3116
8. Standard CRRA	**HJD** Std.Err.	**0.2897** 0.052	γ Std.Err.	2.430167 1.655144		

Notes: The return vector includes 6 assets: the real return on the 90-day t-bill, the real return on Decile 5 of the CRSP value-weighted index, and real return on four managed portfolios in which (z) units are invested in the three-month Treasury bill and (1-z) units in the fifth Decile portfolio. For the third return, the portfolio weight z is the dividend yield on the CRSP equally-weighted stock index minus the dividend yield on the value-weighted index; for the fourth return, z is the three-month Treasury bill rate less the one-month Treasury bill rate; for the fifth return, z is the nominal one-month Treasury bill rate; and for the last return, z is the log consumption-aggregate wealth ratio. All instruments are lagged once and denominated in percentage terms. Everything else holds.

Table 3d Annual Unconditional Models

1. Epstein-Zin-Weil	**HJD** Std.Err.	**0.1238** 0.159	γ Std.Err.	7.076599 4.573401	υ Std.Err.	-27.82324 13.830894
2. Disappointment Aversion	**HJD** Std.Err.	**0.2489** 0.1018	γ Std.Err.	0.729529 11.605748	A Std.Err.	0.812006 0.361901
3. Campbell-Cochrane	**HJD** Std.Err.	**0.2569** 0.1255	γ Std.Err.	0.252988 0.45824	ϕ	0
4. Abel with Kappa	**HJD** Std.Err.	**0.2572** 0.1476	γ Std.Err.	34.48806 110.09586	κ Std.Err.	0.931074 0.450714
5. Capitalist Spirit	**HJD** Std.Err.	**0.2582** 0.1654	γ Std.Err.	30.88205 11.587589	λ Std.Err.	17.63946 18.307798
6. Abel w/o Kappa	**HJD** Std.Err.	**0.2586** 0.1331	γ Std.Err.	26.55075 29.79396		
7. Ferson-Constantinides	**HJD** Std.Err.	**0.2755** 0.1061	γ Std.Err.	1.026201 8.22	θ Std.Err.	-1.010961 334932.19
8. Standard CRRA	**HJD** Std.Err.	**0.2761** 0.1057	γ Std.Err.	0.479489 0.613184		

Notes: The asset vector includes 4 assets: a 90-day Treasury bill plus the CRSP value and equally-weighted indices. For each estimation, we set the subjective rate of discount = 0.9975. The reported HJdistance measure is obtained by choosing the unknown preference parameters in columns 4 and 6 to minimize the Hansen-Jagannathan specification error. A lag length of 4 is used in the computation of the Newey-West (1987a) covariance matrix. The asymptotic standard errors are based on a nonzero null and are computed according to proposition 3.2 in Hansen et al. (1995).

Table 3e Annual Conditional Models

1. Epstein-Zin-Weil	**HJD** Std.Err.	**0.2036** 0.1164	γ Std.Err.	7.58705 2.9562	υ Std.Err.	-27.7731 9.836425
2. Capitalist Spirit	**HJD** Std.Err.	**0.2608** 0.1578	γ Std.Err.	35.3477 8.9985	λ Std.Err.	25.02672 14.90737
3. Campbell-Cochrane	**HJD** Std.Err.	**0.2809** 0.1233	γ Std.Err.	0.24614 0.2083	ø	0
4. Abel with Kappa	**HJD** Std.Err.	**0.2843** 0.1263	γ Std.Err.	29.98027 778.476	κ Std.Err.	0.96615 4.267567
5. Abel w/o Kappa	**HJD** Std.Err.	**0.2846** 0.1209	γ Std.Err.	26.51274 10.1198		
6. Disappointment Aversion	**HJD** Std.Err.	**0.2903** 0.0942	γ Std.Err.	0.14274 9.9158	A Std.Err.	0.882201 0.221866
7. Ferson-Constantinides	**HJD** Std.Err.	**0.3028** 0.0983	γ Std.Err.	0.6533 0.3700	θ Std.Err.	-1.00627 26108.26
8. Standard CRRA	**HJD** Std.Err.	**0.3031** 0.0979	γ Std.Err.	0.14277 0.5459		

Notes: The asset vector includes 3 assets: a 30-day Treasury bill, the CRSP equally-weighted portfolio and a managed portfolio consisting of (1-z) units invested in the first asset and (z) units invested in the second, where (z) is the three month Treasury bill rate less the one-month Treasury bill rate. All instruments are lagged once and denominated in percentage terms. Everything else holds.

Another auxiliary result, which is of independent interest, is the interpretation of the values of our preference parameters which are chosen to minimize the HJ-distance measure. In the next subsection, we turn to a discussion of the sign and magnitude of these parameters.

Preference Parameters

Our empirical results in tables 3a to 3e give the warning that many of our parameter estimates are imprecise and have a relatively large standard error attached to them. This is in fact true of most models, and is confirmed by flat valleys in their objective functions.[28] However, it is also noteworthy to mention that given the shapes of these functions, we perform in most cases iterative rather than two-step GMM. We also experiment with different starting values. Hence, our minima are, to the best of our knowledge, *global*.

Assessing the models on the reasonableness of the estimates of their parameters undoubtedly requires subjective judgments of some sort. While such arguments

28. Figures of all the objective functions can be made available upon request.

are routinely made in the literature, we avoid going in this direction because what is relevant for the purpose in hand is our models' success in asset-pricing exercises *only*. Nonetheless, it is still progressively informative to see if any patterns emerge in our parameter estimates. Hence, we document the following:

(a) For the recursive specification in Epstein and Zin (1990) and Weil (1989), a negative $\hat{v}$ implies that preference for late rather than early resolution of uncertainty is more consistent with a minimum specification error. This is not supportive of the findings in Cochrane and Hansen (1992, figure 3.2) and Bakshi and Naka (1997, figure 4) who find the opposite result using the Hansen and Jagannathan volatility bounds test.

(b) For the standard time and state-separable model and for Abel's (1990) model with and without κ, more accurate asset-pricing requires a higher risk-aversion coefficient in quarterly (tables 3a to 3c) than in annual exercises (tables 3d and 3e).

(c) For the Campbell and Cochrane (1999) model, our estimate of ϕ deserves some explanation. First, as it is chosen over the unit interval, it bears no standard error. Second, its value in quarterly races is between 0.6 and 0.7, implying a fairly moderate habit persistence. This does not generalize to the annual sample where $\phi = 0$, not favoring the authors' formulation of habit.

(d) For the Ferson and Constantidines (1991) habit and durability model to be most empirically accurate, the sign of ϕ implies that habit persistence is consistently more relevant than durability with quarterly data. The converse is true of our annual sample in tables 3d and 3e. In these cases, durability is not only strong but also very imprecise.[29]

(e) For the capitalist-spirit hypothesis, where investors accumulate wealth for the double purpose of consumption and wealth-induced social status, the

29. The empirical literature is still ambiguous on the sign of θ. Using monthly data, Dunn and Singleton (1986), Eichenbaum, Hansen and Singleton (1988), and Gallant and Tauchen (1989) obtain a positive coefficient, suggesting durability in consumption. Conversely, Ferson and Constantidines (FC, 1991) estimate negative coefficients using quarterly and annual data. Like FC, Heaton (1993) finds evidence of habit persistence in his monthly sample.

transversality condition of the infinite horizon problem restricts relative risk aversion in wealth $(\gamma+\lambda)$ to be greater than equal to one. While we make sure the restriction is satisfied, an interesting result emerges: it turns out the model IMRS is better performing when the estimated sum of these two parameters is large enough. In tables 3a, 3c and 3e (where the model ranks a whopping second), $(\gamma+\lambda)$ranges between 60 and 213; conversely, where the model ranks fifth in tables 3b and 3d, this sum is barely 50.

(f) Finally, for the disappointment aversion model, the null hypothesis of first-order risk-neutrality (or A=1) cannot be rejected at any plausible level of significance. This implies that Gul's (year) model is at its best in pricing our assets when more weight is not necessarily attached to disappointing outcomes than to favorable ones.

Concluding Remarks

A model cannot assume superiority based on experimental evidence or the sense of modeling realism that this evidence invokes. Relative performance is appraised in terms of providing better imitations of real asset markets. In this paper, the particular question we wish to answer is motivated by the desire to rank a host of utility-based asset-pricing models based on their accuracy in pricing financial assets. The focus is on tightly parametrized behavioral structures with unknown preference parameters that need to be estimated. Primarily, we find that no single model vastly outperforms the rest. Nevertheless, we document a few patterns where there are clear benefits to using one model over another. We also find that some models perform better at quarterly than annual samples, and vice versa.

Finally, we can suggest a few avenues for future research. Some are more laudable than others, but not one is yet attained in this paper:

(a) Naturally, the validity of these results may depend on the data-set used. Future projects can check the generality of our findings by conducting the same study using international data. Even within the US, one can experiment with novel samples as in Ait-Sahalia, Parker and Yogo (2001) who, in an attempt to fit the first two moments of the equity-premium, make use of a number of data sets on the expenditures of luxury goods. Alternatively, one can apply the same procedure for either one of the vector of risky or risk-less assets. This will provide further insight into the relative usefulness of our models.

(b) An analysis based on markets with frictions can provide an important complementary piece of evidence to this work. Related examples abound in the literature. For example, He and Modest (1995), Luttmer (1996), and Palacios-Huerta (2002) all study the alternate framework in which the SDFs consent to market frictions. Hansen, Heaton and Luttmer (1995) additionally develop the econometric methodology for an asymptotic distribution theory for the HJ-distance measure that can accommodate both short-sale constraints and proportional transaction costs.

(c) The analysis in this paper reveals a number of implementation issues that relate to other applications of SDF models, like performance evaluation. Identifying those managers who possess and use investment information or skills superior to that of the general investing public has long been the enduring goal of *performance evaluation*. In this case, an asset-pricing model of 'normal' investment is asked to indicate investments with 'superior' portfolio returns. This model may be utility-based and may well take on any of the specifications adopted in this paper. Hence, future research should investigate the sensitivity of measures of investment performance of portfolio managers to using any of those models.

Appendix

Data Description

The variables included in our tests and their sources are explained below:

Consumption data, measured as expenditure on non-durables and services (excluding shoes and clothing), are seasonally adjusted at annual rates, in billions of chain-weighted 1996 dollars. The original source is the U.S. Department of Commerce, Bureau of Economic Analysis. Durable goods are excluded from the consumption variable because they represent replacements and additions to an investment or capital stock, rather than a service flow from the existing stock. They are, therefore, counted as part of nonhuman wealth, a component of the overall aggregate wealth. Including 'durables' in the consumption expenditure series is tantamount to ignoring the evolution of the asset over time. Such an evolving asset can be properly accounted for by multiplying the stock by a gross return that is usually less than one, since it consists primarily of depreciation (Lettau and Ludvingson, 2003).

After-tax **labor income** is measured as wages, plus salaries, plus transfer payments, plus other labor income, minus personal contributions for social insurance, minus taxes. Taxes here are defined as wages and salaries divided by (wages and salaries, plus proprietors' income, plus rental income, plus personal dividends, plus personal interest income). Proprietors' income is also with inventory valuation and capital consumption adjustments. The annual data is in current dollars, and the original source is the Bureau of Economic Analysis.

Dividing real total disposable income by real per capita disposable income creates a measure of the U.S. population. This converts total consumption expenditure and total after-tax labor income into per-capita terms. Again, the source to total and per-capita disposable income is the Bureau of Economic Analysis. **Total asset wealth** is household net worth in billions of current dollars (which includes both stock and non-stock market wealth). A time-convention is needed for this a-point-in-time stock variable. Following Lettau and Ludvingson (2001a), consumption data is taken to measure spending at the end of the quarter, so asset wealth for the quarter is measured at the end of the period. Stock market wealth is defined as direct household holdings, mutual fund holdings, holdings of private and public pension plans, personal trusts, and insurance companies. Non-stock market wealth is defined as tangible/real estate wealth, non-stock financial assets, and ownership of privately traded companies in non-corporate equity. Subtracted off are liabilities, including mortgage loans and loans made under home equity lines of credit and secured by junior liens, installment consumer debt and other. The original source is the Board of Governors of the Federal Reserve System.

The nominal after-tax labor income and wealth data are converted to real terms via the personal consumption expenditure chain-type deflator (1996=100), seasonally adjusted. The original source of the price-deflator series is the Bureau of Economic Analysis.

Even as the human capital component of the **consumption-aggregate wealth ratio** goes unobserved, Lettau and Ludvingson (2001a) argue that the important predictive components of consumption to wealth $\left(C_t/W_t\right)$ for future market returns may be expressed in terms of observable variables, namely in terms of consumption (C_t), asset holdings (A_t), and current after-tax labor income (Y_t).[30] The consumption-wealth ratio is defined as $C_t/W = C_t/\left(A_t^{\omega}Y_t^{1-\omega}\right)$. The weight of asset holdings in total wealth (***w***) is also the parameter of a common trend among consumption, asset wealth and after-tax labor income. The ***w*** parameter is unobserved and estimated via a standard OLS, using all observations from 1953:1 to 1998:3 for the quarterly sample and all 53 observations (1949-2001) for the annual sample. The resulting point estimate $\acute{\omega}_{OLS}$ of ***w*** is 0.3054 in the first case and 0.292 in the second. This corresponds to the quarterly and annual logarithms of the wealth to consumption ratio series: $c_t - 0.305a_t - 0.5981y_t$ and $c_t - 0.292a_t - 0.597y_t$.

Since C_t, A_t, and Y_t are cointegrated, $\acute{\omega}_{OLS}$ is *super-consistent* and robust to the presence of regressor endogeneity. This means that this estimate can be treated as if it is a true parameter, in effect making the resulting estimate of C_t/W_t an element of the time-*t* information set.

30. Aggregate wealth is human capital plus current asset holdings.

The source for the returns on **Treasury bills** and **value and equally-weighted portfolios** of common stocks is the Center of Research for Security Prices (CRSP), University of Chicago. The **decile portfolios** are based on the market value of equity outstanding at the beginning of each year. Decile 1 includes the smallest 10 percent stocks and decile 10 the largest ten percent. The nominal returns on all the asset-return data are converted into real returns using the same personal-consumption price-deflator.

References

Abel. 1990. Asset prices under habit formation and catching up with the joneses. *American Economic Review* 80:38–42.

Ait-Sahalia, Yacine, Jonathan A. Parker, and Motohiro Yogo. 2001. Luxury goods and the equity-premium. NBER Working Paper, 8417.

Ahn, Seung C. and Chris Gadarowski. 1999. Small sample properties of the model specification test based on the Hansen-Jagannathan distance measure. Working Paper, Arizona State University.

Ang,Andrew, Geert Beaert, and Jun Liu. 2002. Why stocks may disappoint. Working Paper, Columbia University and NBER.

Bakshi, Gurdip S. and Atsuyuki Naka. 1997. An empirical investigation of asset pricing models using Japanese stock market data. *Journal of International Money and Finance* 16:81–112.

Breen, William, Lawrence R. Glosten, and Ravi Jagannathan. 1989. Economic significance of predictable variations in stock index returns. *Journal of Finance* 44:1177–89.

Bonomo, Marco and Rene Garcia. 1994. Disappointment aversion as a solution to the equity premium and the risk-free rate puzzles. 94s-14. CIRANO, Scientific Series.

Campbell, John Y.. 1996. A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy* 104:572–621.

Campbell, John Y. and John H. Cochrane. 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107:205–52.

Campbell, John Y. and John H. Cochrane. 1995. By force of habit: a consumption-based explanation of aggregate stock market behavior. NBER Working Paper 4995.

Campbell, John Y. and John H. Cochrane. 2000. Explaining the poor performance of consumption-based asset pricing models. *Journal of Finance* LV:6:2865–78.

Chen, Xiaohong and Sydney C. Ludvigson. 2003. Land of addicts? an empirical investigation of habit-based asset pricing models. Working Paper, New York University and NBER.

Cochrane, John H. and Lars Peter Hansen. 1992. Asset pricing explorations for macroeconomics. NBER Working Paper W4088.

Cochrane, John H.. 1996.A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy* 104: 572–621.

Constantinides, George M.. 1990. Habit formation: A resolution of the equity-premium puzzle. *Journal of Political Economy* 98: 519–43.

Daniel, Kenneth and Daniel Marshall. 1997. Equity-premium and risk-free-rate puzzles at long horizons. *Macroeconomic Dynamics* 1:452–84.

Dunn, Kenneth B. and Kenneth J. Singleton. 1986. Modeling the term structure of interest rates under non-separable utility and durability of goods. *Journal of Financial Economics* 17: 27–55.

Eichenbaum, Martin S. Lars P. Hansen, and Kenneth J. Singleton. 1988. A time series analysis of representative agent models of consumption and leisure choices under uncertainty. *Quarterly Journal of Economics* 103: 51–78.

Epstein, L. G. and A. Melino. 1995. A revealed preference analysis of asset pricing under recursive utility. *Review of Economic Studies* 62: 597–618.

Epstein, Larry G. and Stanley E. Zin. 1989. Substitution, risk-aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica* 57: 937–69.

______________________________. 1990. First-order risk aversion and the equity- premium puzzle. *Journal of Monetary Economics* 26:387–407.

______________________________. 1991. Substitution, risk-aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. *Journal of Political Economy* 99:263–86.

Fama, Eugene F. and G. William Schwert. 1977. Human capital and capital market equilibrium. *Journal of Financial Economics*, Elsevier, vol. 4(1): 95–125.

Fama, Eugene F. and Kenneth R. French. 1989 Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics*.

____. 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*.

Farnsworth, Heber, Wayne E. Ferson, David Jackson, and Steven Todd. 2002. Performance evaluation with stochastic discount factors. NBER Working Paper 8791.

Ferson, Wayne and Andrew Siegel. 2002. Stochastic discount factor bounds with conditioning information. Forthcoming. *Review of Financial Studies*.

Ferson, Wayne E. and George M. Constantinides. 1991."Habit persistence and durability in aggregate consumption". *Journal of Financial Economics* 29:199–24.

Gali, Jordi. 1989. Keeping up with the Joneses: consumption externalities, portfolio choice, and asset prices. Columbia University, Graduate School of Business mimeo.

Gallant, Ronald A. and George Tauchen. 1989. Seminonparametric estimation of conditionally constrained heterogeneous processes: Asset pricing applications. *Econometrica* 57:1091–1120.

Gul, Faruk. 1991. A Theory of Disappointment Aversion. *Econometrica* 59: 667–686.

Hansen, Lars P, John Heaton, and Erzo Luttmer. 1995. Econometric evaluation of asset pricing models. *Review of Financial Studies* 8:237–74.

Hansen, Lars P. and Ravi Jagannathan. 1991. Implications of security market data for models of dynamic economies. *Journal of Political Economy* 99:225–62.

____. 1997. Assessing specification errors in stochastic discount factor models. *Journal of Finance* 52:557–90.

Hansen, Lars P. and Kenneth J. Singleton. 1983. Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Journal of Political Economy* 91:249–68.

He, Hua and David M. Modest. 1995. Market frictions and consumption-based asset pricing. *Journal of Political Economy* 103: 94–117.

Heaton, John C. 1993 The interaction between time-nonseparable preferences and time aggregation. *Econometrica* 61:353–85.

________________. 1995. An empirical investigation of asset pricing with temporally dependent preference specifications. *Econometrica* 63:681–717.

Hodrick, Robert J. and Xiaoyan Zhang. 2001. Evaluating the specification errors of asset pricing models. *Journal of Financial Economics* 62:327–76. [86] Ravi Jagannathan, Keiichi Kuota, and Hitoshi Takehara. 1998. Relationship between the labor-income risk and average return: Empirical evidence from the Japanese stock market. *The Journal of Business* 71: 319–47.

Jagannathan, Ravi and Zhenyu Wang. 1996. The conditional CAPM and the cross-section of expected returns. *Journal of Finance* 51:3–53.

Jagannathan, Ravi, keiichi Kuota, and Hitoshi Takehara. 1998. Relationship between the labor-income risk and average return: empirical evidence from the Japanese stock market. *Journal of Business* 71:319–47.

Kan, Raymond and Guofu Zhou. 2002. Hansen-Jagannathan distance: geometry and exact distribution. Working Paper, University of Toronto and Washington University of St. Louis.

Kreps, David M. and E. L. Porteus. 1978. Temporal resolution of uncertainty and dynamic choice theory. *Econometrica* 46:185–200.

Lettau, M. and S. C. Ludvigson. 2001.Consumption, aggregate wealth, and expected stock returns. *Journal of Finance* LVI: 815–48.

_________________________________. 2001. Resurrecting the (c)capm: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy* 109: 1238–87.

__________________________________. 2003. Understanding trend and cycle in asset values: Reevaluating the wealth effect on consumption. Forthcoming. *American Economic Review*.

Luttmer, E.1996. Asset prices in economies with frictions. *Econometrica* 64:1439–67.

Mehra, Rajnish and Edward Prescott. 1985.The equity premium: A puzzle. *Journal of Monetary Economics* 15:145–61.

Melino, Angelino and A. X. Yang. 2003. State dependent preferences can explain the equity premium puzzle. Working Paper, University of Toronto.

Newey, Whitney K. and Kenneth D. West 1987a. A simple positive semi-definite heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55: 703–08.

Ni, Shawn. 1997. Scaling factors in estimation of time-nonseparable utility functions. *The Review of Economics and Statistics* 79:234–40.

Palacios-Huerta, Ignacio. 2002. The determinants of the premium on human capital assets. Working Paper, Brown University.

Weil, Phillip. 1989. The equity-premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics* 24: 401–21.

Wheatley, Simon. 1988. Some tests of the consumption-based asset pricing model. *Journal of Monetary Economics* 22:193–215.

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