Inquiry Oriented Learning– An Eye on Differential Equations

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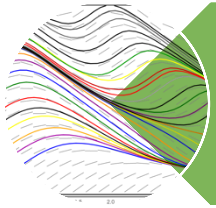
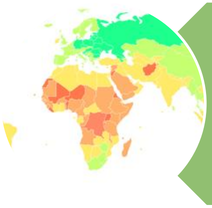
Outline



Differential Equations – Why?



Modeling Real Life Problems

Inquiry Oriented Differential Equations (IODE)

Hands-On Experiments

Why Differential Equations?

Differential equations are used in:

- Physics
- Chemistry
- Biology
- Economics
- Social Sciences

Differential Equations: An Applied Field

We can link Differential Equations to Environmental Concerns

- Modeling climate change
- Modeling the dynamics of a n-species fish population
- Modeling the Refugee Crisis in Lebanon or in other parts of the world, such as Europe

We can link Differential Equations to Health Concerns

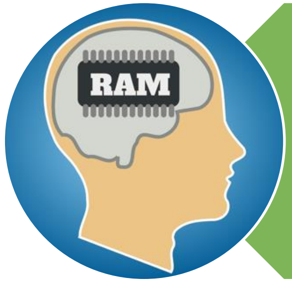
- Modeling the spread of Ebola in Congo and other parts of Africa
- Modeling the spread of COVID during the epidemic
- Any SIR model (S = Susceptible, I = Infected; R = Recovered)

Key Concept in Differential Equations: Modeling

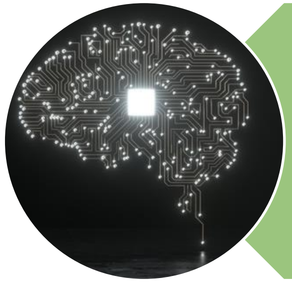
Modeling is the process of re-writing a real-life problem using mathematical terms and equations.

The derivative measures the rate at which a quantity increases or decreases.

Thinking about Differential Equations…



Memorize ready-made equation categories and solution processes?



Discover other methods of solutions?

A **combination** of both?

Modeling Example 1

Introductory and simple

Suppose P(t) represents a population of a certain town.

This population is a function of time: P(t).

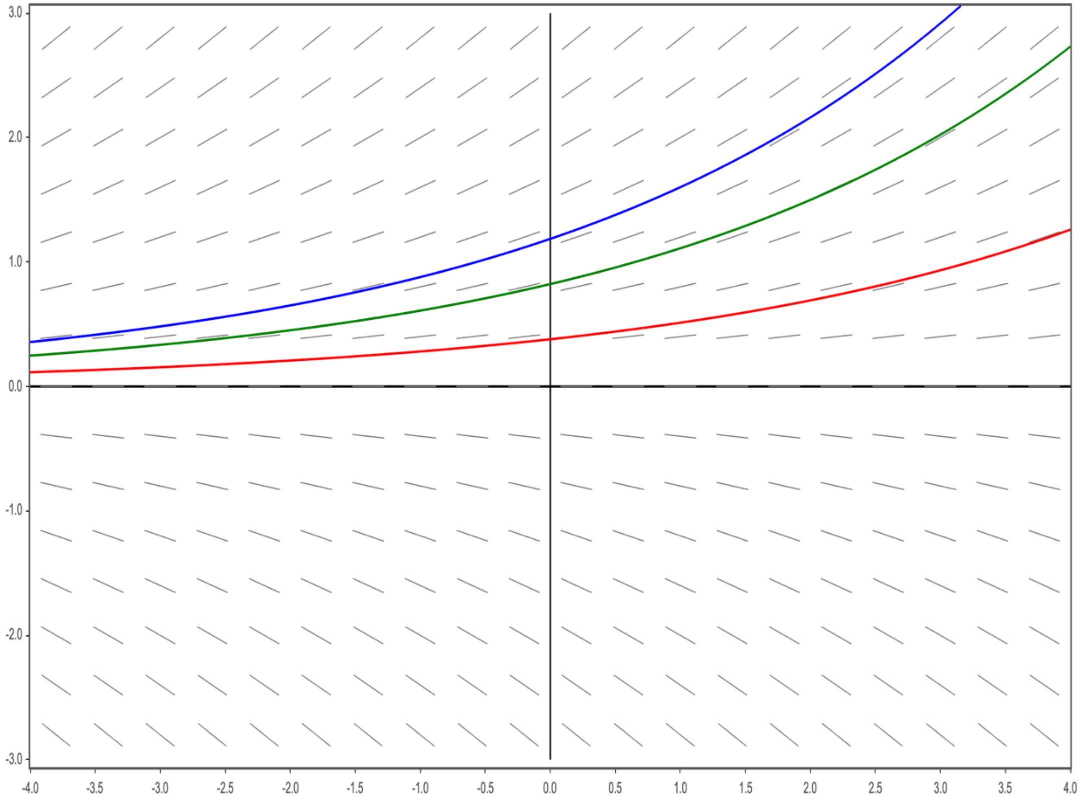
The rate of change is directly proportional to the actual population at time t.

Model????

Modeling Example 1

- The model can be as simple as: $\frac{dP}{dt} = kp(t)$, for some constant k

Population Increase Model

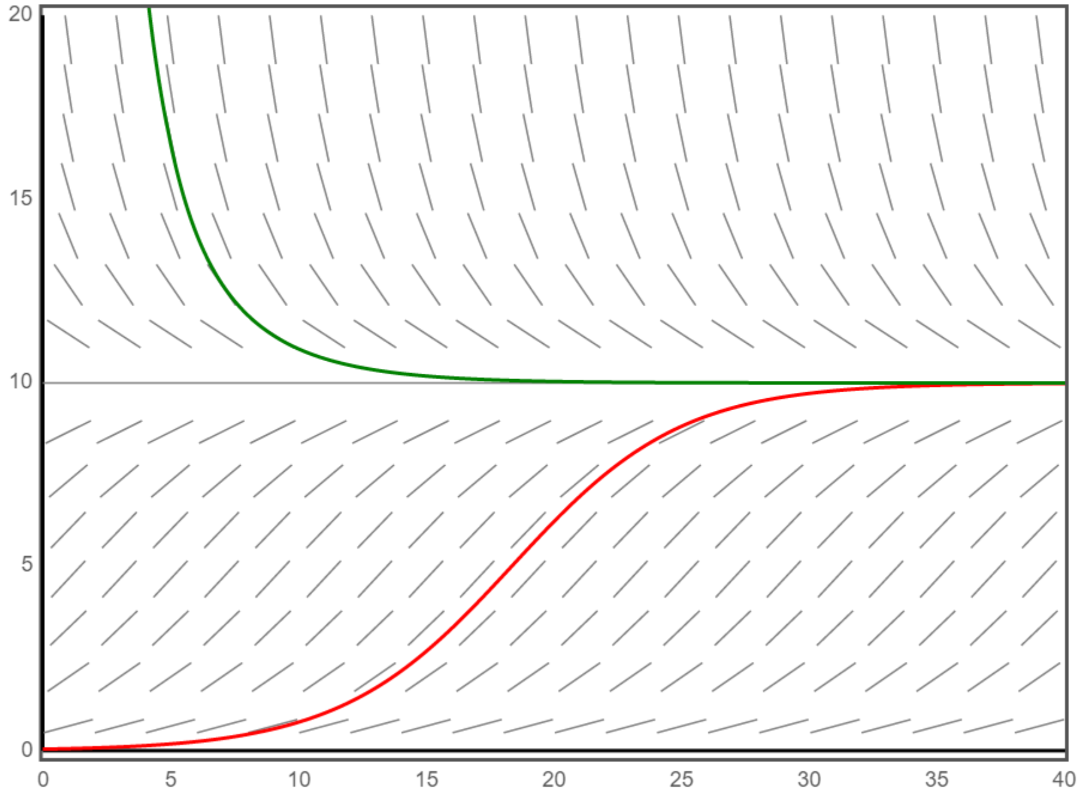


Direction Fields

- A qualitative approach for solving a first-order differential equation

How can we improve this model?

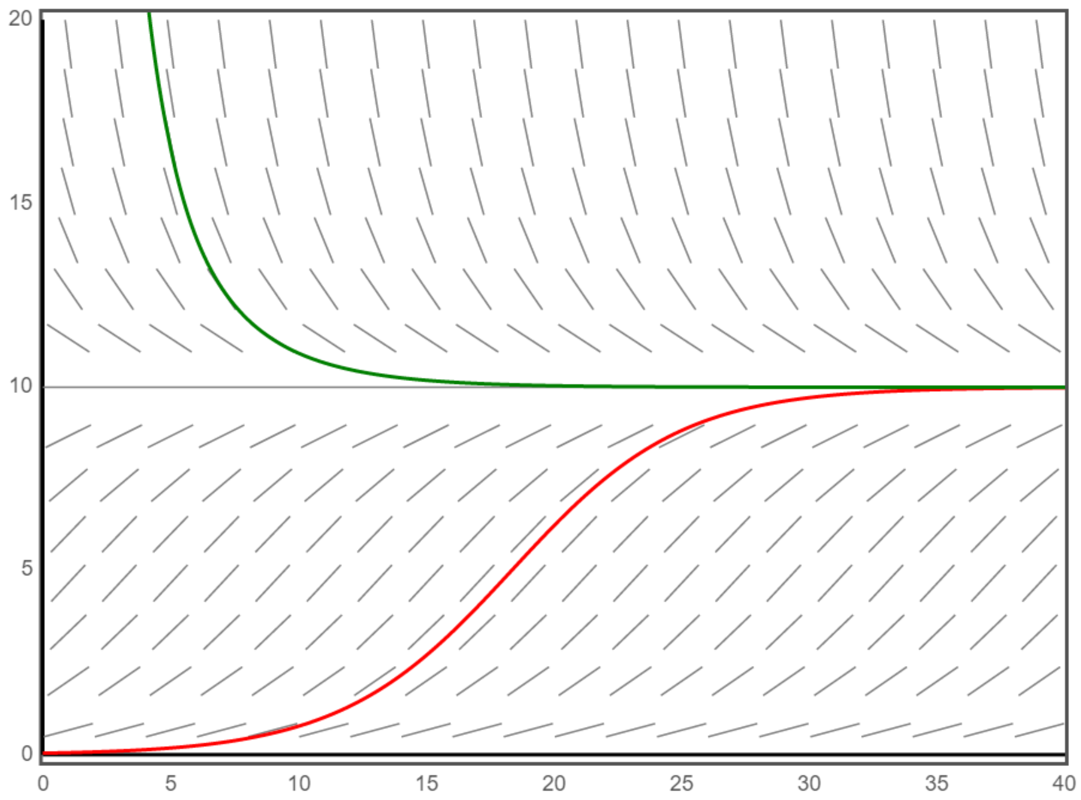
A New Model?



How can we improve this model?

A New Model?

- The population cannot increase with no limit; there has to be a threshold which controls the “growth” of the population.



Logistic Population Growth Model

The Population Growth Model

A Rate of Change Equation for Limited Resources

In a previous problem we saw that the rate of change equation $\frac{dP}{dt} = 0.3P$ can be used to model a situation where there is one species, continuous reproduction, and unlimited resources. In most situations, however, the resources are not unlimited, so to improve the model one has to modify the rate of change equation $\frac{dP}{dt} = 0.3P$ to account for the fact that resources are limited.

1. (a) In what ways does the modified rate of change equation

$$\frac{dP}{dt} = 0.3P\left(1 - \frac{P}{10}\right)$$

account for limited resources? (Think of 10 as scaled to mean 10,000 or 100,000)

(b) How do you interpret the solution with initial condition $P(0) = 10$?

(c) Open the Slope Field Viewer, https://ggbm.at/ZGeeGObp, and plot the slope field for

What has just happened?

Learning Approach

Started exploring examples

Discussed the meaning of the model (visually if possible)

Students **created** their own models

Realistic Mathematics Education

Foundations of RME

A mathematical activity should primarily be a human activity

An activity of discovery

It rejects the predominant idea of mathematics as a ready-made system to be presented and memorized by students

Realistic Mathematics Education

Three principles for RME Instructional Design

Reinvention through progressive mathematization

Didactical phenomenology

Bridging the gap between informal knowledge and formal mathematics

Modeling Example 2: Predator-Prey System

R(t) = Rabbit population at time t; J(t) = Jaguar population at time t

In the absence of Jaguars,

- Rabbit population increases

In the presence of Jaguars,

- Rabbit population is affected negatively

In the absence of rabbits,

- Jaguar population decreases

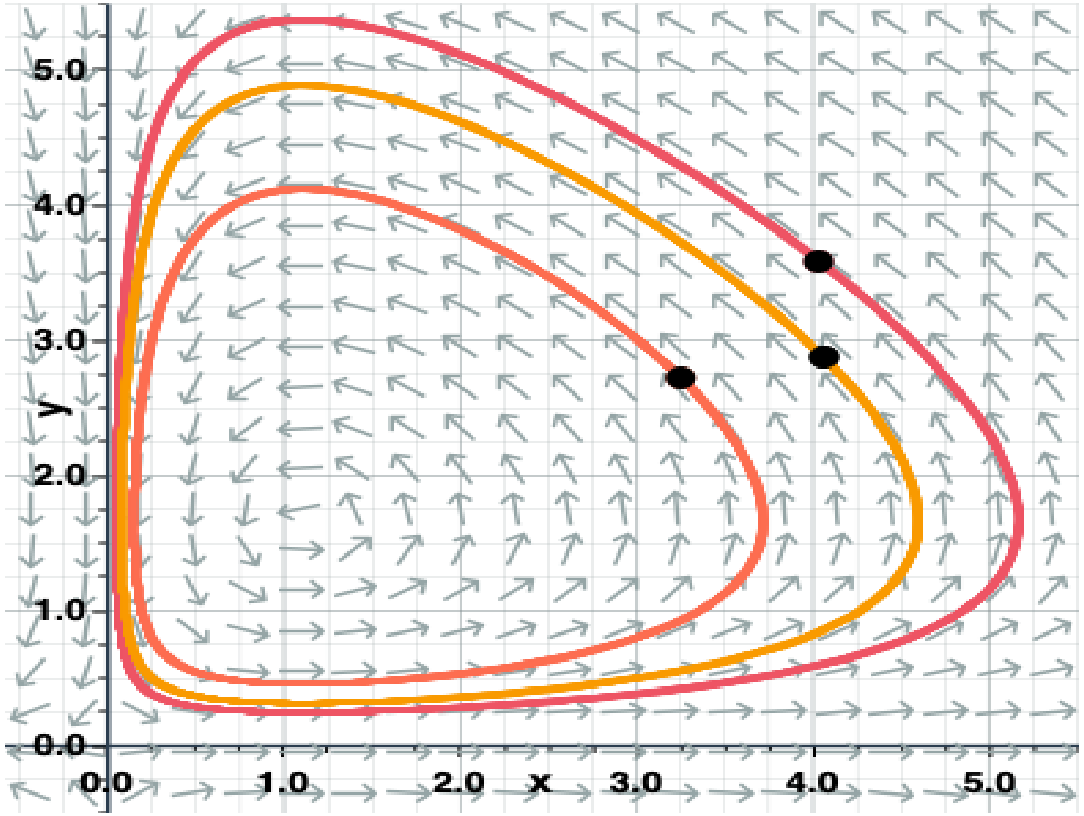
In the presence of rabbits,

- Jaguar population is affected positively

A Possible Predator-Prey Model

$$\begin{cases} \dfrac{dr}{dt} = 2r - 1.2rj \\ \dfrac{dj}{dt} = -j + 0.9rj \end{cases}$$

Predator-Prey Model:



Modeling Example 1

- The previous predator-prey model also says that in the absence of Jaguars, rabbit population increases exponentially, and that in the absence of rabbits, jaguar population decreases exponentially.
- Clearly this is not realistic.
- Gow can you modify the model for a more realistic model?

Other Modeling Examples of Species Interactions

- Consider the following two models:

$$\text{i)} \begin{cases} \dfrac{dx}{dt} = -5x + 2xy \\ \dfrac{dy}{dt} = -4y + 3xy \end{cases} \qquad \text{(ii)} \begin{cases} \dfrac{dx}{dt} = 4x - 2xy \\ \dfrac{dy}{dt} = 2y - xy \end{cases}$$

- What do they represent? Can you give them names? (the one we explored earlier was a predator-prey model?

A Fun Model: Romeo and Juliet

Adapted from Professor Steven Strogatz of Cornell University

- Suppose $r(t)$ measures Romeo's feeling for Juliet, and $j(t)$ Juliet's feeling to Romeo.
- Suppose we have the following system that models the relationship between Romeo and Juliet:

$$\begin{cases} \frac{dr}{dt} = j \\ \frac{dj}{dt} = -r \end{cases}$$

- Which of the following statements are true?
  - The more Romeo loves Juliet, the more strongly Juliet feels for him.
  - The more Juliet loves Romeo, the more strongly Romeo falls for her.
  - Juliet finds Romeo most attractive when he loves her least. When he loves her most, she turns away

- One of the following is a general solution to the system, which one? Assume C_1 and C_1 are both positive.
  - $j(t) = -C_1 e^{-t} + C_2 e^t$; $r(t) = C_1 e^{-t} + C_2 e^t$
  - $j(t) = -C_1 Sin(t) + C_2 Cos(t)$; $r(t) = C_1 Sin(t) + C_2 Cos(t)$
  - $j(t) = C_1 e^{-t} + C_2 e^t$; $r(t) = -C_1 e^{-t} + C_2 e^t$

Inquiry-Oriented Differential Equations

IODE - RME Based Curriculum

IODE is developed by a team of four researchers

Project supported by an NSF grant

Inquiry-Oriented Differential Equations

IODE – RME Based Curriculum

- **Four main instructional principles**
  - Generating student ways of reasoning
  - Building on student contributions
  - Developing a shared understanding
  - Connecting to standard mathematical language

Inquiry-Oriented Differential Equations

The Curriculum

Curriculum does not adopt a textbook

Teacher and student material are in the form of Units (14 in all) developed by the IODE team.

Units designed in accordance with the instructional design theory of Realistic Mathematics Education:

- Intended primarily to foster student reinvention of mathematical concepts.
- Sequence of tasks in each unit builds on student concepts of reasoning as the starting point
- More complex and formal reasoning develops, hence guiding students’ reinvention process.

The IODE website provides all the resources necessary for teaching the course, be it through the units or through the technological resources.

Inquiry-Oriented Differential Equations

A Typical Classroom Environment

- Class divided into groups
- Whiteboards, markers and erasers distributed to each
- Learning is based on the four principles of the IODE
- For every unit taught, students are provided with the Unit's Student Material, and they are asked to be engaged in their own groups on specific tasks.
- During this time, the teacher walks around exploring students' strategies, sometimes answering questions.
- This is followed by an all-class discussion.
- Sometimes, groups are asked to present their work.
- Finally, the teacher introduces language and notation when appropriate to formalize students' ideas and contributions.

LAU Experiment

- Experimental sections – 25 students
- Material covered is similar to a traditional course
- Students divided into 7 groups
- At the completion of every unit, personal notes on the learning process were recorded for my own reflections.
- Many times, students’ work on whiteboards were also photographed for future reference.
- Four online questionnaires were posted on Blackboard

An Example from the Experiemental Section at LAU

Analyzing Autonomous DEs: Spotted Owls

A group of biologists are making predictions about the spotted owl population in a forest in the Pacific Northwest. The autonomous differential equation the scientist use to model the spotted owl population is $\frac{dP}{dt} = \frac{P}{2}\left(1 - \frac{P}{5}\right)\left(\frac{P}{8} - 1\right)$, where P is in hundreds of owls and t is in years. The problem is that the current number of owls is only approximately known.

1. Suppose the scientists estimate that currently P is about 5 (i.e. there are currently about 500 owls in the forest). Since 5 is only an estimate, they make long-term predictions of the owl population for the initial conditions $P = 4.9$, $P = 5.0$, and $P = 5.1$. *Without using a graphing calculator or other software*, determine the long-term predictions for these initial conditions based on the differential equation. Are they similar or different? That is, will slightly different initial conditions yield only slightly different long-term predictions, or will they be radically different? Carry out a similar analysis if the current number of owls is somewhere around 8.

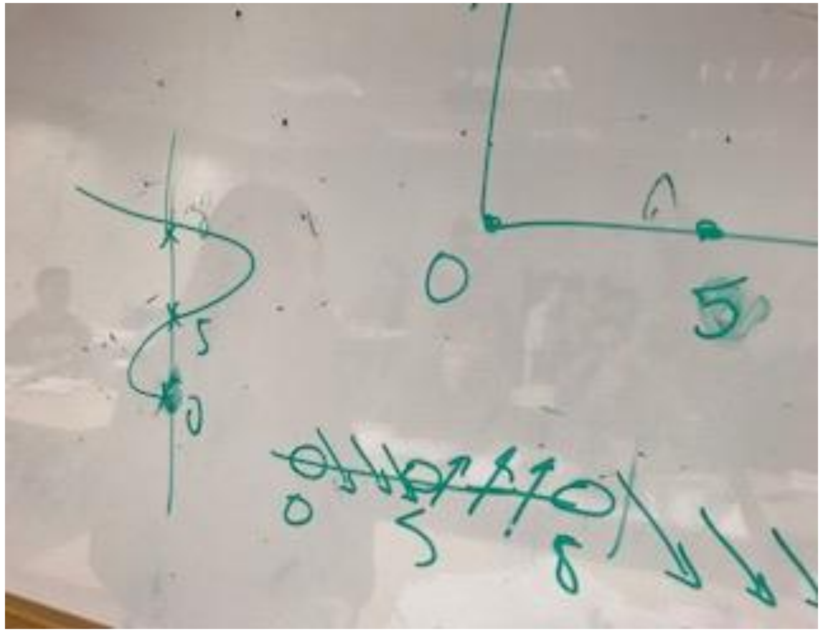
An Example from the Experiemental Section at LAU

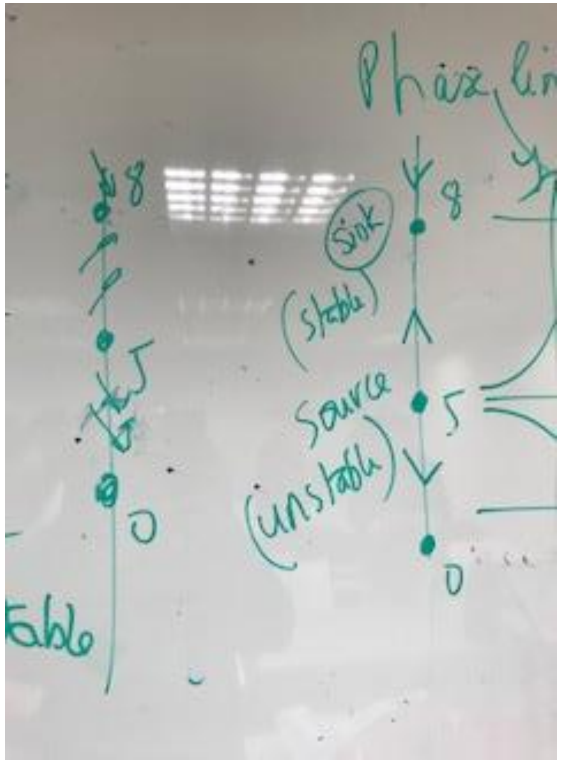
2. Give a one dimensional representation, *without words*, that would describe all solutions to the differential equation.

LAU Experiment

Some results

- What some LAU students came up with:





An Example from the Experiemental Section at LAU

3. A **phase line** is the standard one-dimensional diagram that depicts the qualitative behavior of solutions to an autonomous differential equation. Label the dots and add arrows to the figure below to represent **all** solutions to the differential equation in Problem 1.

4. For the differential equation in problems 1-3 there are three equilibrium solutions. Recall that equilibrium solutions are constant functions that satisfy the differential equation. How do the other solution functions near each equilibrium solution behave in the long term? If you were to label each of these equilibrium solutions based on the way in which nearby solutions behave, what terms would you use and why?

The Challenge

Facing the challenge

A challenge facing undergraduate mathematics education

The Inquiry Oriented Approach

Facing the Challenge

There is a need for teachers to experience innovative ***learning*** (rather than teaching) techniques.

This has created opportunities for mathematicians and undergraduate math education researchers to collaborate together and to explore innovative approaches to teaching and learning.

The Inquiry Oriented Approach

Facing the Challenge

Many undergraduate mathematics instructors are increasingly aware of the value of active learning such as inquiry-based instruction.

In my experience, the guided journey of learning in the IODE class benefited the students in many aspects.

Relevant Publications by Author

- *Inquiry-Oriented Differential Equations; a Guided Journey of Learning*; Teaching Mathematics and its Applications, Vol. 39, 2020, 201-212.

- *Improving Understanding in Ordinary Differential Equations through Writing in a Dynamical Environment*; Teaching Mathematics and its Applications, Oxford University Press, Vol. 31(3), 2012, pp. 153-166.

- *Investigating Students' Approval of a Geometric Approach to Differential Equations and Their Solutions;* International Journal of Mathematical Education in Science and Technology (iJMEST), a Taylor and Francis publication, Vol. 34(5), 2003, pp. 651-662.

- *Exploring Student's Strategies to Solve Ordinary Differential Equations in a Reformed Setting*; Journal of Mathematical Behavior, Vol. 18(4), 2000, pp. 455 – 472.

Technological Resources

For plotting slope fields of first order differential equations, and phase planes for systems of equations.

- https://www.geogebra.org/m/ZGeeGQbp

- The Slopes Application (available in Apple Store and for Android phones)

- https://scholarship.claremont.edu/codee/resources.html

Thank You